Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.3-Tangent/98-4.3.0-a-trg-^m-b-tan-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [387]. This is test number [98].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (387)	0.00 (0)
Mathematica	99.74 (386)	0.26 (1)
Maple	68.99 (267)	31.01 (120)
Fricas	53.23 (206)	46.77 (181)
Maxima	35.40 (137)	64.60 (250)
Mupad	31.52 (122)	68.48 (265)
Giac	21.71 (84)	78.29 (303)
Sympy	4.65 (18)	95.35 (369)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

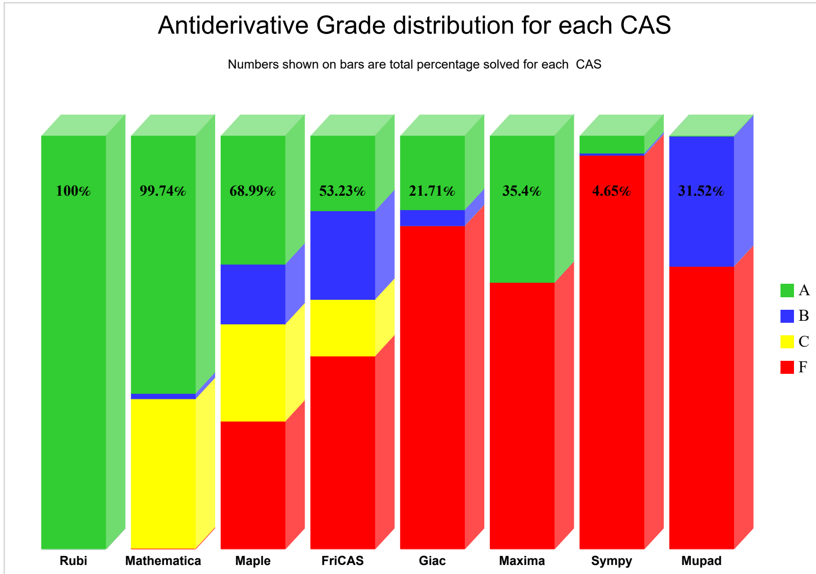
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

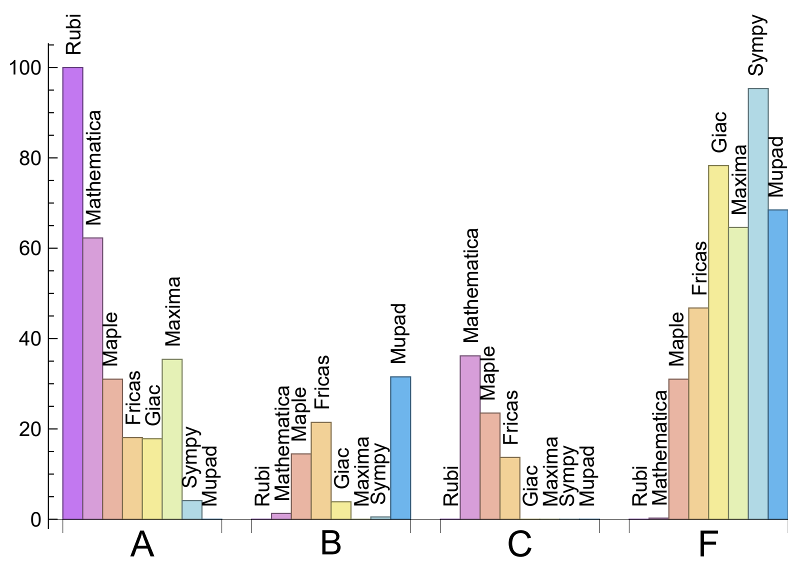
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	62.27	1.29	36.18	0.26
Maxima	35.40	0.00	0.00	64.60
Maple	31.01	14.47	23.51	31.01
Fricas	18.09	21.45	13.70	46.77
Giac	17.83	3.88	0.00	78.29
Sympy	4.13	0.52	0.00	95.35
Mupad	N/A	31.52	0.00	68.48

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	120	100.00 %	0.00 %	0.00 %
Fricas	181	77.35 %	3.31 %	19.34 %
Giac	303	88.45 %	1.65 %	9.90 %
Maxima	250	100.00 %	0.00 %	0.00 %
Sympy	369	70.46 %	17.89 %	11.65 %
Mupad	265	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

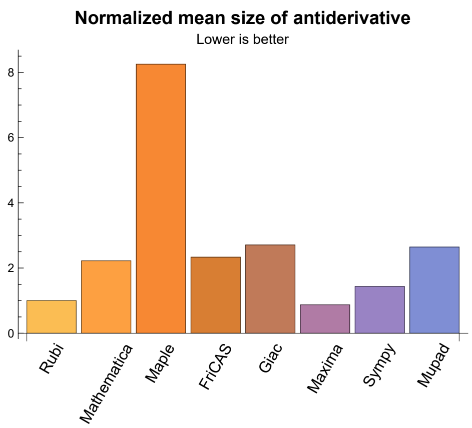
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

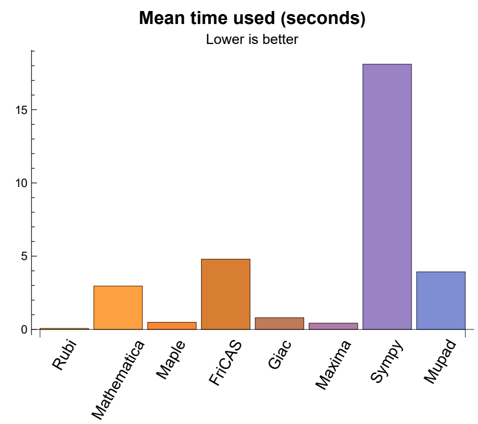
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	103.99	1.00	78.00	1.00
Mathematica	2.96	154.33	2.22	69.00	0.93
Maple	0.48	589.01	8.25	220.00	2.07
Maxima	0.42	117.61	0.87	133.00	0.88
Fricas	4.79	357.63	2.33	112.50	1.57
Sympy	18.11	50.72	1.43	52.00	1.29
Giac	0.80	209.15	2.71	183.00	1.03
Mupad	3.93	146.28	2.64	77.50	1.10

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
5
10
15
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {174, 175, 176, 180, 181, 183, 184, 185, 355, 356, 358, 359, 360, 367, 368, 372, 373, 387}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

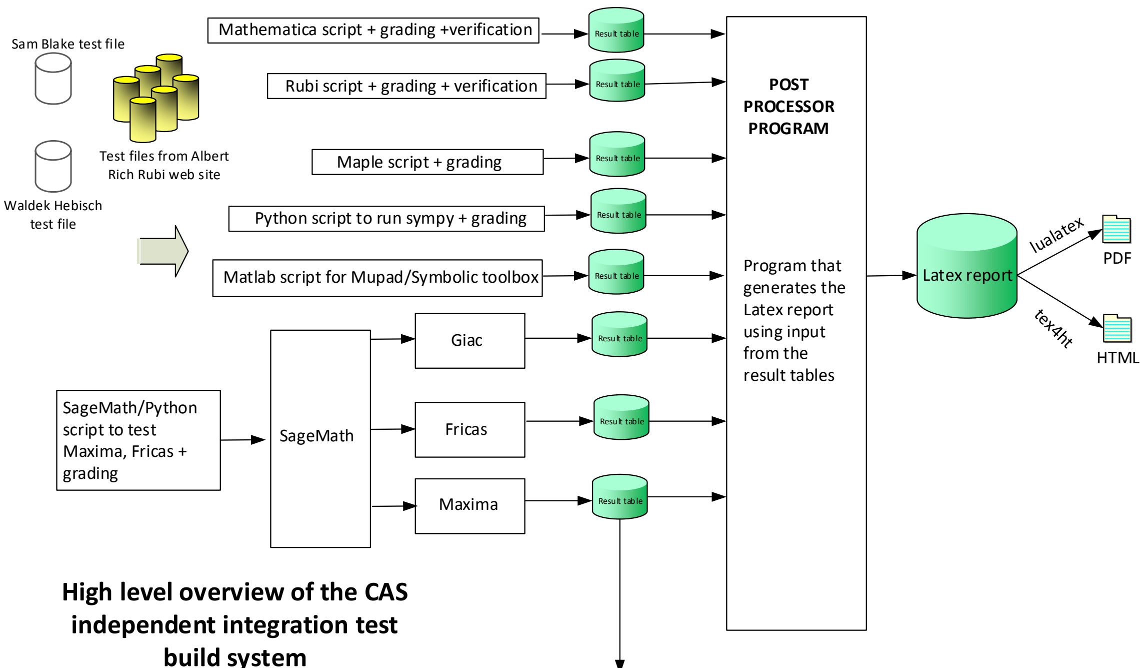
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 20, 23, 24, 25, 26, 27, 28, 29, 30, 32, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 114, 115, 116, 117, 118, 119, 121, 123, 125, 127, 129, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 178, 179, 182, 186, 187, 188, 189, 190, 194, 196, 198, 202, 204, 206, 210, 212, 217, 219, 221, 222, 223, 224, 225, 226, 227, 228, 230, 236, 237, 238, 239, 240, 247, 248, 249, 251, 252, 258, 259, 260, 262, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 295, 297, 300, 302, 306, 307, 309, 311, 313, 315, 317, 319, 321, 322, 324, 326, 328, 330, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 357, 362, 363, 364, 365, 366, 369, 370, 371, 374, 375, 376, 377, 378, 380, 382, 383, 384, 385, 386 }

B grade: { 173, 304, 354, 379, 381 }

C grade: { 10, 12, 14, 15, 16, 17, 18, 19, 21, 22, 31, 33, 34, 35, 39, 40, 41, 59, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 120, 122, 124, 126, 128, 130, 132, 174, 175, 176, 180, 181, 183, 184, 185, 191, 192, 193, 195, 197, 199, 200, 201, 203, 205, 207, 208, 209, 211, 213, 214, 215, 216, 218, 220, 229, 231, 232, 233, 234, 235, 241, 242, 243, 244, 245, 246, 250, 253, 254, 255, 256, 257, 261, 263, 264, 265, 266, 267, 268, 269, 270, 292, 294, 296, 298, 299, 301, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333, 355, 356, 358, 359, 360, 367, 368, 372, 373, 387 }

F grade: { 361 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 57, 58, 59, 67, 68, 76, 77, 78, 79, 87, 88, 97, 98, 99, 106, 107, 125, 127, 129, 131, 134, 135, 136, 137, 163, 195, 196, 197, 198, 204, 205, 206, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 236, 237, 238, 239, 241, 242, 243, 245, 246, 247, 248, 249, 250, 253, 257, 258, 259, 260, 261, 295, 297, 304, 306, 313, 319, 321, 324, 326, 330, 332, 334, 353, 365, 376 }

B grade: { 56, 60, 61, 62, 63, 66, 69, 70, 71, 72, 75, 80, 81, 82, 83, 86, 89, 90, 91, 92, 93, 96, 100, 101, 102, 105, 108, 109, 110, 111, 112, 113, 114, 116, 118, 121, 123, 133, 138, 231, 232, 233, 234, 235, 244, 254, 255, 256, 263, 264, 265, 266, 267, 268, 269, 270 }

C grade: { 52, 54, 55, 64, 65, 73, 74, 84, 85, 94, 95, 103, 104, 115, 117, 119, 120, 122, 124, 126, 128, 130, 132, 139, 140, 141, 142, 143, 144, 164, 165, 177, 178, 179, 191, 192, 193, 194, 199, 200, 201, 202, 203, 207, 208, 209, 214, 215, 230, 240, 251, 252, 262, 291, 292, 293, 294, 296, 298, 299, 300, 301, 302, 303, 305, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 320, 322, 323, 325, 327, 328, 329, 331, 333, 351, 352, 363, 364, 377, 378 }

F grade: { 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 163, 164, 165, 177, 178, 179, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 230, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 258, 259, 260, 261, 262, 351, 352, 353, 363, 364, 365, 376, 377, 378 }

B grade: { }

C grade: { }

F grade: { 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 59, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 231, 232, 233, 234, 235, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 19, 20, 24, 25, 26, 27, 28, 29, 36, 37, 38, 39, 40, 41, 52, 57, 58, 66, 67, 68, 76, 77, 87, 88, 114, 116, 121, 123, 127, 134, 135, 163, 164, 165, 177, 178, 226, 227, 236, 237, 247, 248, 258, 259, 295, 297, 306, 319, 321, 324, 326, 332, 334, 351, 352, 353, 363, 364, 365, 376, 377, 378 }

B grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 22, 54, 55, 56, 64, 65, 73, 74, 75, 84, 85, 86, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 118, 125, 129, 131, 133, 136, 137, 138, 179, 191, 192, 193, 194, 199, 200, 201, 202, 203, 207, 208, 209, 214, 215, 228, 229, 230, 238, 239, 240, 249, 250, 251, 252, 260, 261, 262, 291, 293, 300, 302, 304, 307, 309, 311, 313, 315, 317, 322, 328, 330 }

C grade: { 61, 62, 63, 80, 81, 82, 83, 101, 102, 115, 117, 119, 120, 122, 124, 126, 128, 130, 132, 139, 140, 141, 142, 143, 144, 241, 242, 243, 253, 254, 255, 269, 292, 294, 296, 298, 299, 301, 303, 305, 308, 310, 312, 314, 316, 318, 320, 323, 325, 327, 329, 331, 333 }

F grade: { 23, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 59, 60, 69, 70, 71, 72, 78, 79, 89, 90, 91, 92, 93, 99, 100, 108, 109, 110, 111, 112, 113, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 196, 197, 198, 204, 205, 206, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 231, 232, 233, 234, 235, 244, 245, 246, 256, 257, 263, 264, 265, 266, 267, 268, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 295, 304, 319, 321, 324, 326, 330, 332 }

B grade: { 353, 376 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 322, 323, 325, 327, 328, 329, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

2.1.7 Giac

A grade: { 1, 12, 13, 17, 18, 19, 20, 22, 26, 30, 31, 32, 33, 34, 35, 39, 40, 41, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 163, 226, 227, 228, 229, 230, 236, 237, 238, 240, 247, 248, 249, 250, 251, 252, 258, 259, 260, 262, 376 }

B grade: { 2, 3, 4, 5, 6, 7, 8, 24, 25, 27, 28, 29, 36, 37, 38 }

C grade: { }

F grade: { 9, 10, 11, 14, 15, 16, 21, 23, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 59, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 82, 83, 89, 90, 91, 92, 93, 99, 100, 101, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 231, 232, 233, 234, 235, 239, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 27, 56, 57, 58, 66, 67, 68, 75, 76, 77, 86, 87, 88, 96, 97, 98, 105, 106, 107, 114, 116, 121, 123, 127, 129, 134, 135, 136, 163, 164, 165, 177, 178, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 236, 237, 238, 239, 247, 248, 249, 250, 258, 259, 260, 261, 295, 297, 304, 306, 313, 319, 321, 324, 326, 330, 332, 334, 351, 352, 353, 364, 365, 376, 377, 378 }

C grade: { }

F grade: { 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 113, 115, 117, 118, 119, 120, 122, 124, 125, 126, 128, 130, 131, 132, 133, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 222, 223, 224, 225, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 246, 251, 252, 253, 254, 255, 256, 257, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 298, 299, 300, 301, 302, 303, 305, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 320, 322, 323, 325, 327, 328, 329, 331, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 382, 383, 384, 385, 386, 387 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	11	18	19	13	16
N.S.	1	1.00	1.00	1.42	0.92	1.50	1.58	1.08	1.33
time (sec)	N/A	0.003	0.009	0.029	0.273	0.409	0.043	0.422	2.570

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	23	21	18	17	15	226	14
N.S.	1	1.00	1.64	1.50	1.29	1.21	1.07	16.14	1.00
time (sec)	N/A	0.005	0.009	0.013	0.488	0.376	0.052	0.645	2.506

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	29	31	27	32	246	30
N.S.	1	1.00	0.93	1.07	1.15	1.00	1.19	9.11	1.11
time (sec)	N/A	0.009	0.028	0.025	0.269	0.367	0.069	0.680	2.505

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	38	31	29	26	27	585	24
N.S.	1	1.00	1.36	1.11	1.04	0.93	0.96	20.89	0.86
time (sec)	N/A	0.010	0.012	0.015	0.491	0.356	0.078	0.964	2.503

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	39	54	39	44	512	38
N.S.	1	1.00	0.86	0.91	1.26	0.91	1.02	11.91	0.88
time (sec)	N/A	0.014	0.054	0.031	0.266	0.369	0.100	1.502	2.503

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	53	41	41	38	39	989	35
N.S.	1	1.00	1.20	0.93	0.93	0.86	0.89	22.48	0.80
time (sec)	N/A	0.020	0.019	0.020	0.489	0.369	0.126	2.261	2.532

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	47	49	74	51	56	810	49
N.S.	1	1.00	0.82	0.86	1.30	0.89	0.98	14.21	0.86
time (sec)	N/A	0.020	0.113	0.040	0.271	0.359	0.157	3.253	2.493

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	68	51	51	48	51	1441	44
N.S.	1	1.00	1.17	0.88	0.88	0.83	0.88	24.84	0.76
time (sec)	N/A	0.025	0.015	0.030	0.486	0.375	0.206	5.132	2.521

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	175	169	186	600	0	0	93
N.S.	1	1.00	0.75	0.73	0.80	2.59	0.00	0.00	0.40
time (sec)	N/A	0.144	0.425	0.161	0.493	0.390	0.000	0.000	3.177

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	40	154	176	594	0	0	74
N.S.	1	1.00	0.19	0.73	0.83	2.80	0.00	0.00	0.35
time (sec)	N/A	0.106	0.084	0.064	0.499	0.380	0.000	0.000	2.791

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	159	149	170	533	0	0	73
N.S.	1	1.00	0.76	0.71	0.81	2.54	0.00	0.00	0.35
time (sec)	N/A	0.104	0.173	0.048	0.501	0.363	0.000	0.000	2.738

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	40	136	153	519	0	176	49
N.S.	1	1.00	0.21	0.71	0.80	2.70	0.00	0.92	0.26
time (sec)	N/A	0.087	0.045	0.086	0.501	0.391	0.000	0.465	2.625

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	131	138	155	493	0	184	59
N.S.	1	1.00	0.68	0.72	0.81	2.57	0.00	0.96	0.31
time (sec)	N/A	0.084	0.104	0.082	0.491	0.406	0.000	0.475	2.724

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	38	157	167	652	0	0	76
N.S.	1	1.00	0.18	0.74	0.79	3.08	0.00	0.00	0.36
time (sec)	N/A	0.102	0.069	0.052	0.515	0.379	0.000	0.000	2.728

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	40	157	168	653	0	0	75
N.S.	1	1.00	0.19	0.73	0.79	3.05	0.00	0.00	0.35
time (sec)	N/A	0.104	0.081	0.053	0.494	0.388	0.000	0.000	3.068

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	40	171	195	751	0	0	92
N.S.	1	1.00	0.17	0.73	0.83	3.21	0.00	0.00	0.39
time (sec)	N/A	0.123	0.114	0.054	0.499	0.393	0.000	0.000	3.115

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	38	216	185	588	0	209	247
N.S.	1	1.00	0.16	0.89	0.76	2.42	0.00	0.86	1.02
time (sec)	N/A	0.249	0.029	0.174	0.498	0.440	0.000	0.524	3.076

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	40	191	168	583	0	206	259
N.S.	1	1.00	0.18	0.85	0.75	2.60	0.00	0.92	1.16
time (sec)	N/A	0.282	0.053	0.109	0.496	0.412	0.000	0.468	2.950

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	40	108	98	124	0	127	146
N.S.	1	1.00	0.31	0.82	0.75	0.95	0.00	0.97	1.11
time (sec)	N/A	0.077	0.047	0.083	0.494	0.388	0.000	0.461	2.632

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	100	108	99	299	0	125	128
N.S.	1	1.00	0.76	0.82	0.76	2.28	0.00	0.95	0.98
time (sec)	N/A	0.071	0.156	0.059	0.498	0.353	0.000	0.454	2.732

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	38	203	170	548	0	0	230
N.S.	1	1.00	0.17	0.91	0.76	2.45	0.00	0.00	1.03
time (sec)	N/A	0.222	0.032	0.101	0.497	0.369	0.000	0.000	2.703

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	38	212	182	701	0	227	278
N.S.	1	1.00	0.16	0.87	0.74	2.86	0.00	0.93	1.13
time (sec)	N/A	0.300	0.069	0.074	0.494	0.414	0.000	0.571	2.548

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.050	0.158	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	56	58	47	74	0	696	-1
N.S.	1	1.00	0.57	0.59	0.48	0.76	0.00	7.10	-0.01
time (sec)	N/A	0.028	0.400	0.109	0.506	0.353	0.000	1.161	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	48	34	52	0	256	-1
N.S.	1	1.00	0.77	0.79	0.56	0.85	0.00	4.20	-0.02
time (sec)	N/A	0.019	0.120	0.083	0.498	0.365	0.000	0.739	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	19	38	0	23	-1
N.S.	1	1.00	1.00	1.16	0.59	1.19	0.00	0.72	-0.03
time (sec)	N/A	0.012	0.046	0.076	0.516	0.353	0.000	0.451	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	47	33	50	0	81	34
N.S.	1	1.00	1.26	1.52	1.06	1.61	0.00	2.61	1.10
time (sec)	N/A	0.012	0.095	0.089	0.501	0.360	0.000	0.452	2.446

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	56	64	46	69	0	176	-1
N.S.	1	1.00	0.85	0.97	0.70	1.05	0.00	2.67	-0.02
time (sec)	N/A	0.020	0.407	0.096	0.494	0.348	0.000	0.519	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	68	74	66	82	0	235	-1
N.S.	1	1.00	0.70	0.76	0.68	0.85	0.00	2.42	-0.01
time (sec)	N/A	0.031	0.290	0.096	0.503	0.350	0.000	0.557	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	199	266	178	0	0	291	-1
N.S.	1	1.00	0.55	0.73	0.49	0.00	0.00	0.80	-0.00
time (sec)	N/A	0.108	0.811	0.089	0.512	0.000	0.000	0.604	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	54	236	140	0	0	253	-1
N.S.	1	1.00	0.19	0.83	0.49	0.00	0.00	0.88	-0.00
time (sec)	N/A	0.093	0.071	0.063	0.515	0.000	0.000	0.527	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	161	208	133	0	0	195	-1
N.S.	1	1.00	0.63	0.82	0.52	0.00	0.00	0.76	-0.00
time (sec)	N/A	0.086	0.260	0.075	0.499	0.000	0.000	0.478	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	43	211	126	0	0	251	-1
N.S.	1	1.00	0.17	0.83	0.49	0.00	0.00	0.98	-0.00
time (sec)	N/A	0.087	0.035	0.075	0.497	0.000	0.000	0.557	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	45	236	163	0	0	279	-1
N.S.	1	1.00	0.15	0.79	0.55	0.00	0.00	0.94	-0.00
time (sec)	N/A	0.100	0.078	0.083	0.500	0.000	0.000	0.657	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	45	272	172	0	0	305	-1
N.S.	1	1.00	0.12	0.75	0.47	0.00	0.00	0.84	-0.00
time (sec)	N/A	0.108	0.065	0.065	0.517	0.000	0.000	0.875	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	86	84	79	96	0	960	-1
N.S.	1	1.00	0.47	0.46	0.43	0.53	0.00	5.27	-0.01
time (sec)	N/A	0.046	0.783	0.094	0.490	0.378	0.000	5.604	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	66	64	53	62	0	992	-1
N.S.	1	1.00	0.60	0.58	0.48	0.56	0.00	9.02	-0.01
time (sec)	N/A	0.031	0.808	0.059	0.497	0.361	0.000	2.737	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	42	26	37	0	229	-1
N.S.	1	1.00	0.82	0.84	0.52	0.74	0.00	4.58	-0.02
time (sec)	N/A	0.016	0.106	0.064	0.495	0.357	0.000	0.560	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	27	39	0	45	-1
N.S.	1	1.00	0.84	0.78	0.53	0.76	0.00	0.88	-0.02
time (sec)	N/A	0.017	0.060	0.067	0.487	0.370	0.000	0.542	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	45	63	50	62	0	124	-1
N.S.	1	1.00	0.38	0.53	0.42	0.52	0.00	1.04	-0.01
time (sec)	N/A	0.033	0.055	0.063	0.490	0.348	0.000	0.673	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	45	83	70	82	0	185	-1
N.S.	1	1.00	0.25	0.45	0.38	0.45	0.00	1.01	-0.01
time (sec)	N/A	0.046	0.039	0.089	0.503	0.366	0.000	1.227	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.055	0.192	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	49	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.058	0.257	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.050	0.210	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.038	0.205	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.115	0.431	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.078	0.215	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.045	0.257	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.054	0.236	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	60	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.080	0.218	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	62	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.081	0.236	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	12884	0	23	0	0	-1
N.S.	1	1.00	1.00	402.62	0.00	0.72	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.027	5.160	0.000	0.383	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.061	0.216	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	122	542	225	1916	0	245	-1
N.S.	1	1.00	0.47	2.11	0.88	7.46	0.00	0.95	-0.00
time (sec)	N/A	0.135	0.247	3.177	0.515	67.374	0.000	0.713	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	104	516	194	1903	0	219	-1
N.S.	1	1.00	0.46	2.27	0.85	8.38	0.00	0.96	-0.00
time (sec)	N/A	0.119	0.182	0.323	0.555	66.040	0.000	0.579	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	38	23	37	0	16	48
N.S.	1	1.00	1.00	2.11	1.28	2.06	0.00	0.89	2.67
time (sec)	N/A	0.026	0.085	0.360	0.341	0.371	0.000	0.708	3.047

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	50	33	63	0	43	102
N.S.	1	1.00	0.73	1.22	0.80	1.54	0.00	1.05	2.49
time (sec)	N/A	0.029	0.138	0.388	0.273	0.356	0.000	0.704	6.532

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	50	60	48	82	0	58	356
N.S.	1	1.00	0.79	0.95	0.76	1.30	0.00	0.92	5.65
time (sec)	N/A	0.034	0.181	0.399	0.274	0.380	0.000	0.533	7.017

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	139	216	0	0	0	0	-1
N.S.	1	1.00	1.32	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	12.200	0.376	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	57	188	0	0	0	0	-1
N.S.	1	1.00	0.76	2.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	5.878	0.334	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	73	157	0	52	0	0	-1
N.S.	1	1.00	1.55	3.34	0.00	1.11	0.00	0.00	-0.02
time (sec)	N/A	0.048	10.204	0.378	0.000	0.093	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	115	297	0	111	0	0	-1
N.S.	1	1.00	1.49	3.86	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.074	10.660	0.401	0.000	0.100	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	124	550	0	155	0	0	-1
N.S.	1	1.00	1.18	5.24	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.102	11.513	0.395	0.000	0.110	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	123	702	235	1580	0	252	-1
N.S.	1	1.00	0.44	2.53	0.85	5.70	0.00	0.91	-0.00
time (sec)	N/A	0.138	0.824	3.796	0.491	42.867	0.000	0.528	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	113	676	204	1568	0	226	-1
N.S.	1	1.00	0.46	2.74	0.83	6.35	0.00	0.91	-0.00
time (sec)	N/A	0.124	0.575	0.325	0.502	42.114	0.000	0.533	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	58	23	24	0	16	43
N.S.	1	1.00	1.00	3.22	1.28	1.33	0.00	0.89	2.39
time (sec)	N/A	0.027	0.062	0.334	0.277	0.395	0.000	0.657	2.765

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	50	34	51	0	43	100
N.S.	1	1.00	0.73	1.22	0.83	1.24	0.00	1.05	2.44
time (sec)	N/A	0.032	0.093	0.363	0.274	0.374	0.000	0.641	3.494

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	42	60	58	71	0	64	292
N.S.	1	1.00	0.67	0.95	0.92	1.13	0.00	1.02	4.63
time (sec)	N/A	0.037	0.159	0.384	0.272	0.368	0.000	0.568	5.877

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	90	540	0	0	0	0	-1
N.S.	1	1.00	0.82	4.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	5.439	0.301	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	58	526	0	0	0	0	-1
N.S.	1	1.00	0.76	6.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.304	0.306	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	511	0	0	0	0	-1
N.S.	1	1.00	0.80	6.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.305	0.349	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	71	494	0	0	0	0	-1
N.S.	1	1.00	0.70	4.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.616	0.382	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	142	590	240	1997	0	278	-1
N.S.	1	1.00	0.51	2.13	0.87	7.21	0.00	1.00	-0.00
time (sec)	N/A	0.140	0.609	0.346	0.490	64.934	0.000	0.539	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	126	564	209	1984	0	252	-1
N.S.	1	1.00	0.51	2.28	0.85	8.03	0.00	1.02	-0.00
time (sec)	N/A	0.122	0.419	0.285	0.495	65.682	0.000	0.521	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	38	23	40	0	24	56
N.S.	1	1.00	1.00	1.90	1.15	2.00	0.00	1.20	2.80
time (sec)	N/A	0.031	0.083	0.319	0.272	0.399	0.000	0.613	2.476

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	32	50	36	58	0	42	64
N.S.	1	1.00	0.78	1.22	0.88	1.41	0.00	1.02	1.56
time (sec)	N/A	0.033	0.127	0.318	0.263	0.368	0.000	0.533	2.721

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	42	60	56	82	0	70	134
N.S.	1	1.00	0.67	0.95	0.89	1.30	0.00	1.11	2.13
time (sec)	N/A	0.037	0.239	0.357	0.272	0.394	0.000	0.589	5.464

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	153	246	0	0	0	0	-1
N.S.	1	1.00	1.12	1.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	13.240	0.289	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	133	220	0	0	0	0	-1
N.S.	1	1.00	1.23	2.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	12.373	0.270	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	192	0	101	0	0	-1
N.S.	1	1.00	0.89	2.40	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.440	0.319	0.000	0.121	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	192	0	101	0	0	-1
N.S.	1	1.00	0.89	2.40	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.378	0.347	0.000	0.091	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	316	0	160	0	0	-1
N.S.	1	1.00	1.00	2.87	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.107	10.538	0.359	0.000	0.102	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	130	571	0	207	0	0	-1
N.S.	1	1.00	0.93	4.08	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.137	11.787	0.418	0.000	0.108	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	122	688	220	1456	0	246	-1
N.S.	1	1.00	0.47	2.68	0.86	5.67	0.00	0.96	-0.00
time (sec)	N/A	0.123	0.758	0.350	0.482	38.672	0.000	0.530	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	109	662	188	1442	0	218	-1
N.S.	1	1.00	0.48	2.92	0.83	6.35	0.00	0.96	-0.00
time (sec)	N/A	0.110	0.675	0.329	0.479	37.864	0.000	0.697	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	38	23	46	0	23	102
N.S.	1	1.00	1.00	1.90	1.15	2.30	0.00	1.15	5.10
time (sec)	N/A	0.026	0.108	0.533	0.266	0.423	0.000	0.709	3.326

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	50	35	70	0	45	530
N.S.	1	1.00	0.93	1.16	0.81	1.63	0.00	1.05	12.33
time (sec)	N/A	0.030	0.150	0.610	0.270	0.414	0.000	0.581	7.188

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	60	48	93	0	58	831
N.S.	1	1.00	0.77	0.92	0.74	1.43	0.00	0.89	12.78
time (sec)	N/A	0.034	0.181	0.451	0.267	0.393	0.000	0.542	12.397

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	86	550	0	0	0	0	-1
N.S.	1	1.00	0.80	5.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.884	0.357	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	98	537	0	0	0	0	-1
N.S.	1	1.00	1.24	6.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.775	0.383	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	60	523	0	0	0	0	-1
N.S.	1	1.00	1.28	11.13	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.138	0.353	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	482	0	0	0	0	-1
N.S.	1	1.00	0.96	6.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.327	0.385	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	104	972	0	0	0	0	-1
N.S.	1	1.00	1.02	9.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.715	0.620	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	123	550	225	1871	0	257	-1
N.S.	1	1.00	0.48	2.14	0.88	7.28	0.00	1.00	-0.00
time (sec)	N/A	0.134	0.350	0.316	0.494	62.505	0.000	0.765	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	105	524	193	1856	0	228	-1
N.S.	1	1.00	0.46	2.31	0.85	8.18	0.00	1.00	-0.00
time (sec)	N/A	0.115	0.245	0.297	0.496	60.729	0.000	0.624	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	38	23	58	0	26	381
N.S.	1	1.00	1.00	1.90	1.15	2.90	0.00	1.30	19.05
time (sec)	N/A	0.030	0.122	0.333	0.277	0.366	0.000	0.683	6.447

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	50	35	84	0	45	684
N.S.	1	1.00	0.98	1.16	0.81	1.95	0.00	1.05	15.91
time (sec)	N/A	0.033	0.108	0.366	0.268	0.401	0.000	0.695	8.149

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	60	48	109	0	58	987
N.S.	1	1.00	0.83	0.92	0.74	1.68	0.00	0.89	15.18
time (sec)	N/A	0.038	0.140	0.422	0.266	0.452	0.000	0.810	16.413

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	102	222	0	0	0	0	-1
N.S.	1	1.00	0.91	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	10.433	0.333	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	126	197	0	0	0	0	-1
N.S.	1	1.00	1.59	2.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	10.816	0.322	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	110	302	0	117	0	0	-1
N.S.	1	1.00	1.34	3.68	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.074	10.799	0.328	0.000	0.099	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	136	557	0	161	0	0	-1
N.S.	1	1.00	1.21	4.97	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.105	11.819	0.364	0.000	0.103	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	123	688	219	1558	0	248	-1
N.S.	1	1.00	0.48	2.68	0.85	6.06	0.00	0.96	-0.00
time (sec)	N/A	0.135	0.813	0.309	0.501	39.441	0.000	0.616	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	113	662	189	1545	0	220	-1
N.S.	1	1.00	0.50	2.92	0.83	6.81	0.00	0.97	-0.00
time (sec)	N/A	0.119	0.585	0.297	0.495	38.586	0.000	0.603	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	38	23	63	0	26	530
N.S.	1	1.00	1.00	1.90	1.15	3.15	0.00	1.30	26.50
time (sec)	N/A	0.029	0.176	0.340	0.272	0.390	0.000	0.618	7.397

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	50	50	35	91	0	45	831
N.S.	1	1.00	1.16	1.16	0.81	2.12	0.00	1.05	19.33
time (sec)	N/A	0.037	0.194	0.363	0.281	0.436	0.000	0.816	12.140

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	60	48	114	0	58	1132
N.S.	1	1.00	0.92	0.92	0.74	1.75	0.00	0.89	17.42
time (sec)	N/A	0.039	0.259	0.561	0.277	0.426	0.000	0.810	14.007

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	122	563	0	0	0	0	-1
N.S.	1	1.00	0.85	3.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	1.641	0.378	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	100	550	0	0	0	0	-1
N.S.	1	1.00	0.88	4.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	1.134	0.331	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	97	536	0	0	0	0	-1
N.S.	1	1.00	1.15	6.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.644	0.359	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	503	0	0	0	0	-1
N.S.	1	1.00	0.88	6.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.423	0.325	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	105	964	0	0	0	0	-1
N.S.	1	1.00	0.95	8.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	1.854	0.375	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	116	1455	0	0	0	0	-1
N.S.	1	1.00	0.83	10.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.897	0.403	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	493	0	71	0	0	80
N.S.	1	1.00	0.75	7.25	0.00	1.04	0.00	0.00	1.18
time (sec)	N/A	0.063	0.228	6.007	0.000	0.381	0.000	0.000	4.330

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	80	131	0	110	0	0	-1
N.S.	1	1.00	0.91	1.49	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.076	6.700	0.832	0.000	0.121	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	295	0	52	0	0	60
N.S.	1	1.00	1.00	9.83	0.00	1.73	0.00	0.00	2.00
time (sec)	N/A	0.028	0.152	0.395	0.000	0.350	0.000	0.000	2.898

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	60	88	0	68	0	0	-1
N.S.	1	1.00	1.20	1.76	0.00	1.36	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.137	0.368	0.000	0.097	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	72	185	0	447	0	0	-1
N.S.	1	1.00	0.67	1.73	0.00	4.18	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.318	0.342	0.000	0.660	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	178	0	158	0	0	-1
N.S.	1	1.00	0.92	2.07	0.00	1.84	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.265	0.837	0.000	0.109	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	99	338	0	135	0	0	-1
N.S.	1	1.00	0.79	2.68	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.114	6.691	4.167	0.000	0.110	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	45	492	0	62	0	0	69
N.S.	1	1.00	0.66	7.24	0.00	0.91	0.00	0.00	1.01
time (sec)	N/A	0.072	0.181	0.319	0.000	0.386	0.000	0.000	3.731

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	328	0	107	0	0	-1
N.S.	1	1.00	0.99	3.90	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.224	0.353	0.000	0.098	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	308	0	49	0	0	39
N.S.	1	1.00	1.00	10.27	0.00	1.63	0.00	0.00	1.30
time (sec)	N/A	0.034	0.073	0.347	0.000	0.365	0.000	0.000	3.066

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	92	316	0	110	0	0	-1
N.S.	1	1.00	1.02	3.51	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.296	0.411	0.000	0.109	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	104	247	0	570	0	0	-1
N.S.	1	1.00	0.72	1.70	0.00	3.93	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.403	0.321	0.000	0.637	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	100	349	0	145	0	0	-1
N.S.	1	1.00	0.81	2.84	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.543	0.727	0.000	0.106	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	52	60	0	77	0	0	88
N.S.	1	1.00	0.76	0.88	0.00	1.13	0.00	0.00	1.29
time (sec)	N/A	0.069	0.173	0.596	0.000	0.364	0.000	0.000	4.823

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	87	339	0	125	0	0	-1
N.S.	1	1.00	0.99	3.85	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.253	0.371	0.000	0.100	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	48	0	58	0	0	69
N.S.	1	1.00	1.00	1.50	0.00	1.81	0.00	0.00	2.16
time (sec)	N/A	0.033	0.146	0.311	0.000	0.363	0.000	0.000	3.634

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	69	327	0	75	0	0	-1
N.S.	1	1.00	1.38	6.54	0.00	1.50	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.163	0.692	0.000	0.103	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	80	177	0	453	0	0	-1
N.S.	1	1.00	0.75	1.67	0.00	4.27	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.147	0.333	0.000	0.605	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	89	315	0	168	0	0	-1
N.S.	1	1.00	1.02	3.62	0.00	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.370	0.378	0.000	0.111	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	112	319	0	659	0	0	-1
N.S.	1	1.00	0.77	2.18	0.00	4.51	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.629	0.363	0.000	0.628	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	67	70	0	91	0	0	296
N.S.	1	1.00	0.46	0.48	0.00	0.62	0.00	0.00	2.03
time (sec)	N/A	0.146	0.429	0.318	0.000	0.377	0.000	0.000	8.601

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	57	60	0	77	0	0	94
N.S.	1	1.00	0.52	0.55	0.00	0.71	0.00	0.00	0.86
time (sec)	N/A	0.109	0.235	0.311	0.000	0.358	0.000	0.000	5.480

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	45	48	0	60	0	0	81
N.S.	1	1.00	1.41	1.50	0.00	1.88	0.00	0.00	2.53
time (sec)	N/A	0.037	0.149	0.304	0.000	0.409	0.000	0.000	4.012

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	88	237	0	577	0	0	-1
N.S.	1	1.00	0.62	1.68	0.00	4.09	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.400	0.342	0.000	0.653	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	103	320	0	662	0	0	-1
N.S.	1	1.00	0.68	2.12	0.00	4.38	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.362	0.346	0.000	0.689	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	118	181	0	150	0	0	-1
N.S.	1	1.00	0.71	1.08	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.774	0.838	0.000	0.138	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	97	161	0	135	0	0	-1
N.S.	1	1.00	0.75	1.24	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.387	0.352	0.000	0.111	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	137	0	112	0	0	-1
N.S.	1	1.00	0.86	1.47	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.254	0.334	0.000	0.126	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	185	0	160	0	0	-1
N.S.	1	1.00	0.92	2.15	0.00	1.86	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.209	0.367	0.000	0.121	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	96	337	0	215	0	0	-1
N.S.	1	1.00	0.74	2.59	0.00	1.65	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.374	0.360	0.000	0.116	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	106	487	0	283	0	0	-1
N.S.	1	1.00	0.63	2.92	0.00	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.434	0.405	0.000	0.152	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	10.439	0.283	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.063	10.387	0.225	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	10.354	0.224	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	67	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.070	10.368	0.219	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.071	50.631	0.237	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	50.519	0.200	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.067	50.538	0.193	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.065	50.572	0.219	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	10.487	0.223	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.063	10.407	0.213	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.060	10.382	0.189	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	64	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	10.447	0.208	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	85	0	0	0	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.069	50.706	0.188	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	72	0	0	0	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	10.534	0.195	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	50.847	0.189	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	70	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.067	10.432	0.196	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.071	0.145	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.039	0.091	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	19	18	0	18	17
N.S.	1	1.00	1.00	1.06	1.12	1.06	0.00	1.06	1.00
time (sec)	N/A	0.021	0.011	0.335	0.271	0.414	0.000	0.445	2.454

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	3063	50	60	0	0	91
N.S.	1	1.00	0.80	66.59	1.09	1.30	0.00	0.00	1.98
time (sec)	N/A	0.037	0.065	0.684	0.283	0.350	0.000	0.000	3.312

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	62	7703	76	117	0	0	219
N.S.	1	1.00	0.86	106.99	1.06	1.62	0.00	0.00	3.04
time (sec)	N/A	0.045	0.350	0.658	0.299	0.386	0.000	0.000	7.573

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	71	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.150	0.115	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	71	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.093	0.072	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.091	0.129	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.079	0.098	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	8.401	0.232	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	3.436	0.231	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	3.001	0.247	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	224	0	0	0	0	0	-1
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	5.552	0.201	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	260	0	0	0	0	0	-1
N.S.	1	1.00	3.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	2.066	0.183	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	916	0	0	0	0	0	-1
N.S.	1	1.00	18.32	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	4.994	0.990	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	450	0	0	0	0	0	-1
N.S.	1	1.00	9.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	2.279	0.490	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	1782	30	46	0	0	53
N.S.	1	1.00	0.88	71.28	1.20	1.84	0.00	0.00	2.12
time (sec)	N/A	0.029	0.087	0.762	0.298	0.394	0.000	0.000	2.615

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	46	5437	59	92	0	0	138
N.S.	1	1.00	0.87	102.58	1.11	1.74	0.00	0.00	2.60
time (sec)	N/A	0.037	0.182	0.370	0.300	0.376	0.000	0.000	3.736

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	69	10922	87	152	0	0	-1
N.S.	1	1.00	0.86	136.52	1.09	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.299	0.521	0.319	0.385	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	456	0	0	0	0	0	-1
N.S.	1	1.00	5.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	3.077	0.502	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	252	0	0	0	0	0	-1
N.S.	1	1.00	3.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	1.203	0.011	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.243	0.216	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	1242	0	0	0	0	0	-1
N.S.	1	1.00	15.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	15.623	0.182	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	1516	0	0	0	0	0	-1
N.S.	1	1.00	19.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	17.548	0.190	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	297	0	0	0	0	0	-1
N.S.	1	1.00	3.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	33.328	0.235	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	91	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	11.771	0.214	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	11.419	0.201	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	90	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	12.134	0.194	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.538	0.236	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	66	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.078	0.224	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	45	728	215	643	0	0	97
N.S.	1	1.00	0.19	3.14	0.93	2.77	0.00	0.00	0.42
time (sec)	N/A	0.156	0.092	7.371	0.503	0.401	0.000	0.000	2.583

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	45	548	197	604	0	0	83
N.S.	1	1.00	0.21	2.56	0.92	2.82	0.00	0.00	0.39
time (sec)	N/A	0.131	0.051	0.419	0.511	0.383	0.000	0.000	2.472

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	36	660	196	621	0	0	80
N.S.	1	1.00	0.17	3.14	0.93	2.96	0.00	0.00	0.38
time (sec)	N/A	0.123	0.061	0.366	0.499	0.443	0.000	0.000	2.476

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	132	292	173	515	0	0	61
N.S.	1	1.00	0.69	1.52	0.90	2.68	0.00	0.00	0.32
time (sec)	N/A	0.099	0.204	0.362	0.525	0.376	0.000	0.000	0.213

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	40	136	171	0	0	0	50
N.S.	1	1.00	0.21	0.71	0.89	0.00	0.00	0.00	0.26
time (sec)	N/A	0.085	0.048	0.216	0.509	0.000	0.000	0.000	2.552

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	162	149	185	0	0	0	74
N.S.	1	1.00	0.78	0.71	0.89	0.00	0.00	0.00	0.35
time (sec)	N/A	0.114	0.236	0.071	0.507	0.000	0.000	0.000	2.570

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	42	156	194	0	0	0	76
N.S.	1	1.00	0.20	0.73	0.91	0.00	0.00	0.00	0.36
time (sec)	N/A	0.120	0.056	0.117	0.501	0.000	0.000	0.000	2.597

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	172	171	207	0	0	0	90
N.S.	1	1.00	0.74	0.74	0.90	0.00	0.00	0.00	0.39
time (sec)	N/A	0.138	0.469	0.110	0.551	0.000	0.000	0.000	2.874

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	45	728	215	654	0	0	97
N.S.	1	1.00	0.19	3.11	0.92	2.79	0.00	0.00	0.41
time (sec)	N/A	0.151	0.067	0.411	0.530	0.438	0.000	0.000	2.579

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	45	548	197	623	0	0	83
N.S.	1	1.00	0.21	2.56	0.92	2.91	0.00	0.00	0.39
time (sec)	N/A	0.131	0.052	0.392	0.507	0.382	0.000	0.000	2.518

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	38	660	196	630	0	0	82
N.S.	1	1.00	0.18	3.11	0.92	2.97	0.00	0.00	0.39
time (sec)	N/A	0.125	0.047	0.394	0.517	0.410	0.000	0.000	2.488

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	134	292	173	530	0	0	61
N.S.	1	1.00	0.70	1.52	0.90	2.76	0.00	0.00	0.32
time (sec)	N/A	0.106	0.029	0.360	0.512	0.382	0.000	0.000	2.499

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	37	324	173	553	0	0	54
N.S.	1	1.00	0.19	1.69	0.90	2.88	0.00	0.00	0.28
time (sec)	N/A	0.098	0.012	0.353	0.560	0.399	0.000	0.000	2.528

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	159	149	186	0	0	0	75
N.S.	1	1.00	0.76	0.71	0.89	0.00	0.00	0.00	0.36
time (sec)	N/A	0.102	0.094	0.085	0.538	0.000	0.000	0.000	2.653

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	39	152	191	0	0	0	73
N.S.	1	1.00	0.18	0.72	0.91	0.00	0.00	0.00	0.35
time (sec)	N/A	0.116	0.068	0.063	0.510	0.000	0.000	0.000	2.634

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	172	171	207	0	0	0	91
N.S.	1	1.00	0.74	0.74	0.89	0.00	0.00	0.00	0.39
time (sec)	N/A	0.137	0.246	0.101	0.509	0.000	0.000	0.000	2.938

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	40	728	215	632	0	0	97
N.S.	1	1.00	0.17	3.15	0.93	2.74	0.00	0.00	0.42
time (sec)	N/A	0.146	0.129	0.421	0.505	0.380	0.000	0.000	2.572

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	38	548	197	615	0	0	81
N.S.	1	1.00	0.18	2.58	0.93	2.90	0.00	0.00	0.38
time (sec)	N/A	0.124	0.099	0.382	0.508	0.386	0.000	0.000	2.514

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	35	652	196	610	0	0	79
N.S.	1	1.00	0.17	3.12	0.94	2.92	0.00	0.00	0.38
time (sec)	N/A	0.120	0.068	0.376	0.544	0.402	0.000	0.000	0.194

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	131	138	171	0	0	0	57
N.S.	1	1.00	0.68	0.72	0.89	0.00	0.00	0.00	0.30
time (sec)	N/A	0.081	0.012	0.107	0.548	0.000	0.000	0.000	2.650

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	40	135	170	0	0	0	58
N.S.	1	1.00	0.21	0.70	0.89	0.00	0.00	0.00	0.30
time (sec)	N/A	0.092	0.013	0.099	0.516	0.000	0.000	0.000	2.505

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	159	151	188	0	0	0	77
N.S.	1	1.00	0.75	0.71	0.89	0.00	0.00	0.00	0.36
time (sec)	N/A	0.114	0.117	0.106	0.505	0.000	0.000	0.000	2.657

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	47	156	194	0	0	0	76
N.S.	1	1.00	0.22	0.73	0.91	0.00	0.00	0.00	0.36
time (sec)	N/A	0.113	0.067	0.114	0.504	0.000	0.000	0.000	2.667

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	38	728	215	644	0	0	93
N.S.	1	1.00	0.16	3.14	0.93	2.78	0.00	0.00	0.40
time (sec)	N/A	0.153	0.178	0.555	0.521	0.394	0.000	0.000	2.582

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	37	540	197	639	0	0	80
N.S.	1	1.00	0.18	2.56	0.93	3.03	0.00	0.00	0.38
time (sec)	N/A	0.121	0.032	0.316	0.503	0.389	0.000	0.000	0.199

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	38	157	194	0	0	0	76
N.S.	1	1.00	0.18	0.74	0.92	0.00	0.00	0.00	0.36
time (sec)	N/A	0.104	0.008	0.065	0.504	0.000	0.000	0.000	2.605

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	134	138	170	0	0	0	57
N.S.	1	1.00	0.70	0.72	0.89	0.00	0.00	0.00	0.30
time (sec)	N/A	0.093	0.032	0.083	0.514	0.000	0.000	0.000	2.582

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	40	138	173	0	0	0	58
N.S.	1	1.00	0.21	0.72	0.90	0.00	0.00	0.00	0.30
time (sec)	N/A	0.091	0.012	0.127	0.501	0.000	0.000	0.000	2.478

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	159	151	188	0	0	0	77
N.S.	1	1.00	0.75	0.71	0.89	0.00	0.00	0.00	0.36
time (sec)	N/A	0.116	0.159	0.100	0.510	0.000	0.000	0.000	2.618

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	47	156	194	0	0	0	76
N.S.	1	1.00	0.22	0.73	0.91	0.00	0.00	0.00	0.36
time (sec)	N/A	0.115	0.086	0.092	0.515	0.000	0.000	0.000	2.647

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	172	171	207	0	0	0	93
N.S.	1	1.00	0.74	0.73	0.88	0.00	0.00	0.00	0.40
time (sec)	N/A	0.137	0.355	0.089	0.505	0.000	0.000	0.000	2.978

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.088	0.260	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.078	0.240	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.071	0.256	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	67	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.105	0.230	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	52	60	54	65	0	82	334
N.S.	1	1.00	0.78	0.90	0.81	0.97	0.00	1.22	4.99
time (sec)	N/A	0.038	0.214	3.823	0.281	0.388	0.000	0.486	7.242

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	34	50	38	54	0	57	218
N.S.	1	1.00	0.76	1.11	0.84	1.20	0.00	1.27	4.84
time (sec)	N/A	0.033	0.163	0.395	0.271	0.381	0.000	0.514	6.091

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	19	41	0	23	53
N.S.	1	1.00	1.00	0.86	0.86	1.86	0.00	1.05	2.41
time (sec)	N/A	0.025	0.043	0.147	0.281	0.369	0.000	0.460	2.567

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	40	136	159	547	0	182	49
N.S.	1	1.00	0.21	0.71	0.83	2.85	0.00	0.95	0.26
time (sec)	N/A	0.078	0.047	0.181	0.501	0.370	0.000	0.472	2.499

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	102	522	201	2009	0	227	-1
N.S.	1	1.00	0.45	2.30	0.89	8.85	0.00	1.00	-0.00
time (sec)	N/A	0.119	0.207	0.360	0.497	64.700	0.000	0.505	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	102	559	0	0	0	0	-1
N.S.	1	1.00	0.95	5.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.482	0.372	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	61	513	0	0	0	0	-1
N.S.	1	1.00	0.81	6.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.302	0.290	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	57	523	0	0	0	0	-1
N.S.	1	1.00	1.21	11.13	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.105	0.334	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	94	536	0	0	0	0	-1
N.S.	1	1.00	1.16	6.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.484	0.349	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	86	550	0	0	0	0	-1
N.S.	1	1.00	0.77	4.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.833	0.348	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	52	60	51	68	0	78	392
N.S.	1	1.00	0.78	0.90	0.76	1.01	0.00	1.16	5.85
time (sec)	N/A	0.044	0.165	7.296	0.269	0.400	0.000	0.530	9.188

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	50	36	56	0	55	276
N.S.	1	1.00	0.93	1.11	0.80	1.24	0.00	1.22	6.13
time (sec)	N/A	0.036	0.152	0.410	0.277	0.398	0.000	0.462	6.945

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	45	0	24	100
N.S.	1	1.00	1.00	0.86	0.82	2.05	0.00	1.09	4.55
time (sec)	N/A	0.032	0.062	0.116	0.278	0.383	0.000	0.482	3.568

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	159	149	170	533	0	0	73
N.S.	1	1.00	0.76	0.71	0.81	2.54	0.00	0.00	0.35
time (sec)	N/A	0.104	0.286	0.165	0.496	0.469	0.000	0.000	2.654

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	110	670	188	1558	0	210	-1
N.S.	1	1.00	0.49	2.98	0.84	6.92	0.00	0.93	-0.00
time (sec)	N/A	0.117	0.285	0.278	0.512	42.024	0.000	0.521	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	90	251	0	125	0	0	-1
N.S.	1	1.00	0.66	1.85	0.00	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.910	0.538	0.000	0.111	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	80	225	0	112	0	0	-1
N.S.	1	1.00	0.74	2.08	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.508	0.338	0.000	0.103	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	69	188	0	95	0	0	-1
N.S.	1	1.00	0.86	2.35	0.00	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.320	0.251	0.000	0.091	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	196	0	0	0	0	-1
N.S.	1	1.00	0.74	2.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.147	0.310	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	96	222	0	0	0	0	-1
N.S.	1	1.00	0.89	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	1.199	0.305	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	131	250	0	0	0	0	-1
N.S.	1	1.00	0.96	1.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	12.603	0.303	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	52	60	54	89	0	84	474
N.S.	1	1.00	0.78	0.90	0.81	1.33	0.00	1.25	7.07
time (sec)	N/A	0.042	0.467	2.997	0.276	0.442	0.000	0.568	13.292

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	50	38	75	0	59	352
N.S.	1	1.00	0.93	1.11	0.84	1.67	0.00	1.31	7.82
time (sec)	N/A	0.039	0.297	0.324	0.279	0.456	0.000	0.520	7.224

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	19	60	0	28	230
N.S.	1	1.00	1.00	0.86	0.86	2.73	0.00	1.27	10.45
time (sec)	N/A	0.031	0.060	0.088	0.271	0.410	0.000	0.506	5.584

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	40	154	183	630	0	218	74
N.S.	1	1.00	0.19	0.73	0.86	2.97	0.00	1.03	0.35
time (sec)	N/A	0.104	0.050	0.087	0.487	0.411	0.000	0.455	2.735

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	107	532	202	2030	0	240	-1
N.S.	1	1.00	0.48	2.36	0.90	9.02	0.00	1.07	-0.00
time (sec)	N/A	0.115	0.207	0.583	0.499	64.271	0.000	0.515	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	125	558	235	2047	0	268	-1
N.S.	1	1.00	0.49	2.21	0.93	8.09	0.00	1.06	-0.00
time (sec)	N/A	0.128	0.205	0.296	0.490	64.320	0.000	0.493	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	79	224	0	123	0	0	-1
N.S.	1	1.00	0.72	2.06	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.605	0.382	0.000	0.111	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	68	196	0	104	0	0	-1
N.S.	1	1.00	0.86	2.48	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.294	0.597	0.000	0.104	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	77	167	0	59	0	0	-1
N.S.	1	1.00	1.64	3.55	0.00	1.26	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.148	0.260	0.000	0.112	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	126	198	0	0	0	0	-1
N.S.	1	1.00	1.66	2.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.596	0.333	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	226	0	0	0	0	-1
N.S.	1	1.00	0.86	2.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	11.122	0.329	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	60	54	64	0	80	268
N.S.	1	1.00	0.69	0.92	0.83	0.98	0.00	1.23	4.12
time (sec)	N/A	0.041	0.226	1.732	0.274	0.421	0.000	0.620	6.878

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	50	36	54	0	44	64
N.S.	1	1.00	0.74	1.16	0.84	1.26	0.00	1.02	1.49
time (sec)	N/A	0.035	0.112	0.322	0.271	0.378	0.000	0.597	2.971

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	40	0	18	51
N.S.	1	1.00	1.00	0.95	0.90	2.00	0.00	0.90	2.55
time (sec)	N/A	0.031	0.063	0.101	0.286	0.377	0.000	0.552	2.573

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	38	157	167	652	0	0	76
N.S.	1	1.00	0.18	0.74	0.79	3.08	0.00	0.00	0.36
time (sec)	N/A	0.103	0.044	0.098	0.493	0.391	0.000	0.000	2.685

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	115	982	204	2037	0	252	-1
N.S.	1	1.00	0.46	3.94	0.82	8.18	0.00	1.01	-0.00
time (sec)	N/A	0.132	0.308	0.280	0.491	62.536	0.000	0.508	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	104	535	0	0	0	0	-1
N.S.	1	1.00	0.75	3.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.964	0.324	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	93	502	0	0	0	0	-1
N.S.	1	1.00	0.89	4.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.467	0.339	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	496	0	0	0	0	-1
N.S.	1	1.00	0.88	6.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.408	0.258	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	509	0	0	0	0	-1
N.S.	1	1.00	0.85	6.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.443	0.262	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	77	523	0	0	0	0	-1
N.S.	1	1.00	0.69	4.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.388	0.273	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	89	536	0	0	0	0	-1
N.S.	1	1.00	0.63	3.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.565	0.309	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	113	306	0	117	0	0	-1
N.S.	1	1.00	1.38	3.73	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.476	0.243	0.000	0.099	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	103	987	0	0	0	0	-1
N.S.	1	1.00	0.94	8.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	1.022	0.345	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0	-1
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.154	0.117	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.063	0.092	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.065	0.086	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	56	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.063	0.083	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.056	0.082	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	89	0	0	0	0	0	-1
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.749	0.141	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	78	0	0	0	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.594	0.119	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0	-1
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.520	0.126	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	78	0	0	0	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.140	0.105	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	77	0	0	0	0	0	-1
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.185	0.110	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	80	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.194	0.107	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	80	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.214	0.091	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	80	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.196	0.092	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	80	0	0	0	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.148	0.077	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	79	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.141	0.071	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	92	0	0	0	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.680	0.135	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	69	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.114	0.123	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	69	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.106	0.135	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	69	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.108	0.103	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	67	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.100	0.099	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	174	600	0	858	0	0	-1
N.S.	1	1.00	0.98	3.37	0.00	4.82	0.00	0.00	-0.01
time (sec)	N/A	0.111	1.320	4.489	0.000	0.568	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	71	572	0	113	0	0	-1
N.S.	1	1.00	0.76	6.15	0.00	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.080	11.086	0.612	0.000	0.116	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	136	306	0	710	0	0	-1
N.S.	1	1.00	1.03	2.32	0.00	5.38	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.790	0.405	0.000	0.521	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	62	551	0	70	0	0	-1
N.S.	1	1.00	1.13	10.02	0.00	1.27	0.00	0.00	-0.02
time (sec)	N/A	0.042	10.605	0.505	0.000	0.102	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	50	0	55	53	0	55
N.S.	1	1.00	1.00	1.47	0.00	1.62	1.56	0.00	1.62
time (sec)	N/A	0.034	0.138	0.360	0.000	0.400	12.038	0.000	3.483

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	571	0	121	0	0	-1
N.S.	1	1.00	0.83	6.01	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.079	10.770	0.470	0.000	0.131	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	53	62	0	69	0	0	69
N.S.	1	1.00	0.74	0.86	0.00	0.96	0.00	0.00	0.96
time (sec)	N/A	0.071	0.183	0.544	0.000	0.399	0.000	0.000	3.355

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	92	586	0	135	0	0	-1
N.S.	1	1.00	0.70	4.44	0.00	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.118	11.065	0.392	0.000	0.121	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	95	239	0	151	0	0	-1
N.S.	1	1.00	0.73	1.82	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.120	7.139	0.398	0.000	0.168	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	129	759	0	837	0	0	-1
N.S.	1	1.00	0.76	4.49	0.00	4.95	0.00	0.00	-0.01
time (sec)	N/A	0.121	6.807	0.355	0.000	0.544	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	105	211	0	98	0	0	-1
N.S.	1	1.00	1.19	2.40	0.00	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.825	0.392	0.000	0.112	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	128	719	0	805	0	0	-1
N.S.	1	1.00	0.77	4.31	0.00	4.82	0.00	0.00	-0.01
time (sec)	N/A	0.083	4.908	0.401	0.000	0.669	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	98	207	0	112	0	0	-1
N.S.	1	1.00	1.02	2.16	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.663	0.408	0.000	0.116	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	141	50	0	63	53	0	65
N.S.	1	1.00	4.15	1.47	0.00	1.85	1.56	0.00	1.91
time (sec)	N/A	0.039	1.678	0.333	0.000	0.408	62.857	0.000	3.159

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	105	241	0	125	0	0	-1
N.S.	1	1.00	0.80	1.84	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.122	1.418	0.438	0.000	0.120	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	158	62	0	76	0	0	78
N.S.	1	1.00	1.53	0.60	0.00	0.74	0.00	0.00	0.76
time (sec)	N/A	0.113	3.500	0.358	0.000	0.415	0.000	0.000	3.519

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	189	628	0	924	0	0	-1
N.S.	1	1.00	0.91	3.02	0.00	4.44	0.00	0.00	-0.00
time (sec)	N/A	0.163	3.414	0.382	0.000	0.631	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	93	581	0	167	0	0	-1
N.S.	1	1.00	0.71	4.44	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.118	12.565	0.416	0.000	0.173	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	182	602	0	858	0	0	-1
N.S.	1	1.00	1.08	3.56	0.00	5.08	0.00	0.00	-0.01
time (sec)	N/A	0.109	1.702	0.346	0.000	0.535	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	585	0	122	0	0	-1
N.S.	1	1.00	0.84	6.65	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.078	10.825	0.390	0.000	0.114	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	181	558	0	832	0	0	-1
N.S.	1	1.00	1.08	3.32	0.00	4.95	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.990	0.391	0.000	0.745	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	81	565	0	131	0	0	-1
N.S.	1	1.00	0.84	5.89	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.082	10.849	0.399	0.000	0.107	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	45	50	0	74	0	0	72
N.S.	1	1.00	1.32	1.47	0.00	2.18	0.00	0.00	2.12
time (sec)	N/A	0.038	0.166	0.344	0.000	0.378	0.000	0.000	3.213

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	99	586	0	147	0	0	-1
N.S.	1	1.00	0.76	4.47	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.123	11.082	0.378	0.000	0.115	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	136	758	0	850	0	0	-1
N.S.	1	1.00	0.76	4.26	0.00	4.78	0.00	0.00	-0.01
time (sec)	N/A	0.115	6.818	0.355	0.000	0.592	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	83	208	0	107	0	0	-1
N.S.	1	1.00	0.90	2.26	0.00	1.16	0.00	0.00	-0.01
time (sec)	N/A	0.080	2.434	0.368	0.000	0.105	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	105	344	0	709	0	0	-1
N.S.	1	1.00	0.80	2.63	0.00	5.41	0.00	0.00	-0.01
time (sec)	N/A	0.072	5.011	0.365	0.000	0.571	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	89	175	0	62	0	0	-1
N.S.	1	1.00	1.62	3.18	0.00	1.13	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.526	0.625	0.000	0.099	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	50	0	51	51	0	52
N.S.	1	1.00	1.00	1.56	0.00	1.59	1.59	0.00	1.62
time (sec)	N/A	0.032	0.438	0.339	0.000	0.389	10.992	0.000	2.904

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	213	0	109	0	0	-1
N.S.	1	1.00	0.96	2.24	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.086	1.121	0.374	0.000	0.106	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	112	60	0	63	88	0	64
N.S.	1	1.00	1.56	0.83	0.00	0.88	1.22	0.00	0.89
time (sec)	N/A	0.070	1.275	0.345	0.000	0.373	72.087	0.000	3.022

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	211	1061	0	864	0	0	-1
N.S.	1	1.00	1.23	6.20	0.00	5.05	0.00	0.00	-0.01
time (sec)	N/A	0.117	1.205	0.360	0.000	0.526	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	70	535	0	141	0	0	-1
N.S.	1	1.00	0.72	5.52	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.083	10.701	0.345	0.000	0.111	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	50	0	57	53	0	46
N.S.	1	1.00	1.00	1.56	0.00	1.78	1.66	0.00	1.44
time (sec)	N/A	0.033	0.120	0.328	0.000	0.345	2.191	0.000	2.950

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	67	556	0	140	0	0	-1
N.S.	1	1.00	0.74	6.11	0.00	1.54	0.00	0.00	-0.01
time (sec)	N/A	0.080	10.638	0.376	0.000	0.104	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	67	52	60	0	72	90	0	60
N.S.	1	0.93	0.72	0.83	0.00	1.00	1.25	0.00	0.83
time (sec)	N/A	0.071	0.206	0.311	0.000	0.391	37.905	0.000	3.196

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	81	570	0	153	0	0	-1
N.S.	1	1.00	0.62	4.38	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.122	10.779	0.357	0.000	0.119	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	144	1367	0	920	0	0	-1
N.S.	1	1.00	0.84	7.95	0.00	5.35	0.00	0.00	-0.01
time (sec)	N/A	0.118	1.287	0.358	0.000	0.595	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	116	314	0	165	0	0	-1
N.S.	1	1.00	1.15	3.11	0.00	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.486	0.352	0.000	0.103	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	50	0	66	54	0	55
N.S.	1	1.00	1.00	1.47	0.00	1.94	1.59	0.00	1.62
time (sec)	N/A	0.037	0.173	0.298	0.000	0.365	53.830	0.000	3.174

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	70	322	0	147	0	0	-1
N.S.	1	1.00	0.74	3.39	0.00	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.813	0.361	0.000	0.120	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	110	62	0	81	92	0	81
N.S.	1	1.00	1.59	0.90	0.00	1.17	1.33	0.00	1.17
time (sec)	N/A	0.068	0.907	0.341	0.000	0.430	72.882	0.000	3.563

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	112	336	0	165	0	0	-1
N.S.	1	1.00	0.85	2.55	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.123	2.550	0.369	0.000	0.120	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	159	72	0	96	0	0	93
N.S.	1	1.00	1.50	0.68	0.00	0.91	0.00	0.00	0.88
time (sec)	N/A	0.113	3.320	0.322	0.000	0.430	0.000	0.000	4.348

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.121	0.292	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.091	0.280	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.111	0.244	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.136	0.245	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.168	0.239	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.148	0.234	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	69	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.077	0.228	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	71	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.083	0.225	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.159	0.263	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.096	0.221	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.129	0.194	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.245	0.201	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.160	0.210	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.092	0.213	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.120	0.207	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	64	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.221	0.198	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	47	6797	82	84	0	0	199
N.S.	1	1.00	0.70	101.45	1.22	1.25	0.00	0.00	2.97
time (sec)	N/A	0.045	0.456	0.571	0.291	0.397	0.000	0.000	7.808

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	34	2707	54	53	0	0	87
N.S.	1	1.00	0.79	62.95	1.26	1.23	0.00	0.00	2.02
time (sec)	N/A	0.035	0.135	0.194	0.272	0.481	0.000	0.000	3.390

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	21	20	42	0	19
N.S.	1	1.00	1.00	1.06	1.24	1.18	2.47	0.00	1.12
time (sec)	N/A	0.015	0.027	0.073	0.275	0.374	0.139	0.000	0.118

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	124	0	0	0	0	0	-1
N.S.	1	1.00	3.10	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.930	0.132	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	815	0	0	0	0	0	-1
N.S.	1	1.00	20.90	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.031	12.541	0.118	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	2138	0	0	0	0	0	-1
N.S.	1	1.00	53.45	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	22.173	0.117	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	110	0	0	0	0	0	-1
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.296	0.104	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	6726	0	0	0	0	0	-1
N.S.	1	1.00	106.76	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	25.323	0.076	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	4872	0	0	0	0	0	-1
N.S.	1	1.00	82.58	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	21.602	0.099	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	6532	0	0	0	0	0	-1
N.S.	1	1.00	103.68	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	26.704	0.097	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.693	0.106	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.184	0.234	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	101	10923	77	85	0	0	-1
N.S.	1	1.00	1.36	147.61	1.04	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.048	2.261	1.257	0.282	0.385	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	78	5438	51	61	0	0	139
N.S.	1	1.00	1.59	110.98	1.04	1.24	0.00	0.00	2.84
time (sec)	N/A	0.036	1.258	0.439	0.281	0.376	0.000	0.000	3.845

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	25	24	40	0	0	49
N.S.	1	1.00	1.04	1.04	1.00	1.67	0.00	0.00	2.04
time (sec)	N/A	0.027	0.022	0.148	0.290	0.385	0.000	0.000	2.642

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	53	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.057	0.169	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	939	0	0	0	0	0	-1
N.S.	1	1.00	18.78	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	4.555	0.311	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	1712	0	0	0	0	0	-1
N.S.	1	1.00	34.24	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	13.079	0.386	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.127	0.250	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.099	0.195	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.077	0.122	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	452	0	0	0	0	0	-1
N.S.	1	1.00	6.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	2.541	0.164	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	1340	0	0	0	0	0	-1
N.S.	1	1.00	17.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	8.350	0.296	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	52	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.092	0.148	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	52	0	0	0	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.027	0.055	0.121	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	22	21	54	21	43
N.S.	1	1.00	1.00	1.06	1.22	1.17	3.00	1.17	2.39
time (sec)	N/A	0.017	0.022	0.135	0.285	0.375	0.144	0.467	2.835

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	36	3826	53	63	0	0	92
N.S.	1	1.00	0.84	88.98	1.23	1.47	0.00	0.00	2.14
time (sec)	N/A	0.038	0.093	0.466	0.291	0.374	0.000	0.000	3.439

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	9618	83	120	0	0	222
N.S.	1	1.00	0.91	139.39	1.20	1.74	0.00	0.00	3.22
time (sec)	N/A	0.044	0.316	0.361	0.284	0.393	0.000	0.000	7.629

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	212	0	0	0	0	0	-1
N.S.	1	1.00	3.37	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	1.501	0.145	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	79	0	0	0	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.679	0.098	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	186	0	0	0	0	0	-1
N.S.	1	1.00	2.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	1.254	0.126	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	106	0	0	0	0	0	-1
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.284	0.118	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	5.918	0.305	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	3.364	0.305	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	1.371	0.291	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	87	0	0	0	0	0	-1
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	3.773	0.247	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0	-1
N.S.	1	1.00	3.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	2.124	0.259	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [114] had the largest ratio of [25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	2	1.00	8	0.250
4	A	3	2	1.00	8	0.250
5	A	3	2	1.00	8	0.250
6	A	4	2	1.00	8	0.250
7	A	4	2	1.00	8	0.250
8	A	5	2	1.00	8	0.250
9	A	13	9	1.00	12	0.750
10	A	12	9	1.00	12	0.750
11	A	12	9	1.00	12	0.750
12	A	11	8	1.00	12	0.667
13	A	11	8	1.00	12	0.667
14	A	12	9	1.00	12	0.750
15	A	12	9	1.00	12	0.750
16	A	13	9	1.00	12	0.750
17	A	13	9	1.00	12	0.750
18	A	12	8	1.00	12	0.667
19	A	9	9	1.00	12	0.750
20	A	9	9	1.00	12	0.750
21	A	12	8	1.00	12	0.667
22	A	13	9	1.00	12	0.750
23	A	2	2	1.00	10	0.200
24	A	4	3	1.00	14	0.214
25	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
26	A	2	2	1.00	14	0.143
27	A	2	2	1.00	14	0.143
28	A	3	3	1.00	14	0.214
29	A	4	3	1.00	14	0.214
30	A	16	10	1.00	14	0.714
31	A	14	10	1.00	14	0.714
32	A	13	10	1.00	14	0.714
33	A	13	10	1.00	14	0.714
34	A	14	10	1.00	14	0.714
35	A	16	10	1.00	14	0.714
36	A	7	3	1.00	14	0.214
37	A	5	3	1.00	14	0.214
38	A	3	3	1.00	14	0.214
39	A	3	3	1.00	14	0.214
40	A	5	3	1.00	14	0.214
41	A	7	3	1.00	14	0.214
42	A	3	3	1.00	12	0.250
43	A	3	3	1.00	12	0.250
44	A	3	3	1.00	12	0.250
45	A	3	3	1.00	12	0.250
46	A	3	3	1.00	14	0.214
47	A	3	3	1.00	14	0.214
48	A	3	3	1.00	14	0.214
49	A	3	3	1.00	14	0.214
50	A	3	3	1.00	14	0.214
51	A	3	3	1.00	14	0.214
52	A	2	2	1.00	14	0.143
53	A	3	3	1.00	14	0.214
54	A	13	9	1.00	21	0.429
55	A	12	9	1.00	21	0.429
56	A	2	2	1.00	21	0.095
57	A	3	2	1.00	21	0.095
58	A	3	2	1.00	21	0.095
59	A	5	4	1.00	21	0.190
60	A	4	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	3	1.00	19	0.158
62	A	4	4	1.00	21	0.190
63	A	5	4	1.00	21	0.190
64	A	14	10	1.00	21	0.476
65	A	13	10	1.00	21	0.476
66	A	2	2	1.00	21	0.095
67	A	3	2	1.00	21	0.095
68	A	3	2	1.00	21	0.095
69	A	5	5	1.00	21	0.238
70	A	4	4	1.00	19	0.210
71	A	4	4	1.00	19	0.210
72	A	5	5	1.00	21	0.238
73	A	14	10	1.00	21	0.476
74	A	13	10	1.00	21	0.476
75	A	2	2	1.00	21	0.095
76	A	3	2	1.00	21	0.095
77	A	3	2	1.00	21	0.095
78	A	6	5	1.00	21	0.238
79	A	5	5	1.00	19	0.263
80	A	4	4	1.00	19	0.210
81	A	4	4	1.00	21	0.190
82	A	5	5	1.00	21	0.238
83	A	6	5	1.00	21	0.238
84	A	13	9	1.00	21	0.429
85	A	12	9	1.00	21	0.429
86	A	2	2	1.00	21	0.095
87	A	3	2	1.00	21	0.095
88	A	3	2	1.00	21	0.095
89	A	5	4	1.00	21	0.190
90	A	4	4	1.00	21	0.190
91	A	3	3	1.00	19	0.158
92	A	4	4	1.00	19	0.210
93	A	5	5	1.00	21	0.238
94	A	13	10	1.00	21	0.476
95	A	12	9	1.00	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	2	1.00	21	0.095
97	A	3	2	1.00	21	0.095
98	A	3	2	1.00	21	0.095
99	A	5	5	1.00	21	0.238
100	A	4	4	1.00	19	0.210
101	A	4	4	1.00	19	0.210
102	A	5	5	1.00	21	0.238
103	A	13	10	1.00	21	0.476
104	A	12	9	1.00	21	0.429
105	A	2	2	1.00	21	0.095
106	A	3	2	1.00	21	0.095
107	A	3	2	1.00	21	0.095
108	A	6	5	1.00	21	0.238
109	A	5	5	1.00	21	0.238
110	A	4	4	1.00	21	0.190
111	A	4	4	1.00	19	0.210
112	A	5	5	1.00	19	0.263
113	A	6	6	1.00	21	0.286
114	A	2	2	1.00	25	0.080
115	A	3	3	1.00	25	0.120
116	A	1	1	1.00	25	0.040
117	A	2	2	1.00	25	0.080
118	A	7	7	1.00	25	0.280
119	A	3	3	1.00	25	0.120
120	A	4	4	1.00	25	0.160
121	A	2	2	1.00	25	0.080
122	A	3	3	1.00	25	0.120
123	A	1	1	1.00	25	0.040
124	A	3	3	1.00	25	0.120
125	A	8	8	1.00	25	0.320
126	A	4	3	1.00	25	0.120
127	A	2	2	1.00	25	0.080
128	A	3	3	1.00	25	0.120
129	A	1	1	1.00	25	0.040
130	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	7	7	1.00	25	0.280
132	A	3	3	1.00	25	0.120
133	A	8	8	1.00	25	0.320
134	A	4	3	1.00	25	0.120
135	A	3	3	1.00	25	0.120
136	A	1	1	1.00	25	0.040
137	A	8	8	1.00	25	0.320
138	A	8	8	1.00	25	0.320
139	A	5	4	1.00	25	0.160
140	A	4	4	1.00	25	0.160
141	A	3	3	1.00	25	0.120
142	A	3	3	1.00	25	0.120
143	A	4	4	1.00	25	0.160
144	A	5	4	1.00	25	0.160
145	A	2	2	1.00	25	0.080
146	A	2	2	1.00	25	0.080
147	A	2	2	1.00	25	0.080
148	A	2	2	1.00	25	0.080
149	A	2	2	1.00	25	0.080
150	A	2	2	1.00	25	0.080
151	A	2	2	1.00	25	0.080
152	A	2	2	1.00	25	0.080
153	A	2	2	1.00	25	0.080
154	A	2	2	1.00	25	0.080
155	A	2	2	1.00	25	0.080
156	A	2	2	1.00	25	0.080
157	A	2	2	1.00	25	0.080
158	A	2	2	1.00	25	0.080
159	A	2	2	1.00	25	0.080
160	A	2	2	1.00	25	0.080
161	A	2	2	1.00	19	0.105
162	A	2	2	1.00	17	0.118
163	A	2	2	1.00	17	0.118
164	A	3	2	1.00	19	0.105
165	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	2	2	1.00	19	0.105
167	A	2	2	1.00	19	0.105
168	A	2	2	1.00	19	0.105
169	A	2	2	1.00	19	0.105
170	A	2	2	1.00	23	0.087
171	A	2	2	1.00	23	0.087
172	A	2	2	1.00	23	0.087
173	A	2	2	1.00	23	0.087
174	A	2	2	1.00	21	0.095
175	A	2	2	1.00	19	0.105
176	A	2	2	1.00	19	0.105
177	A	2	2	1.00	19	0.105
178	A	3	2	1.00	19	0.105
179	A	3	2	1.00	19	0.105
180	A	2	2	1.00	19	0.105
181	A	2	2	1.00	17	0.118
182	A	2	2	1.00	17	0.118
183	A	2	2	1.00	19	0.105
184	A	2	2	1.00	19	0.105
185	A	2	2	1.00	23	0.087
186	A	2	2	1.00	23	0.087
187	A	2	2	1.00	23	0.087
188	A	2	2	1.00	23	0.087
189	A	2	2	1.00	21	0.095
190	A	3	3	1.00	21	0.143
191	A	14	10	1.00	21	0.476
192	A	13	10	1.00	21	0.476
193	A	13	10	1.00	21	0.476
194	A	12	9	1.00	19	0.474
195	A	11	8	1.00	12	0.667
196	A	13	10	1.00	19	0.526
197	A	13	10	1.00	21	0.476
198	A	14	10	1.00	21	0.476
199	A	14	10	1.00	21	0.476
200	A	13	10	1.00	21	0.476

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	13	10	1.00	21	0.476
202	A	12	9	1.00	21	0.429
203	A	12	9	1.00	19	0.474
204	A	12	9	1.00	12	0.750
205	A	13	10	1.00	19	0.526
206	A	14	10	1.00	21	0.476
207	A	14	10	1.00	21	0.476
208	A	13	10	1.00	21	0.476
209	A	13	10	1.00	19	0.526
210	A	11	8	1.00	12	0.667
211	A	12	9	1.00	19	0.474
212	A	13	10	1.00	21	0.476
213	A	13	10	1.00	21	0.476
214	A	14	10	1.00	21	0.476
215	A	13	10	1.00	19	0.526
216	A	12	9	1.00	12	0.750
217	A	12	9	1.00	19	0.474
218	A	12	9	1.00	21	0.429
219	A	13	10	1.00	21	0.476
220	A	13	10	1.00	21	0.476
221	A	14	10	1.00	21	0.476
222	A	3	3	1.00	17	0.176
223	A	3	3	1.00	19	0.158
224	A	3	3	1.00	19	0.158
225	A	3	3	1.00	21	0.143
226	A	3	2	1.00	21	0.095
227	A	3	2	1.00	21	0.095
228	A	2	2	1.00	21	0.095
229	A	11	8	1.00	12	0.667
230	A	12	9	1.00	21	0.429
231	A	5	4	1.00	21	0.190
232	A	4	4	1.00	19	0.210
233	A	3	3	1.00	19	0.158
234	A	4	4	1.00	21	0.190
235	A	5	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	2	1.00	21	0.095
237	A	3	2	1.00	21	0.095
238	A	2	2	1.00	21	0.095
239	A	12	9	1.00	12	0.750
240	A	12	9	1.00	21	0.429
241	A	6	5	1.00	21	0.238
242	A	5	5	1.00	21	0.238
243	A	4	4	1.00	19	0.210
244	A	4	4	1.00	19	0.210
245	A	5	5	1.00	21	0.238
246	A	6	5	1.00	21	0.238
247	A	3	2	1.00	21	0.095
248	A	3	2	1.00	21	0.095
249	A	2	2	1.00	21	0.095
250	A	12	9	1.00	12	0.750
251	A	12	9	1.00	21	0.429
252	A	13	10	1.00	21	0.476
253	A	5	4	1.00	21	0.190
254	A	4	4	1.00	21	0.190
255	A	3	3	1.00	19	0.158
256	A	4	4	1.00	19	0.210
257	A	5	4	1.00	21	0.190
258	A	3	2	1.00	21	0.095
259	A	3	2	1.00	21	0.095
260	A	2	2	1.00	21	0.095
261	A	12	9	1.00	12	0.750
262	A	13	10	1.00	21	0.476
263	A	6	5	1.00	21	0.238
264	A	5	5	1.00	21	0.238
265	A	4	4	1.00	19	0.210
266	A	4	4	1.00	19	0.210
267	A	5	5	1.00	21	0.238
268	A	6	5	1.00	21	0.238
269	A	4	4	1.00	19	0.210
270	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	2	2	1.00	19	0.105
272	A	2	2	1.00	19	0.105
273	A	2	2	1.00	19	0.105
274	A	2	2	1.00	19	0.105
275	A	2	2	1.00	19	0.105
276	A	2	2	1.00	19	0.105
277	A	2	2	1.00	19	0.105
278	A	2	2	1.00	19	0.105
279	A	2	2	1.00	19	0.105
280	A	2	2	1.00	19	0.105
281	A	1	1	1.00	21	0.048
282	A	1	1	1.00	21	0.048
283	A	1	1	1.00	21	0.048
284	A	1	1	1.00	21	0.048
285	A	1	1	1.00	21	0.048
286	A	1	1	1.00	21	0.048
287	A	1	1	1.00	21	0.048
288	A	1	1	1.00	21	0.048
289	A	1	1	1.00	21	0.048
290	A	1	1	1.00	21	0.048
291	A	7	7	1.00	25	0.280
292	A	4	4	1.00	25	0.160
293	A	6	6	1.00	25	0.240
294	A	3	3	1.00	25	0.120
295	A	1	1	1.00	25	0.040
296	A	4	4	1.00	25	0.160
297	A	2	2	1.00	25	0.080
298	A	5	4	1.00	25	0.160
299	A	5	5	1.00	25	0.200
300	A	7	7	1.00	25	0.280
301	A	4	4	1.00	25	0.160
302	A	7	7	1.00	25	0.280
303	A	4	4	1.00	25	0.160
304	A	1	1	1.00	25	0.040
305	A	5	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	3	3	1.00	25	0.120
307	A	8	8	1.00	25	0.320
308	A	5	5	1.00	25	0.200
309	A	7	7	1.00	25	0.280
310	A	4	4	1.00	25	0.160
311	A	7	7	1.00	25	0.280
312	A	4	4	1.00	25	0.160
313	A	1	1	1.00	25	0.040
314	A	5	5	1.00	25	0.200
315	A	7	7	1.00	25	0.280
316	A	4	4	1.00	25	0.160
317	A	6	6	1.00	25	0.240
318	A	3	3	1.00	25	0.120
319	A	1	1	1.00	25	0.040
320	A	4	4	1.00	25	0.160
321	A	2	2	1.00	25	0.080
322	A	7	7	1.00	25	0.280
323	A	4	4	1.00	25	0.160
324	A	1	1	1.00	25	0.040
325	A	4	4	1.00	25	0.160
326	A	2	2	0.93	25	0.080
327	A	5	5	1.00	25	0.200
328	A	7	7	1.00	25	0.280
329	A	4	4	1.00	25	0.160
330	A	1	1	1.00	25	0.040
331	A	4	4	1.00	25	0.160
332	A	2	2	1.00	25	0.080
333	A	5	5	1.00	25	0.200
334	A	3	3	1.00	25	0.120
335	A	1	1	1.00	25	0.040
336	A	1	1	1.00	25	0.040
337	A	1	1	1.00	25	0.040
338	A	1	1	1.00	25	0.040
339	A	1	1	1.00	25	0.040
340	A	1	1	1.00	25	0.040

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	1	1	1.00	25	0.040
342	A	1	1	1.00	25	0.040
343	A	1	1	1.00	25	0.040
344	A	1	1	1.00	25	0.040
345	A	1	1	1.00	25	0.040
346	A	1	1	1.00	25	0.040
347	A	1	1	1.00	25	0.040
348	A	1	1	1.00	25	0.040
349	A	1	1	1.00	25	0.040
350	A	1	1	1.00	25	0.040
351	A	3	2	1.00	19	0.105
352	A	3	2	1.00	19	0.105
353	A	2	2	1.00	17	0.118
354	A	2	2	1.00	17	0.118
355	A	2	2	1.00	19	0.105
356	A	2	2	1.00	19	0.105
357	A	1	1	1.00	19	0.053
358	A	1	1	1.00	19	0.053
359	A	1	1	1.00	19	0.053
360	A	1	1	1.00	19	0.053
361	A	1	1	1.00	19	0.053
362	A	1	1	1.00	21	0.048
363	A	3	2	1.00	19	0.105
364	A	3	2	1.00	19	0.105
365	A	2	2	1.00	19	0.105
366	A	2	2	1.00	10	0.200
367	A	2	2	1.00	19	0.105
368	A	2	2	1.00	19	0.105
369	A	1	1	1.00	19	0.053
370	A	1	1	1.00	19	0.053
371	A	1	1	1.00	17	0.059
372	A	1	1	1.00	17	0.059
373	A	1	1	1.00	19	0.053
374	A	2	2	1.00	19	0.105
375	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	2	2	1.00	17	0.118
377	A	3	2	1.00	19	0.105
378	A	3	2	1.00	19	0.105
379	A	1	1	1.00	19	0.053
380	A	1	1	1.00	19	0.053
381	A	1	1	1.00	19	0.053
382	A	1	1	1.00	19	0.053
383	A	3	3	1.00	23	0.130
384	A	3	3	1.00	23	0.130
385	A	3	3	1.00	23	0.130
386	A	3	3	1.00	23	0.130
387	A	3	3	1.00	21	0.143

Chapter 3

Listing of integrals

Local contents

3.1 $\int \tan(c+dx)\,dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(c+dx))}{d}$$

```
[Out] -ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{3556\}$

$$-\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x],x]
```

```
[Out] -(Log[Cos[c + d*x]]/d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(c+dx)\,dx = -\frac{\log(\cos(c+dx))}{d}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x],x]
```

```
[Out] -(Log[Cos[c + d*x]]/d)
```

Maple [A]
time = 0.03, size = 17, normalized size = 1.42

method	result	size
derivativedivides	$\frac{\ln\left(1+\tan^{2}(dx+c)\right)}{2d}$	17
default	$\frac{\ln\left(1+\tan^{2}(dx+c)\right)}{2d}$	17
norman	$\frac{\ln\left(1+\tan^{2}(dx+c)\right)}{2d}$	17
risch	$ix+\frac{2ic}{d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/2/d*ln(1+tan(d*x+c)^2)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.92

$$\frac{\log\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c),x, algorithm="maxima")
[Out] log(sec(d*x + c))/d
```

Fricas [A]
time = 0.41, size = 18, normalized size = 1.50

$$-\frac{\log\left(\frac{1}{\tan\left(dx+c\right)^{2}+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c),x, algorithm="fricas")
[Out] -1/2*log(1/(tan(d*x + c)^2 + 1))/d
```

Sympy [A]
time = 0.04, size = 19, normalized size = 1.58

$$\begin{cases}\frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d} & \text{for}\ d\neq 0\\ x\tan\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c),x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*tan(c), True))
```

Giac [A]
time = 0.42, size = 13, normalized size = 1.08

$$-\frac{\log\left(\left|\cos\left(dx+c\right)\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c),x, algorithm="giac")
[Out] -log(abs(cos(d*x + c)))/d
```

Mupad [B]
time = 2.57, size = 16, normalized size = 1.33

$$\frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x),x)
[Out] log(tan(c + d*x)^2 + 1)/(2*d)
```

3.2 $\int \tan^2(c+dx)\,dx$

Optimal. Leaf size=14

$$-x+\frac{\tan(c+dx)}{d}$$

```
[Out] -x+tan(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 8}

$$\frac{\tan(c+dx)}{d}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2,x]
```

```
[Out] -x + Tan[c + d*x]/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^2(c+dx)\,dx &= \frac{\tan(c+dx)}{d}-\int 1\,dx\\ &= -x+\frac{\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.64

$$-\frac{\text{ArcTan}(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2,x]
[Out] -(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d
```

Maple [A]
time = 0.01, size = 21, normalized size = 1.50

method	result	size
norman	$-x+\frac{\tan(dx+c)}{d}$	15
derivativedivides	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	21
default	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	21
risch	$-x+\frac{2i}{d\left(e^{2i(dx+c)}+1\right)}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(tan(d*x+c)-arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 18, normalized size = 1.29

$$-\frac{dx+c-\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2,x, algorithm="maxima")
[Out] -(d*x + c - tan(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 17, normalized size = 1.21

$$-\frac{dx-\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2,x, algorithm="fricas")
[Out] -(d*x - tan(d*x + c))/d
```

Sympy [A]
time = 0.05, size = 15, normalized size = 1.07

$$\begin{cases} -x+\frac{\tan(c+dx)}{d} & \text{for } d\neq 0 \\ x\tan^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2,x)
```

```
[Out] Piecewise((-x + tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(14) = 28.
time = 0.64, size = 226, normalized size = 16.14

$$\frac{\pi - 4\,dx\tan\left(dx\right)\tan\left(c\right) - \pi \mathrm{sgn}\left(2\,\tan\left(dx\right)^{2}\tan\left(c\right) + 2\,\tan\left(dx\right)\tan\left(c\right)^{2} - 2\,\tan\left(dx\right) - 2\,\tan\left(c\right)\right)\tan\left(dx\right)\tan\left(c\right) - \pi\tan\left(dx\right)\tan\left(c\right) + 2\,\arctan\left(\frac{\tan(dx)\tan(c) - 1}{\tan(dx) + \tan(c)}\right)\tan\left(dx\right)\tan\left(c\right) + 2\,\arctan\left(\frac{\tan(dx) + \tan(c)}{\tan(dx)\tan(c) - 1}\right)\tan\left(dx\right)\tan\left(c\right) + 4\,dx + \pi \mathrm{sgn}\left(2\,\tan\left(dx\right)^{2}\tan\left(c\right) + 2\,\tan\left(dx\right)\tan\left(c\right)^{2} - 2\,\tan\left(dx\right) - 2\,\tan\left(c\right)\right) - 2\,\arctan\left(\frac{\tan(dx)\tan(c) - 1}{\tan(dx) + \tan(c)}\right) - 2\,\arctan\left(\frac{\tan(dx) + \tan(c)}{\tan(dx)\tan(c) - 1}\right) - 4\,\tan\left(dx\right) - 4\,\tan\left(c\right)}{4\,\left(d\tan\left(dx\right)\tan\left(c\right) - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(pi - 4*d*x*tan(d*x)*tan(c) - pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - pi*tan(d*x)*tan(c) + 2*a
rctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 2*arctan
((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 4*d*x + pi*sg
n(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 2*ar
ctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 2*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1)) - 4*tan(d*x) - 4*tan(c))/(d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 2.51, size = 14, normalized size = 1.00

$$\frac{\tan\left(c+d\,x\right)}{d}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2,x)
```

```
[Out] tan(c + d*x)/d - x
```

3.3 $\int \tan^3(c+dx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\cos(c+dx))}{d}+\frac{\tan^2(c+dx)}{2d}$$

```
[Out] ln(cos(d*x+c))/d+1/2*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 3556}

$$\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3,x]
```

```
[Out] Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^3(c+dx)\,dx &= \frac{\tan^2(c+dx)}{2d}-\int \tan(c+dx)\,dx\\ &= \frac{\log(\cos(c+dx))}{d}+\frac{\tan^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 0.93

$$\frac{2\log(\cos(c+dx))+\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3,x]
```

```
[Out] (2*Log[Cos[c + d*x]] + Tan[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.02, size = 29, normalized size = 1.07

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}}{d}$	29
default	$\frac{\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}}{d}$	29
norman	$\frac{\tan^2(dx+c)}{2d}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2d}$	31
risch	$-ix-\frac{2ic}{d}+\frac{2\,e^{2i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)^2))
```

Maxima [A]
time = 0.27, size = 31, normalized size = 1.15

$$-\frac{\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.37, size = 27, normalized size = 1.00

$$\frac{\tan\left(dx+c\right)^2+\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(tan(d*x + c)^2 + log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A]
time = 0.07, size = 32, normalized size = 1.19

$$\begin{cases} -\frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{\tan^{2}\left(c+dx\right)}{2d} & \text{for}\ d\neq 0 \\ x\tan^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3,x)
```

```
[Out] Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**2/(2*d), Ne(d, 0
)), (x*tan(c)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(25) = 50.
time = 0.68, size = 246, normalized size = 9.11

$$\frac{\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-2\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)\tan\left(dx\right)\tan\left(c\right)+\tan\left(dx\right)^{2}+\tan\left(c\right)^{2}+\log\left(\frac{4\left(\tan(dx)^{4}\tan(c)^{2}-2\tan(dx)^{3}\tan(c)+\tan(dx)^{2}\tan(c)^{2}+\tan(dx)^{2}-2\tan(dx)\tan(c)+1\right)}{\tan(c)^{2}+1}\right)+1}{2\left(d\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-2d\tan\left(dx\right)\tan\left(c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2
+ tan(d*x)^2*tan(c)^2 - 2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
+ tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))
*tan(d*x)*tan(c) + tan(d*x)^2 + tan(c)^2 + log(4*(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
)/(tan(c)^2 + 1)) + 1)/(d*tan(d*x)^2*tan(c)^2 - 2*d*tan(d*x)*tan(c) + d)
```

Mupad [B]
time = 2.51, size = 30, normalized size = 1.11

$$\frac{{\mathrm{tan}\left(c+d\,x\right)}^2}{2\,d}-\frac{\ln\left({\mathrm{tan}\left(c+d\,x\right)}^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3,x)
```

```
[Out] tan(c + d*x)^2/(2*d) - log(tan(c + d*x)^2 + 1)/(2*d)
```

3.4 $\int \tan^4(c+dx)\,dx$

Optimal. Leaf size=28

$$x-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}$$

```
[Out] x-tan(d*x+c)/d+1/3*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4,x]
```

```
[Out] x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^4(c+dx)\,dx &= \frac{\tan^3(c+dx)}{3d}-\int \tan^2(c+dx)\,dx\\ &= -\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}+\int 1\,dx\\ &= x-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.36

$$\frac{\text{ArcTan}(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4,x]
```

```
[Out] ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d)
```

Maple [A]
time = 0.02, size = 31, normalized size = 1.11

method	result	size
norman	$x-\frac{\tan(dx+c)}{d}+\frac{\tan^3(dx+c)}{3d}$	27
derivativedivides	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	31
default	$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	31
risch	$x-\frac{4i\left(3\,\mathrm{e}^{4i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}+2\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*tan(d*x+c)^3-tan(d*x+c)+arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 29, normalized size = 1.04

$$\frac{\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 26, normalized size = 0.93

$$\frac{\tan\left(dx+c\right)^3+3\,dx-3\,\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(tan(d*x + c)^3 + 3*d*x - 3*tan(d*x + c))/d
```

Sympy [A]
time = 0.08, size = 27, normalized size = 0.96

$$\begin{cases} x + \frac{\tan^3(c+dx)}{3d} - \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x \tan^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4,x)
```

```
[Out] Piecewise((x + tan(c + d*x)**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)
**4, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 585 vs. 2(26) = 52.
time = 0.96, size = 585, normalized size = 20.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*pi + 12*d*x*tan(d*x)^3*tan(c)^3 - 3*pi*sgn(2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 - 3*pi*tan(d
*x)^3*tan(c)^3 + 6*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*
x)^3*tan(c)^3 + 6*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x
)^3*tan(c)^3 - 36*d*x*tan(d*x)^2*tan(c)^2 + 9*pi*sgn(2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 9*pi*tan
(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan
(d*x)^2*tan(c)^2 - 18*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan
(d*x)^2*tan(c)^2 + 12*tan(d*x)^3*tan(c)^2 + 12*tan(d*x)^2*tan(c)^3 + 36*d*x
*tan(d*x)*tan(c) - 9*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*t
an(d*x) - 2*tan(c))*tan(d*x)*tan(c) - 4*tan(d*x)^3 - 9*pi*tan(d*x)*tan(c) +
 18*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 18*
arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) - 36*tan(
d*x)^2*tan(c) - 36*tan(d*x)*tan(c)^2 - 4*tan(c)^3 - 12*d*x + 3*pi*sgn(2*tan
(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 6*arctan((t
an(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 6*arctan((tan(d*x) + tan(c))/(ta
n(d*x)*tan(c) - 1)) + 12*tan(d*x) + 12*tan(c))/(d*tan(d*x)^3*tan(c)^3 - 3*d
*tan(d*x)^2*tan(c)^2 + 3*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 2.50, size = 24, normalized size = 0.86

$$x - \frac{\tan(c + d\,x) - \frac{\tan(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4,x)
[Out] x - (tan(c + d*x) - tan(c + d*x)^3/3)/d
```

3.5 $\int \tan^5(c+dx)\,dx$

Optimal. Leaf size=43

$$-\frac{\log(\cos(c+dx))}{d}-\frac{\tan^2(c+dx)}{2d}+\frac{\tan^4(c+dx)}{4d}$$

```
[Out] -ln(cos(d*x+c))/d-1/2*tan(d*x+c)^2/d+1/4*tan(d*x+c)^4/d
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 3556}

$$\frac{\tan^4(c+dx)}{4d}-\frac{\tan^2(c+dx)}{2d}-\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5,x]
```

```
[Out] -(Log[Cos[c + d*x]]/d) - Tan[c + d*x]^2/(2*d) + Tan[c + d*x]^4/(4*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^5(c+dx)\,dx&=\frac{\tan^4(c+dx)}{4d}-\int \tan^3(c+dx)\,dx\\&=-\frac{\tan^2(c+dx)}{2d}+\frac{\tan^4(c+dx)}{4d}+\int \tan(c+dx)\,dx\\&=-\frac{\log(\cos(c+dx))}{d}-\frac{\tan^2(c+dx)}{2d}+\frac{\tan^4(c+dx)}{4d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 37, normalized size = 0.86

$$-\frac{4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5,x]
[Out] -1/4*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4)/d
```

Maple [A]
time = 0.03, size = 39, normalized size = 0.91

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^{4}(dx+c)\right)}{4}-\frac{\left(\tan^{2}(dx+c)\right)}{2}+\frac{\ln\left(1+\tan^{2}(dx+c)\right)}{2}}{d}$	39
default	$\frac{\frac{\left(\tan^{4}(dx+c)\right)}{4}-\frac{\left(\tan^{2}(dx+c)\right)}{2}+\frac{\ln\left(1+\tan^{2}(dx+c)\right)}{2}}{d}$	39
norman	$-\frac{\tan^{2}(dx+c)}{2d}+\frac{\tan^{4}(dx+c)}{4d}+\frac{\ln(1+\tan^{2}(dx+c))}{2d}$	44
risch	$ix+\frac{2ic}{d}-\frac{4\left(e^{6i(dx+c)}+e^{4i(dx+c)}+e^{2i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^{4}}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2+1/2*ln(1+tan(d*x+c)^2))
```

Maxima [A]
time = 0.27, size = 54, normalized size = 1.26

$$\frac{\frac{4\,\sin(dx+c)^2-3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-2\,\log\left(\sin\left(dx+c\right)^2-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5,x, algorithm="maxima")
[Out] 1/4*((4*sin(d*x + c)^2 - 3)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 2*log
(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.37, size = 39, normalized size = 0.91

$$\frac{\tan\left(dx+c\right)^4-2\,\tan\left(dx+c\right)^2-2\,\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5,x, algorithm="fricas")
[Out] 1/4*(tan(d*x + c)^4 - 2*tan(d*x + c)^2 - 2*log(1/(tan(d*x + c)^2 + 1)))/d
```

Sympy [A]
time = 0.10, size = 44, normalized size = 1.02

$$\begin{cases} \frac{\log\left(\tan^2\left(c+dx\right)+1\right)}{2d} + \frac{\tan^4\left(c+dx\right)}{4d} - \frac{\tan^2\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\tan^5\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5,x)
```

```
[Out] Piecewise((log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**4/(4*d) - tan(c +
 d*x)**2/(2*d), Ne(d, 0)), (x*tan(c)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(39) = 78.
time = 1.50, size = 512, normalized size = 11.91

$$-\frac{2\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan(dx)^4\tan(c)^4+3\tan(dx)^4\tan(c)^4-8\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan(dx)^3\tan(c)^3+2\tan(dx)^4\tan(c)^2-8\tan(dx)^3\tan(c)^3+2\tan(dx)^2\tan(c)^4+12\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan(dx)^2\tan(c)^2-\tan(dx)^4-8\tan(dx)^3\tan(c)+4\tan(dx)^2\tan(c)^2-8\tan(dx)\tan(c)^3-\tan(c)^4-8\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)\tan(dx)\tan(c)+2\tan(dx)^2-8\tan(dx)\tan(c)+2\tan(c)^2+2\log\left(\frac{4\left(\tan(dx)^4\tan(c)^2-2\tan(dx)^3\tan(c)+\tan(dx)^2\tan(c)^2+\tan(dx)^2-2\tan(dx)\tan(c)+1\right)}{\tan(c)^2+1}\right)+3}{4\left(d\tan(dx)^4\tan(c)^4-4d\tan(dx)^3\tan(c)^3+6d\tan(dx)^2\tan(c)^2-4d\tan(dx)\tan(c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] -1/4*(2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^4*tan(c)
^4 + 3*tan(d*x)^4*tan(c)^4 - 8*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
+ 1))*tan(d*x)^3*tan(c)^3 + 2*tan(d*x)^4*tan(c)^2 - 8*tan(d*x)^3*tan(c)^3 +
 2*tan(d*x)^2*tan(c)^4 + 12*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
))*tan(d*x)^2*tan(c)^2 - tan(d*x)^4 - 8*tan(d*x)^3*tan(c) + 4*tan(d*x)^2*ta
n(c)^2 - 8*tan(d*x)*tan(c)^3 - tan(c)^4 - 8*log(4*(tan(d*x)^4*tan(c)^2 - 2*
tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) +
1)/(tan(c)^2 + 1))*tan(d*x)*tan(c) + 2*tan(d*x)^2 - 8*tan(d*x)*tan(c) + 2*t
an(c)^2 + 2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 3)/(d*tan(d
*x)^4*tan(c)^4 - 4*d*tan(d*x)^3*tan(c)^3 + 6*d*tan(d*x)^2*tan(c)^2 - 4*d*ta
n(d*x)*tan(c) + d)
```

Mupad [B]
time = 2.50, size = 38, normalized size = 0.88

$$\frac{\frac{\ln\left({\tan\left(c+d\,x\right)}^2+1\right)}{2}-\frac{{\tan\left(c+d\,x\right)}^2}{2}+\frac{{\tan\left(c+d\,x\right)}^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5,x)
```

```
[Out] (log(tan(c + d*x)^2 + 1)/2 - tan(c + d*x)^2/2 + tan(c + d*x)^4/4)/d
```

3.6 $\int \tan^6(c+dx)\,dx$

Optimal. Leaf size=44

$$-x+\frac{\tan(c+dx)}{d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}$$

```
[Out] -x+tan(d*x+c)/d-1/3*tan(d*x+c)^3/d+1/5*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\frac{\tan^5(c+dx)}{5d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan(c+dx)}{d}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6,x]
```

```
[Out] -x + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c + d*x]^5/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^6(c+dx)\,dx &= \frac{\tan^5(c+dx)}{5d}-\int \tan^4(c+dx)\,dx\\ &= -\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}+\int \tan^2(c+dx)\,dx\\ &= \frac{\tan(c+dx)}{d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}-\int 1\,dx\\ &= -x+\frac{\tan(c+dx)}{d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.20

$$-\frac{\operatorname{ArcTan}(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}-\frac{\tan^3(c+dx)}{3d}+\frac{\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6,x]
```

```
[Out] -(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d - Tan[c + d*x]^3/(3*d) + Tan[c +
 d*x]^5/(5*d)
```

Maple [A]
time = 0.02, size = 41, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^5(dx+c)\right)}{5}-\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	41
default	$\frac{\frac{\left(\tan^5(dx+c)\right)}{5}-\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	41
norman	$-x+\frac{\tan(dx+c)}{d}-\frac{\tan^3(dx+c)}{3d}+\frac{\tan^5(dx+c)}{5d}$	41
risch	$-x+\frac{2i\left(45\,e^{8i(dx+c)}+90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}+70\,e^{2i(dx+c)}+23\right)}{15d\left(e^{2i(dx+c)}+1\right)^5}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-arctan(tan(d*x+c)))
```

Maxima [A]
time = 0.49, size = 41, normalized size = 0.93

$$\frac{3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx-15\,c+15\,\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c)
)/d
```

Fricas [A]
time = 0.37, size = 38, normalized size = 0.86

$$\frac{3\,\tan\left(dx+c\right)^5-5\,\tan\left(dx+c\right)^3-15\,dx+15\,\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x + 15*tan(d*x + c))/d
```

Sympy [A]
time = 0.13, size = 39, normalized size = 0.89

$$\begin{cases} -x + \frac{\tan^{5}(c+dx)}{5d} - \frac{\tan^{3}(c+dx)}{3d} + \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^{6}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6,x)
```

```
[Out] Piecewise((-x + tan(c + d*x)**5/(5*d) - tan(c + d*x)**3/(3*d) + tan(c + d*x
)/d, Ne(d, 0)), (x*tan(c)**6, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 989 vs. 2(40) = 80.
time = 2.26, size = 989, normalized size = 22.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/60*(15*pi - 60*d*x*tan(d*x)^5*tan(c)^5 - 15*pi*sgn(2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 - 15*pi*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*ta
n(d*x)^5*tan(c)^5 + 300*d*x*tan(d*x)^4*tan(c)^4 + 75*pi*sgn(2*tan(d*x)^2*ta
n(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 7
5*pi*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan
(c)))*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c)
 - 1))*tan(d*x)^4*tan(c)^4 - 60*tan(d*x)^5*tan(c)^4 - 60*tan(d*x)^4*tan(c)^
5 - 600*d*x*tan(d*x)^3*tan(c)^3 - 150*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*
x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 + 20*tan(d*x)^5*ta
n(c)^2 - 150*pi*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x)*tan(c) - 1)/(tan
(d*x) + tan(c)))*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^3*tan(c)^3 + 300*tan(d*x)^4*tan(c)^3 + 300*tan(d
*x)^3*tan(c)^4 + 20*tan(d*x)^2*tan(c)^5 + 600*d*x*tan(d*x)^2*tan(c)^2 + 150
*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*
tan(d*x)^2*tan(c)^2 - 12*tan(d*x)^5 - 100*tan(d*x)^4*tan(c) + 150*pi*tan(d*
x)^2*tan(c)^2 - 300*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d
```

```
*x)^2*tan(c)^2 - 300*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(
d*x)^2*tan(c)^2 - 600*tan(d*x)^3*tan(c)^2 - 600*tan(d*x)^2*tan(c)^3 - 100*t
an(d*x)*tan(c)^4 - 12*tan(c)^5 - 300*d*x*tan(d*x)*tan(c) - 75*pi*sgn(2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c
) + 20*tan(d*x)^3 - 75*pi*tan(d*x)*tan(c) + 150*arctan((tan(d*x)*tan(c) - 1
)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 150*arctan((tan(d*x) + tan(c))/(ta
n(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 300*tan(d*x)^2*tan(c) + 300*tan(d*x)*
tan(c)^2 + 20*tan(c)^3 + 60*d*x + 15*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x
)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 30*arctan((tan(d*x)*tan(c) - 1)/(tan(
d*x) + tan(c))) - 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) - 60
*tan(d*x) - 60*tan(c))/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)^4*tan(c)^4 + 1
0*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan(d*x)*tan(c) -
d)
```

Mupad [B]
time = 2.53, size = 35, normalized size = 0.80

$$\frac{\frac{\tan(c+d\,x)^5}{5}-\frac{\tan(c+d\,x)^3}{3}+\tan(c+d\,x)}{d}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6,x)
```

```
[Out] (tan(c + d*x) - tan(c + d*x)^3/3 + tan(c + d*x)^5/5)/d - x
```

3.7 $\int \tan^7(c+dx)\,dx$

Optimal. Leaf size=57

$$\frac{\log(\cos(c+dx))}{d}+\frac{\tan^2(c+dx)}{2d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^6(c+dx)}{6d}$$

```
[Out] ln(cos(d*x+c))/d+1/2*tan(d*x+c)^2/d-1/4*tan(d*x+c)^4/d+1/6*tan(d*x+c)^6/d
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 3556}

$$\frac{\tan^6(c+dx)}{6d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^2(c+dx)}{2d}+\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7,x]
```

```
[Out] Log[Cos[c + d*x]]/d + Tan[c + d*x]^2/(2*d) - Tan[c + d*x]^4/(4*d) + Tan[c +
 d*x]^6/(6*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^7(c+dx)\,dx &= \frac{\tan^6(c+dx)}{6d}-\int \tan^5(c+dx)\,dx \\
&= -\frac{\tan^4(c+dx)}{4d}+\frac{\tan^6(c+dx)}{6d}+\int \tan^3(c+dx)\,dx \\
&= \frac{\tan^2(c+dx)}{2d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^6(c+dx)}{6d}-\int \tan(c+dx)\,dx \\
&= \frac{\log(\cos(c+dx))}{d}+\frac{\tan^2(c+dx)}{2d}-\frac{\tan^4(c+dx)}{4d}+\frac{\tan^6(c+dx)}{6d}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 47, normalized size = 0.82

$$\frac{12\log(\cos(c+dx))+6\tan^2(c+dx)-3\tan^4(c+dx)+2\tan^6(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7,x]
```

```
[Out] (12*Log[Cos[c + d*x]] + 6*Tan[c + d*x]^2 - 3*Tan[c + d*x]^4 + 2*Tan[c + d*x
]^6)/(12*d)
```

Maple [A]
time = 0.04, size = 49, normalized size = 0.86

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^6(dx+c)\right)}{6}-\frac{\left(\tan^4(dx+c)\right)}{4}+\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}}{d}$	49
default	$\frac{\frac{\left(\tan^6(dx+c)\right)}{6}-\frac{\left(\tan^4(dx+c)\right)}{4}+\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}}{d}$	49
norman	$\frac{\tan^2(dx+c)}{2d}-\frac{\tan^4(dx+c)}{4d}+\frac{\tan^6(dx+c)}{6d}-\frac{\ln(1+\tan^2(dx+c))}{2d}$	57
risch	$-ix-\frac{2ic}{d}+\frac{6\,e^{10i(dx+c)}+12\,e^{8i(dx+c)}+\frac{68\,e^{6i(dx+c)}}{3}+12\,e^{4i(dx+c)}+6\,e^{2i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^6}+\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*tan(d*x+c)^6-1/4*tan(d*x+c)^4+1/2*tan(d*x+c)^2-1/2*ln(1+tan(d*x+c)
^2))
```

Maxima [A]
time = 0.27, size = 74, normalized size = 1.30

$$-\frac{\frac{18\,\sin(dx+c)^4-27\,\sin(dx+c)^2+11}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-6\,\log\left(\sin\left(dx+c\right)^2-1\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] -1/12*((18*sin(d*x + c)^4 - 27*sin(d*x + c)^2 + 11)/(sin(d*x + c)^6 - 3*sin
(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 6*log(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.36, size = 51, normalized size = 0.89

$$\frac{2\,\tan\left(dx+c\right)^6-3\,\tan\left(dx+c\right)^4+6\,\tan\left(dx+c\right)^2+6\,\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(2*tan(d*x + c)^6 - 3*tan(d*x + c)^4 + 6*tan(d*x + c)^2 + 6*log(1/(tan
(d*x + c)^2 + 1)))/d
```

Sympy [A]
time = 0.16, size = 56, normalized size = 0.98

$$\begin{cases} -\frac{\log\left(\tan^{2}\left(c+dx\right)+1\right)}{2d}+\frac{\tan^{6}\left(c+dx\right)}{6d}-\frac{\tan^{4}\left(c+dx\right)}{4d}+\frac{\tan^{2}\left(c+dx\right)}{2d} & \text{for } d \neq 0 \\ x\tan^{7}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7,x)
```

```
[Out] Piecewise((-log(tan(c + d*x)**2 + 1)/(2*d) + tan(c + d*x)**6/(6*d) - tan(c
+ d*x)**4/(4*d) + tan(c + d*x)**2/(2*d), Ne(d, 0)), (x*tan(c)**7, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 810 vs. 2(51) = 102.
time = 3.25, size = 810, normalized size = 14.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/12*(6*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^6*tan(c)
^6 + 11*tan(d*x)^6*tan(c)^6 - 36*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^
2 + 1))*tan(d*x)^5*tan(c)^5 + 6*tan(d*x)^6*tan(c)^4 - 54*tan(d*x)^5*tan(c)^
5 + 6*tan(d*x)^4*tan(c)^6 + 90*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2
+ 1))*tan(d*x)^4*tan(c)^4 - 3*tan(d*x)^6*tan(c)^2 - 36*tan(d*x)^5*tan(c)^3
+ 99*tan(d*x)^4*tan(c)^4 - 36*tan(d*x)^3*tan(c)^5 - 3*tan(d*x)^2*tan(c)^6 -
 120*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^3*tan(c)^3
+ 2*tan(d*x)^6 + 18*tan(d*x)^5*tan(c) + 90*tan(d*x)^4*tan(c)^2 - 72*tan(d*x
)^3*tan(c)^3 + 90*tan(d*x)^2*tan(c)^4 + 18*tan(d*x)*tan(c)^5 + 2*tan(c)^6 +
 90*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 -
 3*tan(d*x)^4 - 36*tan(d*x)^3*tan(c) + 99*tan(d*x)^2*tan(c)^2 - 36*tan(d*x)
*tan(c)^3 - 3*tan(c)^4 - 36*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c
```

```
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1
))*tan(d*x)*tan(c) + 6*tan(d*x)^2 - 54*tan(d*x)*tan(c) + 6*tan(c)^2 + 6*log
(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1)) + 11)/(d*tan(d*x)^6*tan(c)^6
 - 6*d*tan(d*x)^5*tan(c)^5 + 15*d*tan(d*x)^4*tan(c)^4 - 20*d*tan(d*x)^3*tan
(c)^3 + 15*d*tan(d*x)^2*tan(c)^2 - 6*d*tan(d*x)*tan(c) + d)
```

Mupad [B]
time = 2.49, size = 49, normalized size = 0.86

$$-\frac{\frac{\ln\left(\tan(c+d\,x)^2+1\right)}{2}-\frac{\tan(c+d\,x)^2}{2}+\frac{\tan(c+d\,x)^4}{4}-\frac{\tan(c+d\,x)^6}{6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^7,x)
```

```
[Out] -(log(tan(c + d*x)^2 + 1)/2 - tan(c + d*x)^2/2 + tan(c + d*x)^4/4 - tan(c +
 d*x)^6/6)/d
```

3.8 $\int \tan^8(c+dx)\,dx$

Optimal. Leaf size=58

$$x-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^7(c+dx)}{7d}$$

```
[Out] x-tan(d*x+c)/d+1/3*tan(d*x+c)^3/d-1/5*tan(d*x+c)^5/d+1/7*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 8}

$$\frac{\tan^7(c+dx)}{7d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}+x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8,x]
```

```
[Out] x - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d*x]^5/(5*d) + Tan[c + 
d*x]^7/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \tan^8(c+dx)\,dx &= \frac{\tan^7(c+dx)}{7d}-\int \tan^6(c+dx)\,dx\\
&= -\frac{\tan^5(c+dx)}{5d}+\frac{\tan^7(c+dx)}{7d}+\int \tan^4(c+dx)\,dx\\
&= \frac{\tan^3(c+dx)}{3d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^7(c+dx)}{7d}-\int \tan^2(c+dx)\,dx\\
&= -\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^7(c+dx)}{7d}+\int 1\,dx\\
&= x-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 68, normalized size = 1.17

$$\frac{\text{ArcTan}(\tan(c+dx))}{d}-\frac{\tan(c+dx)}{d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan^5(c+dx)}{5d}+\frac{\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8,x]
```

```
[Out] ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d) - Tan[c + d*
x]^5/(5*d) + Tan[c + d*x]^7/(7*d)
```

Maple [A]
time = 0.03, size = 51, normalized size = 0.88

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	51
default	$\frac{\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))}{d}$	51
norman	$x-\frac{\tan(dx+c)}{d}+\frac{\tan^3(dx+c)}{3d}-\frac{\tan^5(dx+c)}{5d}+\frac{\tan^7(dx+c)}{7d}$	53
risch	$x-\frac{8i\left(105\,e^{12i(dx+c)}+315\,e^{10i(dx+c)}+770\,e^{8i(dx+c)}+770\,e^{6i(dx+c)}+609\,e^{4i(dx+c)}+203\,e^{2i(dx+c)}+44\right)}{105d\left(e^{2i(dx+c)}+1\right)^7}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/7*tan(d*x+c)^7-1/5*tan(d*x+c)^5+1/3*tan(d*x+c)^3-tan(d*x+c)+arctan(t
an(d*x+c)))
```

Maxima [A]
time = 0.49, size = 51, normalized size = 0.88

$$\frac{15\,\tan\left(dx+c\right)^7-21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+105\,dx+105\,c-105\,\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x
+ 105*c - 105*tan(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 48, normalized size = 0.83

$$\frac{15\,\tan\left(dx+c\right)^7-21\,\tan\left(dx+c\right)^5+35\,\tan\left(dx+c\right)^3+105\,dx-105\,\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(15*tan(d*x + c)^7 - 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 105*d*x
- 105*tan(d*x + c))/d
```

Sympy [A]
time = 0.21, size = 51, normalized size = 0.88

$$\begin{cases} x + \frac{\tan^{7}(c+dx)}{7d} - \frac{\tan^{5}(c+dx)}{5d} + \frac{\tan^{3}(c+dx)}{3d} - \frac{\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\tan^{8}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8,x)
```

```
[Out] Piecewise((x + tan(c + d*x)**7/(7*d) - tan(c + d*x)**5/(5*d) + tan(c + d*x)
**3/(3*d) - tan(c + d*x)/d, Ne(d, 0)), (x*tan(c)**8, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1441 vs. $2(52) = 104$.
time = 5.13, size = 1441, normalized size = 24.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 1/420*(105*pi + 420*d*x*tan(d*x)^7*tan(c)^7 - 105*pi*sgn(2*tan(d*x)^2*tan(c
) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^7*tan(c)^7 - 105*
pi*tan(d*x)^7*tan(c)^7 + 210*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c
)))*tan(d*x)^7*tan(c)^7 + 210*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -
 1))*tan(d*x)^7*tan(c)^7 - 2940*d*x*tan(d*x)^6*tan(c)^6 + 735*pi*sgn(2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^6*tan
(c)^6 + 735*pi*tan(d*x)^6*tan(c)^6 - 1470*arctan((tan(d*x)*tan(c) - 1)/(tan
(d*x) + tan(c)))*tan(d*x)^6*tan(c)^6 - 1470*arctan((tan(d*x) + tan(c))/(tan
(d*x)*tan(c) - 1))*tan(d*x)^6*tan(c)^6 + 420*tan(d*x)^7*tan(c)^6 + 420*tan(
d*x)^6*tan(c)^7 + 8820*d*x*tan(d*x)^5*tan(c)^5 - 2205*pi*sgn(2*tan(d*x)^2*t
an(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 -
140*tan(d*x)^7*tan(c)^4 - 2205*pi*tan(d*x)^5*tan(c)^5 + 4410*arctan((tan(d*
x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)^5*tan(c)^5 + 4410*arctan((tan(
d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^5*tan(c)^5 - 2940*tan(d*x)^6
*tan(c)^5 - 2940*tan(d*x)^5*tan(c)^6 - 140*tan(d*x)^4*tan(c)^7 - 14700*d*x*
tan(d*x)^4*tan(c)^4 + 3675*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2
 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 84*tan(d*x)^7*tan(c)^2 + 98
```

```
0*tan(d*x)^6*tan(c)^3 + 3675*pi*tan(d*x)^4*tan(c)^4 - 7350*arctan((tan(d*x)
*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)^4*tan(c)^4 - 7350*arctan((tan(d*
x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 + 8820*tan(d*x)^5*t
an(c)^4 + 8820*tan(d*x)^4*tan(c)^5 + 980*tan(d*x)^3*tan(c)^6 + 84*tan(d*x)^
2*tan(c)^7 + 14700*d*x*tan(d*x)^3*tan(c)^3 - 3675*pi*sgn(2*tan(d*x)^2*tan(c
) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 - 60*t
an(d*x)^7 - 588*tan(d*x)^6*tan(c) - 2940*tan(d*x)^5*tan(c)^2 - 3675*pi*tan(
d*x)^3*tan(c)^3 + 7350*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*ta
n(d*x)^3*tan(c)^3 + 7350*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*
tan(d*x)^3*tan(c)^3 - 14700*tan(d*x)^4*tan(c)^3 - 14700*tan(d*x)^3*tan(c)^4
 - 2940*tan(d*x)^2*tan(c)^5 - 588*tan(d*x)*tan(c)^6 - 60*tan(c)^7 - 8820*d*
x*tan(d*x)^2*tan(c)^2 + 2205*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 84*tan(d*x)^5 + 980*tan(d
*x)^4*tan(c) + 2205*pi*tan(d*x)^2*tan(c)^2 - 4410*arctan((tan(d*x)*tan(c) -
 1)/(tan(d*x) + tan(c)))*tan(d*x)^2*tan(c)^2 - 4410*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 + 8820*tan(d*x)^3*tan(c)^2 +
 8820*tan(d*x)^2*tan(c)^3 + 980*tan(d*x)*tan(c)^4 + 84*tan(c)^5 + 2940*d*x*
tan(d*x)*tan(c) - 735*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*
tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - 140*tan(d*x)^3 - 735*pi*tan(d*x)*tan
(c) + 1470*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c
) + 1470*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c)
- 2940*tan(d*x)^2*tan(c) - 2940*tan(d*x)*tan(c)^2 - 140*tan(c)^3 - 420*d*x
+ 105*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan
(c)) - 210*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 210*arctan((
tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) + 420*tan(d*x) + 420*tan(c))/(d*t
an(d*x)^7*tan(c)^7 - 7*d*tan(d*x)^6*tan(c)^6 + 21*d*tan(d*x)^5*tan(c)^5 - 3
5*d*tan(d*x)^4*tan(c)^4 + 35*d*tan(d*x)^3*tan(c)^3 - 21*d*tan(d*x)^2*tan(c)
^2 + 7*d*tan(d*x)*tan(c) - d)
```

Mupad [B]
time = 2.52, size = 44, normalized size = 0.76

$$x-\frac{-\frac{\tan(c+d\,x)^7}{7}+\frac{\tan(c+d\,x)^5}{5}-\frac{\tan(c+d\,x)^3}{3}+\tan(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^8,x)
```

```
[Out] x - (tan(c + d*x) - tan(c + d*x)^3/3 + tan(c + d*x)^5/5 - tan(c + d*x)^7/7)
/d
```

3.9 $\int (b\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=232

$$-\frac{b^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}+\frac{b^{7/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}-\frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\right.}{}$$

```
[Out] -1/2*b^(7/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)+1/2*b
^(7/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/4*b^(7/2)
*ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)+1/4*
b^(7/2)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/
2)-2*b^3*(b*tan(d*x+c))^(1/2)/d+2/5*b*(b*tan(d*x+c))^(5/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^{7/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}+\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\,d}-\frac{b^{7/2}\log\left(\sqrt{b}\,\tan(c+dx)-\sqrt{2}\,\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,d}+\frac{b^{7/2}\log\left(\sqrt{b}\,\tan(c+dx)+\sqrt{2}\,\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,d}-\frac{2b^3\sqrt{b\tan(c+dx)}}{d}+\frac{2b(b\tan(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(7/2),x]
```

```
[Out] -((b^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d))
 + (b^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d)
 - (b^(7/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x
]]])/(2*Sqrt[2]*d) + (b^(7/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*
Sqrt[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) - (2*b^3*Sqrt[b*Tan[c + d*x]])/d + (2*
b*(b*Tan[c + d*x])^(5/2))/(5*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (b\tan(c+dx))^{7/2}\,dx = \frac{2b(b\tan(c+dx))^{5/2}}{5d} - b^2\int (b\tan(c+dx))^{3/2}\,dx$$

$$= -\frac{2b^3\sqrt{b\tan(c+dx)}}{d} + \frac{2b(b\tan(c+dx))^{5/2}}{5d} + b^4\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$

$$= -\frac{2b^3\sqrt{b\tan(c+dx)}}{d} + \frac{2b(b\tan(c+dx))^{5/2}}{5d} + \frac{b^5\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(b^2+x^2)}\,dx, x, b\tan\right.}{d}$$

$$= -\frac{2b^3\sqrt{b\tan(c+dx)}}{d} + \frac{2b(b\tan(c+dx))^{5/2}}{5d} + \frac{(2b^5)\,\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan}\right.}{d}$$

$$= -\frac{2b^3\sqrt{b\tan(c+dx)}}{d} + \frac{2b(b\tan(c+dx))^{5/2}}{5d} + \frac{b^4\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c}\right.}{d}$$

$$= -\frac{2b^3\sqrt{b\tan(c+dx)}}{d} + \frac{2b(b\tan(c+dx))^{5/2}}{5d} - \frac{b^{7/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}\,x-x^2}\,dx,\right.}{2\sqrt{2}\,d}$$

$$= -\frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\,\tan(c+dx)-\sqrt{2}\,\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\,d} + \frac{b^{7/2}\log\left(\sqrt{b}+\sqrt{b}\right.}{}$$

$$= -\frac{b^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d} + \frac{b^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right.}{\sqrt{2}\,d}$$

Mathematica [A]
time = 0.43, size = 175, normalized size = 0.75

$$\frac{b^3\sqrt{b\tan(c+dx)}\left(-10\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)+10\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}\right)-5\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+5\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-40\sqrt{\tan(c+dx)}+8\tan^{\frac{5}{2}}(c+dx)\right)}{20d\sqrt{\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(7/2),x]
```

```
[Out] (b^3*Sqrt[b*Tan[c + d*x]]*(-10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]
]] + 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - 5*Sqrt[2]*Log[1 -
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt
[Tan[c + d*x]] + Tan[c + d*x]] - 40*Sqrt[Tan[c + d*x]] + 8*Tan[c + d*x]^(5/
2)))/(20*d*Sqrt[Tan[c + d*x]])
```

Maple [A]
time = 0.16, size = 169, normalized size = 0.73

method	result
derivativedivides	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{5}{2}}}{5}-b^2\sqrt{b\tan\left(dx+c\right)}+\frac{b^2\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\sqrt{2}+\sqrt{b}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\sqrt{2}+\sqrt{b}}\right.\right.}{}\right.}{d}$
default	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{5}{2}}}{5}-b^2\sqrt{b\tan\left(dx+c\right)}+\frac{b^2\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\sqrt{2}+\sqrt{b}}{b\tan(dx+c)-\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\sqrt{2}+\sqrt{b}}\right.\right.}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(1/5*(b*tan(d*x+c))^(5/2)-b^2*(b*tan(d*x+c))^(1/2)+1/8*b^2*(b^2)^(1/4
)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^
(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))
+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^
2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)))
```

Maxima [A]
time = 0.49, size = 186, normalized size = 0.80

$$\frac{10\sqrt{2}\,b^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{b}}\right)+10\sqrt{2}\,b^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{b}}\right)+5\sqrt{2}\,b^{\frac{9}{2}}\log\left(b\tan\left(dx+c\right)+\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{b}+b\right)-5\sqrt{2}\,b^{\frac{9}{2}}\log\left(b\tan\left(dx+c\right)-\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{b}+b\right)+8\left(b\tan\left(dx+c\right)\right)^{\frac{5}{2}}b^2-40\sqrt{b\tan\left(dx+c\right)}\,b^4}{20\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/20*(10*sqrt(2)*b^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan
(d*x + c)))/sqrt(b)) + 10*sqrt(2)*b^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt
(b) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + 5*sqrt(2)*b^(9/2)*log(b*tan(d*x +
c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - 5*sqrt(2)*b^(9/2)*log(b*ta
n(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) + 8*(b*tan(d*x + c))
^(5/2)*b^2 - 40*sqrt(b*tan(d*x + c))*b^4)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(179) = 358$.
time = 0.39, size = 600, normalized size = 2.59

$$-\frac{20\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{1}{4}}d\arctan\left(-\frac{b^{14}+\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{3}{4}}b^3d^3\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}-\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{3}{4}}d^3\sqrt{\frac{b^7\sin\left(dx+c\right)+\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{1}{4}}b^3d\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)+\sqrt{\frac{b^{14}}{d^4}}d^2\cos\left(dx+c\right)}{\cos\left(dx+c\right)}}}{b^{14}}\right)\cos\left(dx+c\right)^2+20\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{1}{4}}d\arctan\left(\frac{b^{14}-\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{3}{4}}b^3d^3\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}+\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{3}{4}}d^3\sqrt{\frac{b^7\sin\left(dx+c\right)-\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{1}{4}}b^3d\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)+\sqrt{\frac{b^{14}}{d^4}}d^2\cos\left(dx+c\right)}{\cos\left(dx+c\right)}}}{b^{14}}\right)\cos\left(dx+c\right)^2-5\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{1}{4}}d\cos\left(dx+c\right)^2\log\left(\frac{b^7\sin(dx+c)+\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{1}{4}}b^3d\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\cos(dx+c)+\sqrt{\frac{b^{14}}{d^4}}d^2\cos(dx+c)}{\cos(dx+c)}\right)+5\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{1}{4}}d\cos\left(dx+c\right)^2\log\left(\frac{b^7\sin(dx+c)-\sqrt{2}\left(\frac{b^{14}}{d^4}\right)^{\frac{1}{4}}b^3d\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\cos(dx+c)+\sqrt{\frac{b^{14}}{d^4}}d^2\cos(dx+c)}{\cos(dx+c)}\right)+8\left(6b^3\cos\left(dx+c\right)^2-b^3\right)\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}}{20\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(20*sqrt(2)*(b^14/d^4)^(1/4)*d*arctan(-(b^14 + sqrt(2)*(b^14/d^4)^(3/
4)*b^3*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c)) - sqrt(2)*(b^14/d^4)^(3/4)*d^3
*sqrt((b^7*sin(d*x + c) + sqrt(2)*(b^14/d^4)^(1/4)*b^3*d*sqrt(b*sin(d*x + c
)/cos(d*x + c))*cos(d*x + c) + sqrt(b^14/d^4)*d^2*cos(d*x + c))/cos(d*x + c
)))/b^14)*cos(d*x + c)^2 + 20*sqrt(2)*(b^14/d^4)^(1/4)*d*arctan((b^14 - sqr
t(2)*(b^14/d^4)^(3/4)*b^3*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c)) + sqrt(2)*(
b^14/d^4)^(3/4)*d^3*sqrt((b^7*sin(d*x + c) - sqrt(2)*(b^14/d^4)^(1/4)*b^3*d
*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c) + sqrt(b^14/d^4)*d^2*cos(d*
x + c))/cos(d*x + c)))/b^14)*cos(d*x + c)^2 - 5*sqrt(2)*(b^14/d^4)^(1/4)*d*
cos(d*x + c)^2*log((b^7*sin(d*x + c) + sqrt(2)*(b^14/d^4)^(1/4)*b^3*d*sqrt(
b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c) + sqrt(b^14/d^4)*d^2*cos(d*x + c)
)/cos(d*x + c)) + 5*sqrt(2)*(b^14/d^4)^(1/4)*d*cos(d*x + c)^2*log((b^7*sin(
d*x + c) - sqrt(2)*(b^14/d^4)^(1/4)*b^3*d*sqrt(b*sin(d*x + c)/cos(d*x + c))
*cos(d*x + c) + sqrt(b^14/d^4)*d^2*cos(d*x + c))/cos(d*x + c)) + 8*(6*b^3*c
os(d*x + c)^2 - b^3)*sqrt(b*sin(d*x + c)/cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(c+dx))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(7/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 3.18, size = 93, normalized size = 0.40

$$\frac{2\,b\,{\left(b\tan(c+d\,x)\right)}^{5/2}}{5\,d}-\frac{2\,b^3\,\sqrt{b\tan\left(c+d\,x\right)}}{d}-\frac{{\left(-1\right)}^{1/4}\,b^{7/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{b\tan\left(c+d\,x\right)}}{\sqrt{b}}\right)\,1{}\mathrm{i}}{d}-\frac{{\left(-1\right)}^{1/4}\,b^{7/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{b\tan\left(c+d\,x\right)}\,1{}\mathrm{i}}{\sqrt{b}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x))^(7/2),x)
```

```
[Out] (2*b*(b*tan(c + d*x))^(5/2))/(5*d) - (2*b^3*(b*tan(c + d*x))^(1/2))/d - ((-
1)^(1/4)*b^(7/2)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/d -
((-1)^(1/4)*b^(7/2)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2)*1i)/b^(1/2)))/d
```

3.10 $\int (b\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=212

$$\frac{b^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}-\frac{b^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}-\frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\,\tan\right.}{}$$

```
[Out] 1/2*b^(5/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/2*b^
(5/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/4*b^(5/2)*
ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)+1/4*b
^(5/2)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2
)+2/3*b*(b*tan(d*x+c))^(3/2)/d
```

Rubi [A]

time = 0.11, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{b^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\,d}-\frac{b^{5/2}\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,d}+\frac{b^{5/2}\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,d}+\frac{2b(b\tan(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (b^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(5/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]
)/(2*Sqrt[2]*d) + (b^(5/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqr
t[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (2*b*(b*Tan[c + d*x])^(3/2))/(3*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int (b\tan(c+dx))^{5/2}\,dx &= \frac{2b(b\tan(c+dx))^{3/2}}{3d} - b^2\int \sqrt{b\tan(c+dx)}\,dx \\
&= \frac{2b(b\tan(c+dx))^{3/2}}{3d} - \frac{b^3\mathrm{Subst}\left(\int \frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d} \\
&= \frac{2b(b\tan(c+dx))^{3/2}}{3d} - \frac{(2b^3)\,\mathrm{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} \\
&= \frac{2b(b\tan(c+dx))^{3/2}}{3d} + \frac{b^3\mathrm{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} - \frac{b^3\mathrm{Subst}\left(\int \frac{b+x}{b^2+x}\right.}{} \\
&= \frac{2b(b\tan(c+dx))^{3/2}}{3d} - \frac{b^{5/2}\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} - \frac{b^5}{} \\
&= -\frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}d} + \frac{b^{5/2}\log\left(\sqrt{b}+\sqrt{b}\right.}{} \\
&= \frac{b^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d} - \frac{b^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 40, normalized size = 0.19

$$
-\frac{2b\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)\right)(b\tan(c+dx))^{3/2}}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2])*(b*Tan[c + d*x
])^(3/2))/(3*d)
```

Maple [A]
time = 0.06, size = 154, normalized size = 0.73

method	result

derivativedivides	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{b^2\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan}}{(b^2)}\right.\right.}{8(b^2)^{\frac{1}{4}}}\right.}{d}$
default	$\frac{2b\left(\frac{(b\tan(dx+c))^{\frac{3}{2}}}{3}-\frac{b^2\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan}}{(b^2)}\right.\right.}{8(b^2)^{\frac{1}{4}}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(1/3*(b*tan(d*x+c))^(3/2)-1/8*b^2/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+
c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2
)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(
1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(
1/2)+1)))
```

Maxima [A]
time = 0.50, size = 176, normalized size = 0.83

$$-\frac{3\,b^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{\sqrt{b}}+\frac{\sqrt{2}\log\left(b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{\sqrt{b}}\right)-8\,(b\tan(dx+c))^{\frac{3}{2}}\,b^2}{12\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(
d*x + c)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b
) - 2*sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) +
 sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x
+ c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b)) - 8*(b*tan(d*x +
c))^(3/2)*b^2)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(161) = 322$.
time = 0.38, size = 594, normalized size = 2.80

$$\frac{12\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{1}{4}}d\arctan\left(-\frac{b^{10}+\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{1}{4}}b^7d\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}-\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{1}{4}}d\sqrt{\frac{b^{15}\sin(dx+c)+\sqrt{\frac{b^{10}}{d^4}}b^{10}d^2\cos(dx+c)+\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{3}{4}}b^7d^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\cos(dx+c)}{\cos(dx+c)}}}{b^{10}}\right)\cos(dx+c)+12\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{1}{4}}d\arctan\left(\frac{b^{10}-\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{1}{4}}b^7d\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}+\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{1}{4}}d\sqrt{\frac{b^{15}\sin(dx+c)+\sqrt{\frac{b^{10}}{d^4}}b^{10}d^2\cos(dx+c)-\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{3}{4}}b^7d^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\cos(dx+c)}{\cos(dx+c)}}}{b^{10}}\right)\cos(dx+c)+3\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{1}{4}}d\cos(dx+c)\log\left(\frac{b^{15}\sin(dx+c)+\sqrt{\frac{b^{10}}{d^4}}b^{10}d^2\cos(dx+c)+\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{3}{4}}b^7d^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\cos(dx+c)}{\cos(dx+c)}\right)-3\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{1}{4}}d\cos(dx+c)\log\left(\frac{b^{15}\sin(dx+c)+\sqrt{\frac{b^{10}}{d^4}}b^{10}d^2\cos(dx+c)-\sqrt{2}\left(\frac{b^{10}}{d^4}\right)^{\frac{3}{4}}b^7d^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\cos(dx+c)}{\cos(dx+c)}\right)+8b^2\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\sin(dx+c)}{12\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*(b^10/d^4)^(1/4)*d*arctan(-(b^10 + sqrt(2)*(b^10/d^4)^(1/4
)*b^7*d*sqrt(b*sin(d*x + c)/cos(d*x + c)) - sqrt(2)*(b^10/d^4)^(1/4)*d*sqrt
((b^15*sin(d*x + c) + sqrt(b^10/d^4)*b^10*d^2*cos(d*x + c) + sqrt(2)*(b^10/
d^4)^(3/4)*b^7*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c))/cos(d*x
+ c)))/b^10)*cos(d*x + c) + 12*sqrt(2)*(b^10/d^4)^(1/4)*d*arctan((b^10 - sq
rt(2)*(b^10/d^4)^(1/4)*b^7*d*sqrt(b*sin(d*x + c)/cos(d*x + c)) + sqrt(2)*(b
^10/d^4)^(1/4)*d*sqrt((b^15*sin(d*x + c) + sqrt(b^10/d^4)*b^10*d^2*cos(d*x
+ c) - sqrt(2)*(b^10/d^4)^(3/4)*b^7*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*c
os(d*x + c))/cos(d*x + c)))/b^10)*cos(d*x + c) + 3*sqrt(2)*(b^10/d^4)^(1/4)
*d*cos(d*x + c)*log((b^15*sin(d*x + c) + sqrt(b^10/d^4)*b^10*d^2*cos(d*x +
c) + sqrt(2)*(b^10/d^4)^(3/4)*b^7*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos
(d*x + c))/cos(d*x + c)) - 3*sqrt(2)*(b^10/d^4)^(1/4)*d*cos(d*x + c)*log((b
^15*sin(d*x + c) + sqrt(b^10/d^4)*b^10*d^2*cos(d*x + c) - sqrt(2)*(b^10/d^4
)^(3/4)*b^7*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c))/cos(d*x + c
)) + 8*b^2*sqrt(b*sin(d*x + c)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(c+dx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 2.79, size = 74, normalized size = 0.35

$$\frac{2\,b\,(b\tan(c+d\,x))^{3/2}}{3\,d}-\frac{(-1)^{1/4}\,b^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{d}+\frac{(-1)^{1/4}\,b^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x))^(5/2),x)
```

```
[Out] (2*b*(b*tan(c + d*x))^(3/2))/(3*d) - ((-1)^(1/4)*b^(5/2)*atan(((-1)^(1/4)*(
b*tan(c + d*x))^(1/2))/b^(1/2)))/d + ((-1)^(1/4)*b^(5/2)*atanh(((-1)^(1/4)*
(b*tan(c + d*x))^(1/2))/b^(1/2)))/d
```

3.11 $\int (b\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=210

$$\frac{b^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}-\frac{b^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}+\frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\,\text{ta}\right.}{}$$

```
[Out] 1/2*b^(3/2)*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)-1/2*b^
(3/2)*arctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)+1/4*b^(3/2)*
ln(b^(1/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)-1/4*b
^(3/2)*ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2
)+2*b*(b*tan(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{b^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\,d}+\frac{b^{3/2}\log\left(\sqrt{b}\,\tan(c+dx)-\sqrt{2}\,\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,d}-\frac{b^{3/2}\log\left(\sqrt{b}\,\tan(c+dx)+\sqrt{2}\,\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,d}+\frac{2b\sqrt{b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(3/2),x]
```

```
[Out] (b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) -
(b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d) +
(b^(3/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]
)/(2*Sqrt[2]*d) - (b^(3/2)*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqr
t[b*Tan[c + d*x]]])/(2*Sqrt[2]*d) + (2*b*Sqrt[b*Tan[c + d*x]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (b\tan(c+dx))^{3/2}\,dx = \frac{2b\sqrt{b\tan(c+dx)}}{d} - b^2\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$

$$= \frac{2b\sqrt{b\tan(c+dx)}}{d} - \frac{b^3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{2b\sqrt{b\tan(c+dx)}}{d} - \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$

$$= \frac{2b\sqrt{b\tan(c+dx)}}{d} - \frac{b^2\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} - \frac{b^2\text{Subst}\left(\int \frac{b+x}{b^2+}\right.}{}$$

$$= \frac{2b\sqrt{b\tan(c+dx)}}{d} + \frac{b^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\,d} + \frac{b^3}{}$$

$$= \frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\,d} - \frac{b^{3/2}\log\left(\sqrt{b}+\sqrt{b}\right.}{}$$

$$= \frac{b^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d} - \frac{b^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}$$

Mathematica [A]
time = 0.17, size = 159, normalized size = 0.76

$$\frac{\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+8\sqrt{\tan(c+dx)}\right)(b\tan(c+dx))^{3/2}}{4d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*Sqrt[Tan[c + d*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] +
 8*Sqrt[Tan[c + d*x]])*(b*Tan[c + d*x])^(3/2))/(4*d*Tan[c + d*x]^(3/2))
```

Maple [A]
time = 0.05, size = 149, normalized size = 0.71

method	result

derivativedivides	$\frac{2b\left(\sqrt{b\tan(dx+c)}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}}{\cdots}\right.\right.}{8}\cdots}{d}$
default	$\frac{2b\left(\sqrt{b\tan(dx+c)}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}}{\cdots}\right.\right.}{8}\cdots}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*((b*tan(d*x+c))^(1/2)-1/8*(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2)
^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*
(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*
tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))
)
```

Maxima [A]
time = 0.50, size = 170, normalized size = 0.81

$$-\frac{2\sqrt{2}\,b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+2\sqrt{2}\,b^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+\sqrt{2}\,b^{\frac{5}{2}}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)-\sqrt{2}\,b^{\frac{5}{2}}\log\left(b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)-8\sqrt{b\tan(dx+c)}\,b^2}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*b^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(
d*x + c)))/sqrt(b)) + 2*sqrt(2)*b^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b
) - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + sqrt(2)*b^(5/2)*log(b*tan(d*x + c) +
 sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - sqrt(2)*b^(5/2)*log(b*tan(d*x
+ c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - 8*sqrt(b*tan(d*x + c))*b
^2)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(161) = 322$.
time = 0.36, size = 533, normalized size = 2.54

$$\frac{4\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{1}{4}}d\arctan\left(-\frac{b^6+\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{3}{4}}bd^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}-\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{3}{4}}d^3\sqrt{\frac{\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{1}{4}}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\cos(dx+c)+b^3\sin(dx+c)+\sqrt{\frac{b^6}{d^4}}d^2\cos(dx+c)}{\cos(dx+c)}}}{b^6}\right)+4\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{1}{4}}d\arctan\left(\frac{b^6-\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{3}{4}}bd^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}+\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{3}{4}}d^3\sqrt{-\frac{\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{1}{4}}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\cos(dx+c)-b^3\sin(dx+c)-\sqrt{\frac{b^6}{d^4}}d^2\cos(dx+c)}{\cos(dx+c)}}}{b^6}\right)-\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{1}{4}}d\log\left(\frac{\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{1}{4}}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\cos(dx+c)+b^3\sin(dx+c)+\sqrt{\frac{b^6}{d^4}}d^2\cos(dx+c)}{\cos(dx+c)}\right)+\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{1}{4}}d\log\left(-\frac{\sqrt{2}\left(\frac{b^6}{d^4}\right)^{\frac{1}{4}}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\cos(dx+c)-b^3\sin(dx+c)-\sqrt{\frac{b^6}{d^4}}d^2\cos(dx+c)}{\cos(dx+c)}\right)+8b\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(b^6/d^4)^(1/4)*d*arctan(-(b^6 + sqrt(2)*(b^6/d^4)^(3/4)*b*d
^3*sqrt(b*sin(d*x + c)/cos(d*x + c)) - sqrt(2)*(b^6/d^4)^(3/4)*d^3*sqrt((sq
rt(2)*(b^6/d^4)^(1/4)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c) +
b^3*sin(d*x + c) + sqrt(b^6/d^4)*d^2*cos(d*x + c))/cos(d*x + c)))/b^6) + 4*
sqrt(2)*(b^6/d^4)^(1/4)*d*arctan((b^6 - sqrt(2)*(b^6/d^4)^(3/4)*b*d^3*sqrt(
b*sin(d*x + c)/cos(d*x + c)) + sqrt(2)*(b^6/d^4)^(3/4)*d^3*sqrt(-(sqrt(2)*(
b^6/d^4)^(1/4)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c) - b^3*sin
(d*x + c) - sqrt(b^6/d^4)*d^2*cos(d*x + c))/cos(d*x + c)))/b^6) - sqrt(2)*(
b^6/d^4)^(1/4)*d*log((sqrt(2)*(b^6/d^4)^(1/4)*b*d*sqrt(b*sin(d*x + c)/cos(d
*x + c))*cos(d*x + c) + b^3*sin(d*x + c) + sqrt(b^6/d^4)*d^2*cos(d*x + c))/
cos(d*x + c)) + sqrt(2)*(b^6/d^4)^(1/4)*d*log(-(sqrt(2)*(b^6/d^4)^(1/4)*b*d
*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c) - b^3*sin(d*x + c) - sqrt(b
^6/d^4)*d^2*cos(d*x + c))/cos(d*x + c)) + 8*b*sqrt(b*sin(d*x + c)/cos(d*x +
 c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan{(c+dx)})^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c))^(3/2), x)
```

Mupad [B]
time = 2.74, size = 73, normalized size = 0.35

$$\frac{2\,b\,\sqrt{b\tan{(c+d\,x)}}}{d}+\frac{{(-1)}^{1/4}\,b^{3/2}\,\mathrm{atan}\left(\frac{{(-1)}^{1/4}\,\sqrt{b\tan{(c+d\,x)}}}{\sqrt{b}}\right)\,1\mathrm{i}}{d}+\frac{{(-1)}^{1/4}\,b^{3/2}\,\mathrm{atanh}\left(\frac{{(-1)}^{1/4}\,\sqrt{b\tan{(c+d\,x)}}}{\sqrt{b}}\right)\,1\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x))^(3/2),x)
```

```
[Out] (2*b*(b*tan(c + d*x))^(1/2))/d + ((-1)^(1/4)*b^(3/2)*atan(((-1)^(1/4)*(b*ta
n(c + d*x))^(1/2))/b^(1/2))*1i)/d + ((-1)^(1/4)*b^(3/2)*atanh(((-1)^(1/4)*(
b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/d
```

3.12 $\int \sqrt{b\tan(c+dx)}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}+\frac{\sqrt{b}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}+\frac{\sqrt{b}\,\log\left(\sqrt{b}+\sqrt{b}\,\text{t}\right.}{}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d*2^(1/2)+1/4*ln(b^(1
/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))*b^(1/2)/d*2^(1/2)-1/4*
ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))*b^(1/2)/d*2^(1/
2)
```

Rubi [A]
time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\,d}+\frac{\sqrt{b}\,\log\left(\sqrt{b}\,\tan(c+dx)-\sqrt{2}\,\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,d}-\frac{\sqrt{b}\,\log\left(\sqrt{b}\,\tan(c+dx)+\sqrt{2}\,\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] -((Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d))
 + (Sqrt[b]*ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]])/(Sqrt[2]*d)
 + (Sqrt[b]*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x
]]])/(2*Sqrt[2]*d) - (Sqrt[b]*Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*
Sqrt[b*Tan[c + d*x]]])/(2*Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \sqrt{b\tan(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{(2b)\text{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$

$$= -\frac{b\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} + \frac{b\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right.}{d}$$

$$= \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}\,x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\,d} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-}{-b+\sqrt{2}\sqrt{b}}\right.}{2}$$

$$= \frac{\sqrt{b}\,\log\left(\sqrt{b}+\sqrt{b}\,\tan(c+dx)-\sqrt{2}\,\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\,d} - \frac{\sqrt{b}\,\log\left(\sqrt{b}+\sqrt{b}\,\text{ta}\right.}{}$$

$$= -\frac{\sqrt{b}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d} + \frac{\sqrt{b}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 40, normalized size = 0.21

$$\frac{2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)(b\tan(c+dx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(3/2))/
(3*b*d)
```

Maple [A]
time = 0.09, size = 136, normalized size = 0.71

method	result
derivativedivides	$\frac{b\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-2\right.}{4d(b^2)^{\frac{1}{4}}}$

default	$\frac{b\sqrt{2}\,\left(\ln\left(\frac{b\tan(dx+c)-\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\,\sqrt{2}+\sqrt{b^{2}}}{b\tan(dx+c)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan\left(dx+c\right)}}{\left(b^{2}\right)^{\frac{1}{4}}}+1\right)\right.}{4d\left(b^{2}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*b/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1
/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(
1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))
```

Maxima [A]
time = 0.50, size = 153, normalized size = 0.80

$$\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{b}+b\right)}{\sqrt{b}}+\frac{\sqrt{2}\log\left(b\tan(dx+c)-\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{b}+b\right)}{\sqrt{b}}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x + c
)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2*s
qrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + sqrt(2
)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x + c) -
sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(145) = 290$.
time = 0.39, size = 519, normalized size = 2.70

$$-\sqrt{2}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,bd\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}-\sqrt{2}\,d\sqrt{\frac{\sqrt{2}\,bd^{3}\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{3}{4}}\cos\left(dx+c\right)+b^{2}d^{2}\sqrt{\frac{b^{2}}{d^{4}}}\cos\left(dx+c\right)+b^{3}\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}+b^{2}}{b^{2}}\right)-\sqrt{2}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,bd\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}-\sqrt{2}\,d\sqrt{-\frac{\sqrt{2}\,bd^{3}\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{3}{4}}\cos\left(dx+c\right)-b^{2}d^{2}\sqrt{\frac{b^{2}}{d^{4}}}\cos\left(dx+c\right)-b^{3}\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}-b^{2}}{b^{2}}\right)-\frac{1}{4}\sqrt{2}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}\log\left(\frac{\sqrt{2}\,bd^{3}\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{3}{4}}\cos\left(dx+c\right)+b^{2}d^{2}\sqrt{\frac{b^{2}}{d^{4}}}\cos\left(dx+c\right)+b^{3}\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)+\frac{1}{4}\sqrt{2}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{1}{4}}\log\left(-\frac{\sqrt{2}\,bd^{3}\sqrt{\frac{b\sin\left(dx+c\right)}{\cos\left(dx+c\right)}}\left(\frac{b^{2}}{d^{4}}\right)^{\frac{3}{4}}\cos\left(dx+c\right)-b^{2}d^{2}\sqrt{\frac{b^{2}}{d^{4}}}\cos\left(dx+c\right)-b^{3}\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(b^2/d^4)^(1/4)*arctan(-(sqrt(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x +
 c))*(b^2/d^4)^(1/4) - sqrt(2)*d*sqrt((sqrt(2)*b*d^3*sqrt(b*sin(d*x + c)/co
s(d*x + c))*(b^2/d^4)^(3/4)*cos(d*x + c) + b^2*d^2*sqrt(b^2/d^4)*cos(d*x +
c) + b^3*sin(d*x + c))/cos(d*x + c))*(b^2/d^4)^(1/4) + b^2)/b^2) - sqrt(2)*
(b^2/d^4)^(1/4)*arctan(-(sqrt(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(b^2
/d^4)^(1/4) - sqrt(2)*d*sqrt(-(sqrt(2)*b*d^3*sqrt(b*sin(d*x + c)/cos(d*x +
c))*(b^2/d^4)^(3/4)*cos(d*x + c) - b^2*d^2*sqrt(b^2/d^4)*cos(d*x + c) - b^3
```

```
*sin(d*x + c))/cos(d*x + c))*(b^2/d^4)^(1/4) - b^2)/b^2) - 1/4*sqrt(2)*(b^2
/d^4)^(1/4)*log((sqrt(2)*b*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(b^2/d^4)^
(3/4)*cos(d*x + c) + b^2*d^2*sqrt(b^2/d^4)*cos(d*x + c) + b^3*sin(d*x + c))
/cos(d*x + c)) + 1/4*sqrt(2)*(b^2/d^4)^(1/4)*log(-(sqrt(2)*b*d^3*sqrt(b*sin
(d*x + c)/cos(d*x + c))*(b^2/d^4)^(3/4)*cos(d*x + c) - b^2*d^2*sqrt(b^2/d^4
)*cos(d*x + c) - b^3*sin(d*x + c))/cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tan{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 176, normalized size = 0.92

$$\frac{\frac{2\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{2\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{|b|}}\right)}{d}-\frac{\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(dx+c\right)+\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{|b|}+|b|\right)}{d}+\frac{\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(dx+c\right)-\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{|b|}+|b|\right)}{d}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*sq
rt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 2*sqrt(2)*abs(b)^(3/2)*arctan(-1/2*sq
rt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d - sqr
t(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(ab
s(b)) + abs(b))/d + sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) - sqrt(2)*sqrt(
b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d)/b
```

Mupad [B]
time = 2.62, size = 49, normalized size = 0.26

$$\frac{{\left(-1\right)}^{1/4}\,\sqrt{b}\,\left(\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{b\,\mathrm{tan}\left(c+d\,x\right)}}{\sqrt{b}}\right)-\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{b\,\mathrm{tan}\left(c+d\,x\right)}}{\sqrt{b}}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*b^(1/2)*(atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)) - at
anh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))))/d
```

3.13 $\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$

Optimal. Leaf size=192

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d}-\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)\right.}{2\sqrt{2}\,\sqrt{}}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/2)+1/2*a
rctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/d*2^(1/2)/b^(1/2)-1/4*ln(b^(1
/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)/b^(1/2)+1/4*
ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/d*2^(1/2)/b^(1/
2)
```

Rubi [A]
time = 0.08, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$,
Rules used = {3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\sqrt{b}\,d}-\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\sqrt{b}\,d}+\frac{\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*Sqrt[b]*d)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*Sqrt[b]*d) - L
og[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2
]*Sqrt[b]*d) + Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c +
d*x]]]/(2*Sqrt[2]*Sqrt[b]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{(2b)\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{b-\sqrt{2}\sqrt{b}\,x+x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{b+\sqrt{2}\sqrt{b}\,x+x^2}\,dx, x\right.}{2d}$$

$$= -\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}\sqrt{b}\,d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c-\right.}{2}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\sqrt{b}\,d} - \frac{\log}{}$$

Mathematica [A]

time = 0.10, size = 131, normalized size = 0.68

$$\frac{\left(-2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\right)\sqrt{\tan(c+dx)}}{2\sqrt{2}\,d\sqrt{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]],x]
```

```
[Out] ((-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[Tan
[c + d*x]]] - Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + Log[1 +
Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]])*Sqrt[Tan[c + d*x]])/(2*Sqrt[2]*
d*Sqrt[b*Tan[c + d*x]])
```

Maple [A]

time = 0.08, size = 138, normalized size = 0.72

method	result
derivativedivides	$\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}\right.\right.}{4db}$

default	$\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right.\right.}{4db}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/b*(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1
/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(
1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*a
rctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))
```

Maxima [**A**]
time = 0.49, size = 155, normalized size = 0.81

$$\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)+\sqrt{2}\sqrt{b}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)-\sqrt{2}\sqrt{b}\log\left(b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d
*x + c)))/sqrt(b)) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b)
 - 2*sqrt(b*tan(d*x + c)))/sqrt(b)) + sqrt(2)*sqrt(b)*log(b*tan(d*x + c) +
sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b) - sqrt(2)*sqrt(b)*log(b*tan(d*x +
 c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b))/(b*d)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(145) = 290.
time = 0.41, size = 493, normalized size = 2.57

$$-\sqrt{2}\left(\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}bd^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^2d^4}\right)^{\frac{3}{4}}+\sqrt{2}bd^3\sqrt{\frac{b^2d^2\sqrt{\frac{1}{b^2d^4}}\cos(dx+c)+\sqrt{2}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^2d^4}\right)^{\frac{3}{4}}-1\right)-\sqrt{2}\left(\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}bd^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^2d^4}\right)^{\frac{3}{4}}+\sqrt{2}bd^3\sqrt{\frac{b^2d^2\sqrt{\frac{1}{b^2d^4}}\cos(dx+c)-\sqrt{2}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^2d^4}\right)^{\frac{3}{4}}+1\right)+\frac{1}{4}\sqrt{2}\left(\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2d^2\sqrt{\frac{1}{b^2d^4}}\cos(dx+c)+\sqrt{2}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)-\frac{1}{4}\sqrt{2}\left(\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2d^2\sqrt{\frac{1}{b^2d^4}}\cos(dx+c)-\sqrt{2}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^2d^4}\right)^{\frac{1}{4}}\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(1/(b^2*d^4))^(1/4)*arctan(-sqrt(2)*b*d^3*sqrt(b*sin(d*x + c)/cos(
d*x + c))*(1/(b^2*d^4))^(3/4) + sqrt(2)*b*d^3*sqrt((b^2*d^2*sqrt(1/(b^2*d^4
))*cos(d*x + c) + sqrt(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^2*d^4
))^(1/4)*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/(b^2*d^4))^(3/4) -
 1) - sqrt(2)*(1/(b^2*d^4))^(1/4)*arctan(-sqrt(2)*b*d^3*sqrt(b*sin(d*x + c)
/cos(d*x + c))*(1/(b^2*d^4))^(3/4) + sqrt(2)*b*d^3*sqrt((b^2*d^2*sqrt(1/(b^
2*d^4))*cos(d*x + c) - sqrt(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^
2*d^4))^(1/4)*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/(b^2*d^4))^(3
/4) + 1) + 1/4*sqrt(2)*(1/(b^2*d^4))^(1/4)*log((b^2*d^2*sqrt(1/(b^2*d^4))*c
```

```
os(d*x + c) + sqrt(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^2*d^4))^(
1/4)*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c)) - 1/4*sqrt(2)*(1/(b^2*d^4
))^(1/4)*log((b^2*d^2*sqrt(1/(b^2*d^4))*cos(d*x + c) - sqrt(2)*b*d*sqrt(b*s
in(d*x + c)/cos(d*x + c))*(1/(b^2*d^4))^(1/4)*cos(d*x + c) + b*sin(d*x + c)
)/cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 184, normalized size = 0.96

$$\frac{\sqrt{2}\,\sqrt{|b|}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2\,bd}+\frac{\sqrt{2}\,\sqrt{|b|}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{2\,bd}+\frac{\sqrt{2}\,\sqrt{|b|}\,\log\left(b\tan(dx+c)+\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\sqrt{|b|}+|b|\right)}{4\,bd}-\frac{\sqrt{2}\,\sqrt{|b|}\,\log\left(b\tan(dx+c)-\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\sqrt{|b|}+|b|\right)}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*sqrt(
b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 1/2*sqrt(2)*sqrt(abs(b))*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d
) + 1/4*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x +
c))*sqrt(abs(b)) + abs(b))/(b*d) - 1/4*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x +
 c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d)
```

Mupad [B]
time = 2.72, size = 59, normalized size = 0.31

$$-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{b\,\tan\left(c+d\,x\right)}}{\sqrt{b}}\right)\,1{}\mathrm{i}}{\sqrt{b}\,d}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{b\,\tan\left(c+d\,x\right)}}{\sqrt{b}}\right)\,1{}\mathrm{i}}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x))^(1/2),x)
```

```
[Out] - ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(1/2
)*d) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/(
b^(1/2)*d)
```

3.14 $\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,b^{3/2}d}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,b^{3/2}d}-\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\right.}{2\sqrt{2}\,b^{3/2}d}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d*2^(1/2)-1/4*ln(b^(1/
2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(3/2)/d*2^(1/2)+1/4*l
n(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(3/2)/d*2^(1/2
)-2/b/d/(b*tan(d*x+c))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,b^{3/2}d}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\,b^{3/2}d}-\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,b^{3/2}d}+\frac{\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,b^{3/2}d}-\frac{2}{bd\sqrt{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(3/2)*d) - Ar
cTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(3/2)*d) - Log[
Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2]*b
^(3/2)*d) + Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c + d*x
]]]/(2*Sqrt[2]*b^(3/2)*d) - 2/(b*d*Sqrt[b*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{\int \sqrt{b\tan(c+dx)}\,dx}{b^2}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{bd} - \frac{\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\right.}{}$$

$$= -\frac{2}{bd\sqrt{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} - \frac{\text{Sub}}{}$$

$$= -\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{3/2}d} + \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+\right.}{2}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{3/2}d} - \frac{\log\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 38, normalized size = 0.18

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)}{bd\sqrt{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2])/(b*d*Sqrt[b*Tan[c + d
*x]])
```

Maple [A]
time = 0.05, size = 157, normalized size = 0.74

method	result

derivativedivides	$\frac{2b\left(-\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}{b\tan(dx+c)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^{2}\right)^{\frac{1}{4}}}+1\right)}{8b^{2}\left(b^{2}\right)^{\frac{1}{4}}}\right.}{d}$
default	$\frac{2b\left(-\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}{b\tan(dx+c)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{b\tan(dx+c)}}{\left(b^{2}\right)^{\frac{1}{4}}}+1\right)}{8b^{2}\left(b^{2}\right)^{\frac{1}{4}}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(-1/8/b^2/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*
x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(
1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2
)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))-1/b^2/(b*tan(d*
x+c))^(1/2))
```

Maxima [A]
time = 0.51, size = 167, normalized size = 0.79

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{\sqrt{b}}+\frac{\sqrt{2}\log\left(b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{\sqrt{b}}+\frac{8}{\sqrt{b\tan(dx+c)}}}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x + c)
))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2*sq
rt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + sqrt(2)
*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x + c) - s
qrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + 8/sqrt(b*tan(d*x + c)))/
(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(163) = 326$.
time = 0.38, size = 652, normalized size = 3.08

$$\frac{8\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+4\left(\sqrt{2}b^{2}d\cos(dx+c)^{2}-\sqrt{2}b^{2}d\right)\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{1}{4}}+\sqrt{2}bd\sqrt{\frac{\sqrt{2}b^{5}d^{3}\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{3}{4}}\cos(dx+c)+b^{4}d^{2}\sqrt{\frac{1}{b^{6}d^{4}}}\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{1}{4}}-1\right)+4\left(\sqrt{2}b^{2}d\cos(dx+c)^{2}-\sqrt{2}b^{2}d\right)\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}bd\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{1}{4}}+\sqrt{2}bd\sqrt{-\frac{\sqrt{2}b^{5}d^{3}\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{3}{4}}\cos(dx+c)-b^{4}d^{2}\sqrt{\frac{1}{b^{6}d^{4}}}\cos(dx+c)-b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{1}{4}}+1\right)+\left(\sqrt{2}b^{2}d\cos(dx+c)^{2}-\sqrt{2}b^{2}d\right)\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{1}{4}}\log\left(\frac{\sqrt{2}b^{5}d^{3}\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{3}{4}}\cos(dx+c)+b^{4}d^{2}\sqrt{\frac{1}{b^{6}d^{4}}}\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)-\left(\sqrt{2}b^{2}d\cos(dx+c)^{2}-\sqrt{2}b^{2}d\right)\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{1}{4}}\log\left(-\frac{\sqrt{2}b^{5}d^{3}\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{6}d^{4}}\right)^{\frac{3}{4}}\cos(dx+c)-b^{4}d^{2}\sqrt{\frac{1}{b^{6}d^{4}}}\cos(dx+c)-b\sin(dx+c)}{\cos(dx+c)}\right)}{4\left(b^{2}d\cos(dx+c)^{2}-b^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(8*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 4*(sqr
t(2)*b^2*d*cos(d*x + c)^2 - sqrt(2)*b^2*d)*(1/(b^6*d^4))^(1/4)*arctan(-sqrt
(2)*b*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^6*d^4))^(1/4) + sqrt(2)*b*d
*sqrt((sqrt(2)*b^5*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^6*d^4))^(3/4
)*cos(d*x + c) + b^4*d^2*sqrt(1/(b^6*d^4))*cos(d*x + c) + b*sin(d*x + c))/c
os(d*x + c))*(1/(b^6*d^4))^(1/4) - 1) + 4*(sqrt(2)*b^2*d*cos(d*x + c)^2 - s
qrt(2)*b^2*d)*(1/(b^6*d^4))^(1/4)*arctan(-sqrt(2)*b*d*sqrt(b*sin(d*x + c)/c
os(d*x + c))*(1/(b^6*d^4))^(1/4) + sqrt(2)*b*d*sqrt(-(sqrt(2)*b^5*d^3*sqrt(
b*sin(d*x + c)/cos(d*x + c))*(1/(b^6*d^4))^(3/4)*cos(d*x + c) - b^4*d^2*sqr
t(1/(b^6*d^4))*cos(d*x + c) - b*sin(d*x + c))/cos(d*x + c))*(1/(b^6*d^4))^(
1/4) + 1) + (sqrt(2)*b^2*d*cos(d*x + c)^2 - sqrt(2)*b^2*d)*(1/(b^6*d^4))^(1
/4)*log((sqrt(2)*b^5*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^6*d^4))^(3
/4)*cos(d*x + c) + b^4*d^2*sqrt(1/(b^6*d^4))*cos(d*x + c) + b*sin(d*x + c))
/cos(d*x + c)) - (sqrt(2)*b^2*d*cos(d*x + c)^2 - sqrt(2)*b^2*d)*(1/(b^6*d^4
))^(1/4)*log(-(sqrt(2)*b^5*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^6*d^
4))^(3/4)*cos(d*x + c) - b^4*d^2*sqrt(1/(b^6*d^4))*cos(d*x + c) - b*sin(d*x
 + c))/cos(d*x + c)))/(b^2*d*cos(d*x + c)^2 - b^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\tan(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 2.73, size = 76, normalized size = 0.36

$$\frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d}-\frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{b\tan(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d}-\frac{2}{b\,d\,\sqrt{b\tan(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(3/2)*d)
 - ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(3/2)*
d) - 2/(b*d*(b*tan(c + d*x))^(1/2))
```

3.15 $\int \frac{1}{(b\tan(c+dx))^{5/2}} \, dx$

Optimal. Leaf size=214

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,b^{5/2}d}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,b^{5/2}d}+\frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\right.}{2\sqrt{2}\,b^{5/2}d}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d*2^(1/2)+1/4*ln(b^(1/
2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(5/2)/d*2^(1/2)-1/4*l
n(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(5/2)/d*2^(1/2
)-2/3/b/d/(b*tan(d*x+c))^(3/2)
```

Rubi [A]

time = 0.10, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,b^{5/2}d}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\,b^{5/2}d}+\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,b^{5/2}d}-\frac{\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,b^{5/2}d}-\frac{2}{3bd(b\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-5/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(5/2)*d) - Ar
cTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(5/2)*d) + Log[
Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2]*b
^(5/2)*d) - Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c + d*x
]]]/(2*Sqrt[2]*b^(5/2)*d) - 2/(3*b*d*(b*Tan[c + d*x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(b\tan(c+dx))^{5/2}}\,dx = -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{\int \frac{1}{\sqrt{b\tan(c+dx)}}\,dx}{b^2}$$

$$= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{bd}$$

$$= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{b^2d} - \frac{\text{Subst}\left(\int \frac{b+x}{b^2+x}\right.}{}$$

$$= -\frac{2}{3bd(b\tan(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} + \frac{\text{S}}{}$$

$$= \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{5/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+\right.}{2\sqrt{}}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{5/2}d} + \frac{\log\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 40, normalized size = 0.19

$$-\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\tan^2(c+dx)\right)}{3bd(b\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/4, 1, 1/4, -Tan[c + d*x]^2])/(3*b*d*(b*Tan[c + d*x
])^(3/2))
```

Maple [A]
time = 0.05, size = 157, normalized size = 0.73

method	result

derivativedivides	$\frac{2b\left(-\frac{1}{3b^2(b\tan(dx+c))^{\frac{3}{2}}}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\ \sqrt{}}{}\right.\right.}{8b^4}\right.}{d}$
default	$\frac{2b\left(-\frac{1}{3b^2(b\tan(dx+c))^{\frac{3}{2}}}-\frac{(b^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\ \sqrt{}}{}\right.\right.}{8b^4}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(-1/3/b^2/(b*tan(d*x+c))^(3/2)-1/8/b^4*(b^2)^(1/4)*2^(1/2)*(ln((b*tan
(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)
-(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b
^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+
c))^(1/2)+1)))
```

Maxima [A]
time = 0.49, size = 168, normalized size = 0.79

$$-\frac{\frac{6\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{6\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{3\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{b^{\frac{3}{2}}}-\frac{3\sqrt{2}\log\left(b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{b^{\frac{3}{2}}}+\frac{8}{(b\tan(dx+c))^{\frac{3}{2}}}}{12\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(6*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x + c
)))/sqrt(b))/b^(3/2) + 6*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2*s
qrt(b*tan(d*x + c)))/sqrt(b))/b^(3/2) + 3*sqrt(2)*log(b*tan(d*x + c) + sqrt
(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/b^(3/2) - 3*sqrt(2)*log(b*tan(d*x + c
) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/b^(3/2) + 8/(b*tan(d*x + c))^
(3/2))/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(163) = 326$.
time = 0.39, size = 653, normalized size = 3.05

$$\frac{8\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\cos(dx+c)^2+12\left(\sqrt{2}b^3d\cos(dx+c)^2-\sqrt{2}b^3d\right)\left(\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}b^7d^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{10}d^4}\right)^{\frac{3}{4}}+\sqrt{2}b^7d^3\sqrt{\frac{b^6d^2\sqrt{\frac{1}{b^{10}d^4}}\cos(dx+c)+\sqrt{2}b^3d\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{10}d^4}\right)^{\frac{3}{4}}-1\right)+12\left(\sqrt{2}b^3d\cos(dx+c)^2-\sqrt{2}b^3d\right)\left(\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}b^7d^3\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{10}d^4}\right)^{\frac{3}{4}}+\sqrt{2}b^7d^3\sqrt{\frac{b^6d^2\sqrt{\frac{1}{b^{10}d^4}}\cos(dx+c)-\sqrt{2}b^3d\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{10}d^4}\right)^{\frac{3}{4}}+1\right)-3\left(\sqrt{2}b^3d\cos(dx+c)^2-\sqrt{2}b^3d\right)\left(\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\log\left(\frac{b^6d^2\sqrt{\frac{1}{b^{10}d^4}}\cos(dx+c)+\sqrt{2}b^3d\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)+3\left(\sqrt{2}b^3d\cos(dx+c)^2-\sqrt{2}b^3d\right)\left(\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\log\left(\frac{b^6d^2\sqrt{\frac{1}{b^{10}d^4}}\cos(dx+c)-\sqrt{2}b^3d\sqrt{\frac{b\sin(dx+c)}{\cos(dx+c)}}\left(\frac{1}{b^{10}d^4}\right)^{\frac{1}{4}}\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)}{12\left(b^3d\cos(dx+c)^2-b^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*sqrt(b*sin(d*x + c)/cos(d*x + c))*cos(d*x + c)^2 + 12*(sqrt(2)*b^3*
d*cos(d*x + c)^2 - sqrt(2)*b^3*d)*(1/(b^10*d^4))^(1/4)*arctan(-sqrt(2)*b^7*
d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^10*d^4))^(3/4) + sqrt(2)*b^7*d^
3*sqrt((b^6*d^2*sqrt(1/(b^10*d^4))*cos(d*x + c) + sqrt(2)*b^3*d*sqrt(b*sin(
d*x + c)/cos(d*x + c))*(1/(b^10*d^4))^(1/4)*cos(d*x + c) + b*sin(d*x + c))/
cos(d*x + c))*(1/(b^10*d^4))^(3/4) - 1) + 12*(sqrt(2)*b^3*d*cos(d*x + c)^2
- sqrt(2)*b^3*d)*(1/(b^10*d^4))^(1/4)*arctan(-sqrt(2)*b^7*d^3*sqrt(b*sin(d*
x + c)/cos(d*x + c))*(1/(b^10*d^4))^(3/4) + sqrt(2)*b^7*d^3*sqrt((b^6*d^2*s
qrt(1/(b^10*d^4))*cos(d*x + c) - sqrt(2)*b^3*d*sqrt(b*sin(d*x + c)/cos(d*x
+ c))*(1/(b^10*d^4))^(1/4)*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))*(1/
(b^10*d^4))^(3/4) + 1) - 3*(sqrt(2)*b^3*d*cos(d*x + c)^2 - sqrt(2)*b^3*d)*(
1/(b^10*d^4))^(1/4)*log((b^6*d^2*sqrt(1/(b^10*d^4))*cos(d*x + c) + sqrt(2)*
b^3*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^10*d^4))^(1/4)*cos(d*x + c) +
 b*sin(d*x + c))/cos(d*x + c)) + 3*(sqrt(2)*b^3*d*cos(d*x + c)^2 - sqrt(2)*
b^3*d)*(1/(b^10*d^4))^(1/4)*log((b^6*d^2*sqrt(1/(b^10*d^4))*cos(d*x + c) -
sqrt(2)*b^3*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^10*d^4))^(1/4)*cos(d*
x + c) + b*sin(d*x + c))/cos(d*x + c)))/(b^3*d*cos(d*x + c)^2 - b^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c))^(-5/2), x)
```

Mupad [B]
time = 3.07, size = 75, normalized size = 0.35

$$-\frac{2}{3\,b\,d\,{\left(b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{3/2}}+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{b\,\mathrm{tan}\left(c+d\,x\right)}}{\sqrt{b}}\right)\,1{}\mathrm{i}}{b^{5/2}\,d}+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{b\,\mathrm{tan}\left(c+d\,x\right)}}{\sqrt{b}}\right)\,1{}\mathrm{i}}{b^{5/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*tan(c + d*x))^(5/2),x)

[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(5/2)*d) - 2/(3*b*d*(b*tan(c + d*x))^(3/2)) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2))*1i)/(b^(5/2)*d)

3.16 $\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=234

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,b^{7/2}d}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,b^{7/2}d}+\frac{\log\left(\sqrt{b}+\sqrt{b}\,\tan(c+dx)-\right.}{2\sqrt{2}\,b^{7/}}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(b*tan(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d*2^(1/2)+1/4*ln(b^(1
/2)-2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(7/2)/d*2^(1/2)-1/4*
ln(b^(1/2)+2^(1/2)*(b*tan(d*x+c))^(1/2)+b^(1/2)*tan(d*x+c))/b^(7/2)/d*2^(1/
2)+2/b^3/d/(b*tan(d*x+c))^(1/2)-2/5/b/d/(b*tan(d*x+c))^(5/2)
```

Rubi [A]
time = 0.12, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}\,b^{7/2}d}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}+1\right)}{\sqrt{2}\,b^{7/2}d}+\frac{\log\left(\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,b^{7/2}d}-\frac{\log\left(\sqrt{b}\tan(c+dx)+\sqrt{2}\sqrt{b\tan(c+dx)}+\sqrt{b}\right)}{2\sqrt{2}\,b^{7/2}d}+\frac{2}{b^3d\sqrt{b\tan(c+dx)}}-\frac{2}{5bd(b\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-7/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(7/2)*d)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[b*Tan[c + d*x]])/Sqrt[b]]/(Sqrt[2]*b^(7/2)*d) + L
og[Sqrt[b] + Sqrt[b]*Tan[c + d*x] - Sqrt[2]*Sqrt[b*Tan[c + d*x]]]/(2*Sqrt[2
]*b^(7/2)*d) - Log[Sqrt[b] + Sqrt[b]*Tan[c + d*x] + Sqrt[2]*Sqrt[b*Tan[c +
d*x]]]/(2*Sqrt[2]*b^(7/2)*d) - 2/(5*b*d*(b*Tan[c + d*x])^(5/2)) + 2/(b^3*d*
Sqrt[b*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(b\tan(c+dx))^{7/2}}\,dx = -\frac{2}{5bd(b\tan(c+dx))^{5/2}} - \frac{\int \frac{1}{(b\tan(c+dx))^{3/2}}\,dx}{b^2}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{\int \sqrt{b\tan(c+dx)}\,dx}{b^4}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{b^2+x^2}\,dx, x, b\tan(c+d\right.}{b^3d}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c\right.}{b^3d}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \sqrt{b\tan(c+\right.}{b^3d}$$

$$= -\frac{2}{5bd(b\tan(c+dx))^{5/2}} + \frac{2}{b^3d\sqrt{b\tan(c+dx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x,\right.}{2\sqrt{2}b^{7/2}d}$$

$$= \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+dx)-\sqrt{2}\sqrt{b\tan(c+dx)}\right)}{2\sqrt{2}b^{7/2}d} - \frac{\log\left(\sqrt{b}+\sqrt{b}\tan(c+\right.}{2\sqrt{}}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\tan(c+dx)}}{\sqrt{b}}\right)}{\sqrt{2}b^{7/2}d} + \frac{\log}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.11, size = 40, normalized size = 0.17

$$-\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\tan^2(c+dx)\right)}{5bd(b\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-7/2),x]
[Out] (-2*Hypergeometric2F1[-5/4, 1, -1/4, -Tan[c + d*x]^2])/(5*b*d*(b*Tan[c + d*
x])^(5/2))
```

Maple [A]
time = 0.05, size = 171, normalized size = 0.73

method	result
derivativedivides	$\frac{2b\left(\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-\right.}{8b^4(b^2)^{\frac{1}{4}}}\right.}{d}$
default	$\frac{2b\left(\frac{\sqrt{2}\left(\ln\left(\frac{b\tan(dx+c)-(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{b\tan(dx+c)}}{(b^2)^{\frac{1}{4}}}+1\right)-\right.}{8b^4(b^2)^{\frac{1}{4}}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*b*(1/8/b^4/(b^2)^(1/4)*2^(1/2)*(ln((b*tan(d*x+c)-(b^2)^(1/4)*(b*tan(d*x
+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1
/2)*2^(1/2)+(b^2)^(1/2)))+2*arctan(2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)
+1)-2*arctan(-2^(1/2)/(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)+1))-1/5/b^2/(b*tan(d
*x+c))^(5/2)+1/b^4/(b*tan(d*x+c))^(1/2))
```

Maxima [A]
time = 0.50, size = 195, normalized size = 0.83

$$\frac{\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{b}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{b}}\right)}{\sqrt{b}}-\frac{\sqrt{2}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{\sqrt{b}}+\frac{\sqrt{2}\log\left(b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{b}+b\right)}{\sqrt{b}}\right)}{b^2}+\frac{8\left(5b^2\tan(dx+c)^2-b^2\right)}{(b\tan(dx+c))^{\frac{5}{2}}b^2}}{20\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/20*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(b) + 2*sqrt(b*tan(d*x +
 c)))/sqrt(b))/sqrt(b) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(b) - 2
*sqrt(b*tan(d*x + c)))/sqrt(b))/sqrt(b) - sqrt(2)*log(b*tan(d*x + c) + sqrt
(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b) + sqrt(2)*log(b*tan(d*x + c)
- sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(b) + b)/sqrt(b))/b^2 + 8*(5*b^2*tan(d*x
 + c)^2 - b^2)/((b*tan(d*x + c))^(5/2)*b^2))/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(181) = 362$.
time = 0.39, size = 751, normalized size = 3.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(8*(6*cos(d*x + c)^3 - 5*cos(d*x + c))*sqrt(b*sin(d*x + c)/cos(d*x +
c))*sin(d*x + c) + 20*(sqrt(2)*b^4*d*cos(d*x + c)^4 - 2*sqrt(2)*b^4*d*cos(d
*x + c)^2 + sqrt(2)*b^4*d)*(1/(b^14*d^4))^(1/4)*arctan(-sqrt(2)*b^3*d*sqrt(
b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^4))^(1/4) + sqrt(2)*b^3*d*sqrt((sqr
t(2)*b^11*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^4))^(3/4)*cos(d*
x + c) + b^8*d^2*sqrt(1/(b^14*d^4))*cos(d*x + c) + b*sin(d*x + c))/cos(d*x
+ c))*(1/(b^14*d^4))^(1/4) - 1) + 20*(sqrt(2)*b^4*d*cos(d*x + c)^4 - 2*sqrt
(2)*b^4*d*cos(d*x + c)^2 + sqrt(2)*b^4*d)*(1/(b^14*d^4))^(1/4)*arctan(-sqrt
(2)*b^3*d*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^4))^(1/4) + sqrt(2)*
b^3*d*sqrt(-(sqrt(2)*b^11*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^
4))^(3/4)*cos(d*x + c) - b^8*d^2*sqrt(1/(b^14*d^4))*cos(d*x + c) - b*sin(d*
x + c))/cos(d*x + c))*(1/(b^14*d^4))^(1/4) + 1) + 5*(sqrt(2)*b^4*d*cos(d*x
+ c)^4 - 2*sqrt(2)*b^4*d*cos(d*x + c)^2 + sqrt(2)*b^4*d)*(1/(b^14*d^4))^(1/
4)*log((sqrt(2)*b^11*d^3*sqrt(b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^4))^(
3/4)*cos(d*x + c) + b^8*d^2*sqrt(1/(b^14*d^4))*cos(d*x + c) + b*sin(d*x + c
))/cos(d*x + c)) - 5*(sqrt(2)*b^4*d*cos(d*x + c)^4 - 2*sqrt(2)*b^4*d*cos(d*
x + c)^2 + sqrt(2)*b^4*d)*(1/(b^14*d^4))^(1/4)*log(-(sqrt(2)*b^11*d^3*sqrt(
b*sin(d*x + c)/cos(d*x + c))*(1/(b^14*d^4))^(3/4)*cos(d*x + c) - b^8*d^2*sq
rt(1/(b^14*d^4))*cos(d*x + c) - b*sin(d*x + c))/cos(d*x + c)))/(b^4*d*cos(d
*x + c)^4 - 2*b^4*d*cos(d*x + c)^2 + b^4*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\tan(c+dx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-7/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 3.11, size = 92, normalized size = 0.39

$$\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan\left(c+d\,x\right)}}{\sqrt{b}}\right)}{b^{7/2}\,d}-\frac{\frac{2}{5\,b}-\frac{2\tan(c+d\,x)^2}{b}}{d\,\left(b\tan\left(c+d\,x\right)\right)^{5/2}}-\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{b\tan\left(c+d\,x\right)}}{\sqrt{b}}\right)}{b^{7/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x))^(7/2),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(7/2)*d)
- (2/(5*b) - (2*tan(c + d*x)^2)/b)/(d*(b*tan(c + d*x))^(5/2)) - ((-1)^(1/4)
*atanh(((-1)^(1/4)*(b*tan(c + d*x))^(1/2))/b^(1/2)))/(b^(7/2)*d)
```

3.17 $\int (b\tan(c+dx))^{4/3}\,dx$

Optimal. Leaf size=243

$$-\frac{b^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d}+\frac{b^{4/3}\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d}-\frac{b^{4/3}\text{ArcTan}\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c}{\sqrt[3]{b}}\right.}{2d}$$

```
[Out] -b^(4/3)*arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/d-1/2*b^(4/3)*arctan(-3^(1/2)
+2*(b*tan(d*x+c))^(1/3)/b^(1/3))/d-1/2*b^(4/3)*arctan(3^(1/2)+2*(b*tan(d*x+
c))^(1/3)/b^(1/3))/d+1/4*b^(4/3)*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c))^
(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d-1/4*b^(4/3)*ln(b^(2/3)+b^(1/3)*3^(1/2
)*(b*tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d+3*b*(b*tan(d*x+c))^(
1/3)/d
```

Rubi [A]

time = 0.25, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3554, 3557, 335, 215, 648, 632, 210, 642, 209}

$$-\frac{b^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d}+\frac{b^{4/3}\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d}-\frac{b^{4/3}\text{ArcTan}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2d}+\frac{\sqrt{3}\,b^{4/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}-\frac{\sqrt{3}\,b^{4/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}+\frac{3b\sqrt[3]{b\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(4/3),x]
```

```
[Out] -((b^(4/3)*ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)])/d) + (b^(4/3)*ArcTan[Sqr
t[3] - (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) - (b^(4/3)*ArcTan[Sqrt[3]
 + (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) + (Sqrt[3]*b^(4/3)*Log[b^(2/3
) - Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*d)
 - (Sqrt[3]*b^(4/3)*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) +
(b*Tan[c + d*x])^(2/3)])/(4*d) + (3*b*(b*Tan[c + d*x])^(1/3))/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int (b\tan(c+dx))^{4/3}\,dx &= \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - b^2\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx\\
&= \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - \frac{b^3\mathrm{Subst}\left(\int \frac{1}{x^{2/3}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - \frac{(3b^3)\,\mathrm{Subst}\left(\int \frac{1}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}\\
&= \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} - \frac{b^{4/3}\mathrm{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{\sqrt{3}}{2}x}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d} - \frac{b^4
\\
&= -\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{3b\sqrt[3]{b\tan(c+dx)}}{d} + \frac{\left(\sqrt{3}\,b^{4/3}\right)\mathrm{Subst}\left(\int \frac{-}{b^{2/3}-}\right.
\\
&= -\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{\sqrt{3}\,b^{4/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+\right.}{4d}
\\
&= -\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{b^{4/3}\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2d} -
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 38, normalized size = 0.16

$$
-\frac{3b\left(-1+{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\tan^2(c+dx)\right)\right)\sqrt[3]{b\tan(c+dx)}}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(4/3),x]
```

```
[Out] (-3*b*(-1 + Hypergeometric2F1[1/6, 1, 7/6, -Tan[c + d*x]^2])*(b*Tan[c + d*x
])^(1/3))/d
```

Maple [A]
time = 0.17, size = 216, normalized size = 0.89

method	result

derivativedivides	$\frac{3b\left((b\tan(dx+c))^{\frac{1}{3}}-\left(-\frac{\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{(b^2)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan(dx\ldots}{(b^2)^{\frac{1}{6}}}\ldots\right.}{6b^2}\ldots\right.\right.}{\ldots}$
default	$\frac{3b\left((b\tan(dx+c))^{\frac{1}{3}}-\left(-\frac{\sqrt{3}\,(b^2)^{\frac{1}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{(b^2)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan(dx\ldots}{(b^2)^{\frac{1}{6}}}\ldots\right.}{6b^2}\ldots\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*((b*tan(d*x+c))^(1/3)-(-1/12/b^2*3^(1/2)*(b^2)^(1/6)*ln((b*tan(d*x+c)
)^(2/3)-3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/b^2*(b^2)
^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))+1/12/b^2*3^(1/2)*
(b^2)^(1/6)*ln((b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3
)+(b^2)^(1/3))+1/6/b^2*(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6
)+3^(1/2))+1/3/b^2*(b^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))*b^
2)
```

Maxima [A]
time = 0.50, size = 185, normalized size = 0.76

$$-\frac{\sqrt{3}\,b^{\frac{7}{3}}\log\left(\sqrt{3}\,(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)-\sqrt{3}\,b^{\frac{7}{3}}\log\left(-\sqrt{3}\,(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)+2\,b^{\frac{7}{3}}\arctan\left(\frac{\sqrt{3}\,b^{\frac{1}{3}}+2\,(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)+2\,b^{\frac{7}{3}}\arctan\left(-\frac{\sqrt{3}\,b^{\frac{1}{3}}-2\,(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)+4\,b^{\frac{7}{3}}\arctan\left(\frac{(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)-12\,(b\tan(dx+c))^{\frac{1}{3}}b^2}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(3)*b^(7/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d
*x + c))^(2/3) + b^(2/3)) - sqrt(3)*b^(7/3)*log(-sqrt(3)*(b*tan(d*x + c))^(
1/3)*b^(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3)) + 2*b^(7/3)*arctan((sqrt(3
)*b^(1/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 2*b^(7/3)*arctan(-(sqrt(3)
*b^(1/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 4*b^(7/3)*arctan((b*tan(d*x
 + c))^(1/3)/b^(1/3)) - 12*(b*tan(d*x + c))^(1/3)*b^2)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(185) = 370.
time = 0.44, size = 588, normalized size = 2.42

$$-\frac{\sqrt{3}\left(\frac{b^8}{d^6}\right)^{\frac{1}{6}}d\log\left(\sqrt{3}\left(\frac{b^8}{d^6}\right)^{\frac{1}{6}}bd\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}+b^2\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}+\left(\frac{b^8}{d^6}\right)^{\frac{1}{3}}d^2\right)-\sqrt{3}\left(\frac{b^8}{d^6}\right)^{\frac{1}{6}}d\log\left(-\sqrt{3}\left(\frac{b^8}{d^6}\right)^{\frac{1}{6}}bd\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}+b^2\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}+\left(\frac{b^8}{d^6}\right)^{\frac{1}{3}}d^2\right)-4\left(\frac{b^8}{d^6}\right)^{\frac{1}{6}}d\arctan\left(-\frac{\sqrt{3}\,b^8+2\left(\frac{b^8}{d^6}\right)^{\frac{5}{6}}bd^5\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}-2\sqrt{\sqrt{3}\left(\frac{b^8}{d^6}\right)^{\frac{1}{6}}bd\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}+b^2\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}+\left(\frac{b^8}{d^6}\right)^{\frac{1}{3}}d^2}\left(\frac{b^8}{d^6}\right)^{\frac{5}{6}}d^5}{b^8}\right)-4\left(\frac{b^8}{d^6}\right)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\,b^8-2\left(\frac{b^8}{d^6}\right)^{\frac{5}{6}}bd^5\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}+2\sqrt{-\sqrt{3}\left(\frac{b^8}{d^6}\right)^{\frac{1}{6}}bd\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}+b^2\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}+\left(\frac{b^8}{d^6}\right)^{\frac{1}{3}}d^2}\left(\frac{b^8}{d^6}\right)^{\frac{5}{6}}d^5}{b^8}\right)-8\left(\frac{b^8}{d^6}\right)^{\frac{1}{6}}d\arctan\left(-\frac{\left(\frac{b^8}{d^6}\right)^{\frac{5}{6}}bd^5\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}-\sqrt{b^2\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}+\left(\frac{b^8}{d^6}\right)^{\frac{1}{3}}d^2}\left(\frac{b^8}{d^6}\right)^{\frac{5}{6}}d^5}{b^8}\right)-12b\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(3)*(b^8/d^6)^(1/6)*d*log(sqrt(3)*(b^8/d^6)^(1/6)*b*d*(b*sin(d*x
+ c)/cos(d*x + c))^(1/3) + b^2*(b*sin(d*x + c)/cos(d*x + c))^(2/3) + (b^8/d
^6)^(1/3)*d^2) - sqrt(3)*(b^8/d^6)^(1/6)*d*log(-sqrt(3)*(b^8/d^6)^(1/6)*b*d
*(b*sin(d*x + c)/cos(d*x + c))^(1/3) + b^2*(b*sin(d*x + c)/cos(d*x + c))^(2
/3) + (b^8/d^6)^(1/3)*d^2) - 4*(b^8/d^6)^(1/6)*d*arctan(-(sqrt(3)*b^8 + 2*(
b^8/d^6)^(5/6)*b*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3) - 2*sqrt(sqrt(3)*(
b^8/d^6)^(1/6)*b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3) + b^2*(b*sin(d*x + c
)/cos(d*x + c))^(2/3) + (b^8/d^6)^(1/3)*d^2)*(b^8/d^6)^(5/6)*d^5)/b^8) - 4*
(b^8/d^6)^(1/6)*d*arctan((sqrt(3)*b^8 - 2*(b^8/d^6)^(5/6)*b*d^5*(b*sin(d*x
+ c)/cos(d*x + c))^(1/3) + 2*sqrt(-sqrt(3)*(b^8/d^6)^(1/6)*b*d*(b*sin(d*x +
 c)/cos(d*x + c))^(1/3) + b^2*(b*sin(d*x + c)/cos(d*x + c))^(2/3) + (b^8/d^
6)^(1/3)*d^2)*(b^8/d^6)^(5/6)*d^5)/b^8) - 8*(b^8/d^6)^(1/6)*d*arctan(-((b^8
/d^6)^(5/6)*b*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3) - sqrt(b^2*(b*sin(d*x
 + c)/cos(d*x + c))^(2/3) + (b^8/d^6)^(1/3)*d^2)*(b^8/d^6)^(5/6)*d^5)/b^8)
- 12*b*(b*sin(d*x + c)/cos(d*x + c))^(1/3))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(c+dx\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(4/3), x)
```

Giac [A]
time = 0.52, size = 209, normalized size = 0.86

$$-\frac{1}{4}\,b\left(\frac{\sqrt{3}\,|b|^{\frac{1}{3}}\log\left(\sqrt{3}\left(b\tan\left(dx+c\right)\right)^{\frac{1}{3}}|b|^{\frac{1}{3}}+\left(b\tan\left(dx+c\right)\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{d}-\frac{\sqrt{3}\,|b|^{\frac{1}{3}}\log\left(-\sqrt{3}\left(b\tan\left(dx+c\right)\right)^{\frac{1}{3}}|b|^{\frac{1}{3}}+\left(b\tan\left(dx+c\right)\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{d}+\frac{2\,|b|^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,|b|^{\frac{1}{3}}+2\left(b\tan(dx+c)\right)^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{d}+\frac{2\,|b|^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,|b|^{\frac{1}{3}}-2\left(b\tan(dx+c)\right)^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{d}+\frac{4\,|b|^{\frac{1}{3}}\arctan\left(\frac{\left(b\tan(dx+c)\right)^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{d}-\frac{12\left(b\tan\left(dx+c\right)\right)^{\frac{1}{3}}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] -1/4*b*(sqrt(3)*abs(b)^(1/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3
) + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/d - sqrt(3)*abs(b)^(1/3)*log(-sq
rt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + abs(b)
^(2/3))/d + 2*abs(b)^(1/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*tan(d*x + c)
)^(1/3))/abs(b)^(1/3))/d + 2*abs(b)^(1/3)*arctan(-(sqrt(3)*abs(b)^(1/3) - 2
*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/d + 4*abs(b)^(1/3)*arctan((b*tan(d*x
 + c))^(1/3)/abs(b)^(1/3))/d - 12*(b*tan(d*x + c))^(1/3)/d)
```

Mupad [B]
time = 3.08, size = 247, normalized size = 1.02

$$\frac{3\,b\,{\left(b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{1/3}}{d}-\frac{{\left(-1\right)}^{1/6}\,b^{4/3}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{5/6}\,{\left(b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{1/3}\,1{}\mathrm{i}}{b^{1/3}}\right)\,1{}\mathrm{i}}{d}-\frac{{\left(-1\right)}^{1/6}\,b^{4/3}\,\ln\left({\left(-1\right)}^{1/6}\,b^{1/3}+2\,{\left(b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{1/3}+{\left(-1\right)}^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,d}-\frac{{\left(-1\right)}^{1/6}\,b^{4/3}\,\ln\left(2\,{\left(b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{1/3}-{\left(-1\right)}^{1/6}\,b^{1/3}+{\left(-1\right)}^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,d}+\frac{{\left(-1\right)}^{1/6}\,b^{4/3}\,\ln\left({\left(-1\right)}^{1/6}\,b^{1/3}-2\,{\left(b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{1/3}+{\left(-1\right)}^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}{d}+\frac{{\left(-1\right)}^{1/6}\,b^{4/3}\,\ln\left({\left(-1\right)}^{1/6}\,b^{1/3}+2\,{\left(b\,\mathrm{tan}\left(c+d\,x\right)\right)}^{1/3}-{\left(-1\right)}^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x))^(4/3),x)
```

```
[Out] (3*b*(b*tan(c + d*x))^(1/3))/d - ((-1)^(1/6)*b^(4/3)*atan(((-1)^(5/6)*(b*ta
n(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/d - ((-1)^(1/6)*b^(4/3)*log((-1)^(1/6)*b
^(1/3) + 2*(b*tan(c + d*x))^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1
i)/2 + 1/2))/(2*d) - ((-1)^(1/6)*b^(4/3)*log(2*(b*tan(c + d*x))^(1/3) - (-1
)^(1/6)*b^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*d)
 + ((-1)^(1/6)*b^(4/3)*log((-1)^(1/6)*b^(1/3) - 2*(b*tan(c + d*x))^(1/3) +
(-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/4 + 1/4))/d + ((-1)^(1/6)*b^(4/3)
*log((-1)^(1/6)*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) - (-1)^(2/3)*3^(1/2)*b^(
1/3))*((3^(1/2)*1i)/4 - 1/4))/d
```

3.18 $\int (b\tan(c+dx))^{2/3}\,dx$

Optimal. Leaf size=224

$$\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d}-\frac{b^{2/3}\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d}+\frac{b^{2/3}\text{ArcTan}\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d}$$

```
[Out] b^(2/3)*arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/d+1/2*b^(2/3)*arctan(-3^(1/2)+
2*(b*tan(d*x+c))^(1/3)/b^(1/3))/d+1/2*b^(2/3)*arctan(3^(1/2)+2*(b*tan(d*x+c
))^(1/3)/b^(1/3))/d+1/4*b^(2/3)*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c))^(
1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d-1/4*b^(2/3)*ln(b^(2/3)+b^(1/3)*3^(1/2)
*(b*tan(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/d
```

Rubi [A]
time = 0.28, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3557, 335, 301, 648, 632, 210, 642, 209}

$$\frac{b^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d}-\frac{b^{2/3}\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2d}+\frac{b^{2/3}\text{ArcTan}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2d}+\frac{\sqrt{3}\,b^{2/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}-\frac{\sqrt{3}\,b^{2/3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(2/3),x]
```

```
[Out] (b^(2/3)*ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)])/d - (b^(2/3)*ArcTan[Sqrt[3
] - (2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) + (b^(2/3)*ArcTan[Sqrt[3] +
(2*(b*Tan[c + d*x])^(1/3))/b^(1/3)])/(2*d) + (Sqrt[3]*b^(2/3)*Log[b^(2/3) -
 Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*d) -
(Sqrt[3]*b^(2/3)*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*
Tan[c + d*x])^(2/3)])/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
```

```
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (b\tan(c+dx))^{2/3}\,dx = \frac{b\text{Subst}\left(\int \frac{x^{2/3}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{(3b)\text{Subst}\left(\int \frac{x^4}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$
$$= \frac{b^{2/3}\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\frac{\sqrt{3}x}{2}}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d} + \frac{b^{2/3}\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\sqrt{}}{b^{2/3}+\sqrt{3}\sqrt[3]{}}\right.}{}$$
$$= \frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{\left(\sqrt{3}b^{2/3}\right)\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\,\mathrm{t}}\right.}{4d}$$
$$= \frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} + \frac{\sqrt{3}b^{2/3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(\right.}{4d}$$
$$= \frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2d} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 40, normalized size = 0.18

$$\frac{3\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\tan^2(c+dx)\right)(b\tan(c+dx))^{5/3}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(2/3),x]
[Out] (3*Hypergeometric2F1[5/6, 1, 11/6, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(5/3))
/(5*b*d)
```

Maple [A]
time = 0.11, size = 191, normalized size = 0.85

method	result
derivativedivides	$\frac{3b\left(\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^2)^{\frac{1}{6}}}-\sqrt{3}\right)}{6(b^2)^{\frac{1}{6}}}-\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\,\mathrm{l}}{}\right.}{d}$

default	$\frac{3b\left(\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^2)^{\frac{1}{6}}}-\sqrt{3}\right)}{6(b^2)^{\frac{1}{6}}}-\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}} \ldots}{\ldots}\right.}{d \ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c))^(2/3)-3^(1/2)*(b^2)^(
1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/(b^2)^(1/6)*arctan(2*(b*tan(d*x+
c))^(1/3)/(b^2)^(1/6)-3^(1/2))-1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c
))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/(b^2)^(1
/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))+1/3/(b^2)^(1/6)*arct
an((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))
```

Maxima [A]
time = 0.50, size = 168, normalized size = 0.75

$$-\frac{\left(\frac{\sqrt{3}\log\left(\sqrt{3}\,(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\,(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}-\frac{2\arctan\left(\frac{\sqrt{3}\,b^{\frac{1}{3}}+2(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{2\arctan\left(-\frac{\sqrt{3}\,b^{\frac{1}{3}}-2(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{4\arctan\left(\frac{(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}\right)b}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d*x + c))
^(2/3) + b^(2/3))/b^(1/3) - sqrt(3)*log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(
1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3))/b^(1/3) - 2*arctan((sqrt(3)*b^(1/3
) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 2*arctan(-(sqrt(3)*b^(1/3)
 - 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 4*arctan((b*tan(d*x + c))^(
1/3)/b^(1/3))/b^(1/3))*b/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(168) = 336$.
time = 0.41, size = 583, normalized size = 2.60

$$-\frac{1}{4}\sqrt{3}\left(\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left(\sqrt{3}\,b^3d^5\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{b^4}{d^6}\right)^{\frac{5}{6}}+b^4d^4\left(\frac{b^4}{d^6}\right)^{\frac{2}{3}}+b^6\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}\right)+\frac{1}{4}\sqrt{3}\left(\frac{b^4}{d^6}\right)^{\frac{1}{6}}\log\left(-\sqrt{3}\,b^3d^5\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{b^4}{d^6}\right)^{\frac{5}{6}}+b^4d^4\left(\frac{b^4}{d^6}\right)^{\frac{2}{3}}+b^6\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}\right)-\left(\frac{b^4}{d^6}\right)^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{3}\,b^4+2b^3d\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{b^4}{d^6}\right)^{\frac{1}{6}}-2\sqrt{\sqrt{3}\,b^3d^5\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{b^4}{d^6}\right)^{\frac{5}{6}}+b^4d^4\left(\frac{b^4}{d^6}\right)^{\frac{2}{3}}+b^6\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}}\,d\left(\frac{b^4}{d^6}\right)^{\frac{1}{6}}}{b^4}\right)-\left(\frac{b^4}{d^6}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\,b^4-2b^3d\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{b^4}{d^6}\right)^{\frac{1}{6}}+2\sqrt{-\sqrt{3}\,b^3d^5\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{b^4}{d^6}\right)^{\frac{5}{6}}+b^4d^4\left(\frac{b^4}{d^6}\right)^{\frac{2}{3}}+b^6\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}}\,d\left(\frac{b^4}{d^6}\right)^{\frac{1}{6}}}{b^4}\right)-2\left(\frac{b^4}{d^6}\right)^{\frac{1}{6}}\arctan\left(-\frac{b^3d\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{b^4}{d^6}\right)^{\frac{1}{6}}-\sqrt{b^4d^4\left(\frac{b^4}{d^6}\right)^{\frac{2}{3}}+b^6\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}}\,d\left(\frac{b^4}{d^6}\right)^{\frac{1}{6}}}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*(b^4/d^6)^(1/6)*log(sqrt(3)*b^3*d^5*(b*sin(d*x + c)/cos(d*x +
c))^(1/3)*(b^4/d^6)^(5/6) + b^4*d^4*(b^4/d^6)^(2/3) + b^6*(b*sin(d*x + c)/c
os(d*x + c))^(2/3)) + 1/4*sqrt(3)*(b^4/d^6)^(1/6)*log(-sqrt(3)*b^3*d^5*(b*s
in(d*x + c)/cos(d*x + c))^(1/3)*(b^4/d^6)^(5/6) + b^4*d^4*(b^4/d^6)^(2/3) +
```

```
 b^6*(b*sin(d*x + c)/cos(d*x + c))^(2/3)) - (b^4/d^6)^(1/6)*arctan(-(sqrt(3
)*b^4 + 2*b^3*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(b^4/d^6)^(1/6) - 2*sqr
t(sqrt(3)*b^3*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(b^4/d^6)^(5/6) + b^4
*d^4*(b^4/d^6)^(2/3) + b^6*(b*sin(d*x + c)/cos(d*x + c))^(2/3))*d*(b^4/d^6)
^(1/6))/b^4) - (b^4/d^6)^(1/6)*arctan((sqrt(3)*b^4 - 2*b^3*d*(b*sin(d*x + c
)/cos(d*x + c))^(1/3)*(b^4/d^6)^(1/6) + 2*sqrt(-sqrt(3)*b^3*d^5*(b*sin(d*x
+ c)/cos(d*x + c))^(1/3)*(b^4/d^6)^(5/6) + b^4*d^4*(b^4/d^6)^(2/3) + b^6*(b
*sin(d*x + c)/cos(d*x + c))^(2/3))*d*(b^4/d^6)^(1/6))/b^4) - 2*(b^4/d^6)^(1
/6)*arctan(-(b^3*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(b^4/d^6)^(1/6) - sq
rt(b^4*d^4*(b^4/d^6)^(2/3) + b^6*(b*sin(d*x + c)/cos(d*x + c))^(2/3))*d*(b^
4/d^6)^(1/6))/b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan (c+dx))^{\frac{2}{3}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(2/3), x)
```

Giac [A]
time = 0.47, size = 206, normalized size = 0.92

$$-\frac{\sqrt{3}\,|b|^{\frac{5}{3}}\log\left(\sqrt{3}\,(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{4\,bd}+\frac{\sqrt{3}\,|b|^{\frac{5}{3}}\log\left(-\sqrt{3}\,(b\tan(dx+c))^{\frac{1}{3}}|b|^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{4\,bd}+\frac{|b|^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}\,|b|^{\frac{1}{3}}+2\,(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{2\,bd}+\frac{|b|^{\frac{5}{3}}\arctan\left(-\frac{\sqrt{3}\,|b|^{\frac{1}{3}}-2\,(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{2\,bd}+\frac{|b|^{\frac{5}{3}}\arctan\left(\frac{(b\tan(dx+c))^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*abs(b)^(5/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) +
 (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b*d) + 1/4*sqrt(3)*abs(b)^(5/3)*lo
g(-sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + a
bs(b)^(2/3))/(b*d) + 1/2*abs(b)^(5/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*t
an(d*x + c))^(1/3))/abs(b)^(1/3))/(b*d) + 1/2*abs(b)^(5/3)*arctan(-(sqrt(3)
*abs(b)^(1/3) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b*d) + abs(b)^(5/3
)*arctan((b*tan(d*x + c))^(1/3)/abs(b)^(1/3))/(b*d)
```

Mupad [B]
time = 2.95, size = 259, normalized size = 1.16

$$\frac{{(-1)}^{1/6}\,b^{2/3}\,\mathrm{atan}\left(\frac{{(-1)}^{2/3}\,{(b\,\tan(c+d\,x))}^{1/3}}{b^{1/3}}\right)\,1{}\mathrm{i}}{d}-\frac{{(-1)}^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{486\,{(-1)}^{1/6}\,b^{26/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{(b\,\tan(c+d\,x))}^{1/3}}{d^3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,d}-\frac{{(-1)}^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{486\,{(-1)}^{1/6}\,b^{26/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{(b\,\tan(c+d\,x))}^{1/3}}{d^3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,d}+\frac{{(-1)}^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{486\,{(-1)}^{1/6}\,b^{26/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,{(b\,\tan(c+d\,x))}^{1/3}}{d^3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}{d}+\frac{{(-1)}^{1/6}\,b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{486\,{(-1)}^{1/6}\,b^{26/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)\,{(b\,\tan(c+d\,x))}^{1/3}}{d^3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x))^(2/3),x)
```

```
[Out] ((-1)^(1/6)*b^(2/3)*atan(((-1)^(2/3)*(b*tan(c + d*x))^(1/3))/b^(1/3))*1i)/d
 - ((-1)^(1/6)*b^(2/3)*log((972*b^9)/d^3 + (486*(-1)^(1/6)*b^(26/3)*(3^(1/2
)*1i - 1)*(b*tan(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 - 1/2))/(2*d) - ((-1
)^(1/6)*b^(2/3)*log((972*b^9)/d^3 + (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1i +
1)*(b*tan(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-1)^(1/6)
*b^(2/3)*log((972*b^9)/d^3 - (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1i - 1)*(b*t
an(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/4 - 1/4))/d + ((-1)^(1/6)*b^(2/3)*lo
g((972*b^9)/d^3 - (486*(-1)^(1/6)*b^(26/3)*(3^(1/2)*1i + 1)*(b*tan(c + d*x)
)^(1/3))/d^3)*((3^(1/2)*1i)/4 + 1/4))/d
```

3.19 $\int \sqrt[3]{b\tan(c+dx)}\,dx$

Optimal. Leaf size=131

$$-\frac{\sqrt{3}\sqrt[3]{b}\,\text{ArcTan}\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}\,b^{2/3}}\right)}{2d}-\frac{\sqrt[3]{b}\,\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\,\log\left(b^{4/3}-b^{2/3}(b\tan(c+\right.}{4d}$$

```
[Out] -1/2*b^(1/3)*ln(b^(2/3)+(b*tan(d*x+c))^(2/3))/d+1/4*b^(1/3)*ln(b^(4/3)-b^(2
/3)*(b*tan(d*x+c))^(2/3)+(b*tan(d*x+c))^(4/3))/d-1/2*b^(1/3)*arctan(1/3*(b^
(2/3)-2*(b*tan(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3557, 335, 281, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt{3}\sqrt[3]{b}\,\text{ArcTan}\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}\,b^{2/3}}\right)}{2d}-\frac{\sqrt[3]{b}\,\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\,\log\left(-b^{2/3}(b\tan(c+dx))^{2/3}+b^{4/3}+(b\tan(c+dx))^{4/3}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*b^(1/3)*ArcTan[(b^(2/3) - 2*(b*Tan[c + d*x])^(2/3))/(Sqrt[3]*
b^(2/3))])/d - (b^(1/3)*Log[b^(2/3) + (b*Tan[c + d*x])^(2/3)])/(2*d) + (b^(
1/3)*Log[b^(4/3) - b^(2/3)*(b*Tan[c + d*x])^(2/3) + (b*Tan[c + d*x])^(4/3)]
)/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \sqrt[3]{b\tan(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{\sqrt[3]{x}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{(3b)\text{Subst}\left(\int \frac{x^3}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$
$$= \frac{(3b)\text{Subst}\left(\int \frac{x}{b^2+x^3}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2d}$$
$$= -\frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{1}{b^{2/3}+x}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{b^{2/3}+x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\right.}{2d}$$
$$= -\frac{\sqrt[3]{b}\,\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\,\text{Subst}\left(\int \frac{-b^{2/3}+2x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+d\right.}{4d}$$
$$= -\frac{\sqrt[3]{b}\,\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\,\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\,\text{ta}\right.}{4d}$$
$$= -\frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2(b\tan(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt[3]{b}\,\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\,\text{log}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 40, normalized size = 0.31

$$\frac{3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\tan^2(c+dx)\right)(b\tan(c+dx))^{4/3}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[2/3, 1, 5/3, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(4/3))/
(4*b*d)
```

Maple [A]
time = 0.08, size = 108, normalized size = 0.82

method	result

derivativedivides	$\frac{3b\left(-\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{1}{3}}\right)}{6\left(b^2\right)^{\frac{1}{3}}}+\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-\left(b^2\right)^{\frac{1}{3}}(b\tan(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)}{12\left(b^2\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c}{\left(b^2\right)^{\frac{1}{3}}}\right.}{3}\right.}{6\left(b^2\right)^{\frac{1}{3}}}\right.}{d}$
default	$\frac{3b\left(-\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{1}{3}}\right)}{6\left(b^2\right)^{\frac{1}{3}}}+\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-\left(b^2\right)^{\frac{1}{3}}(b\tan(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)}{12\left(b^2\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c}{\left(b^2\right)^{\frac{1}{3}}}\right.}{3}\right.}{6\left(b^2\right)^{\frac{1}{3}}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(-1/6/(b^2)^(1/3)*ln((b*tan(d*x+c))^(2/3)+(b^2)^(1/3))+1/12/(b^2)^(1/
3)*ln((b*tan(d*x+c))^(4/3)-(b^2)^(1/3)*(b*tan(d*x+c))^(2/3)+(b^2)^(2/3))+1/
6*3^(1/2)/(b^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*tan(d*x+c))^(2/3
)-1)))
```

Maxima [A]
time = 0.49, size = 98, normalized size = 0.75

$$\frac{2\sqrt{3}\,b^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-b^{\frac{2}{3}}\right)}{3\,b^{\frac{2}{3}}}\right)+b^{\frac{4}{3}}\log\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)-2\,b^{\frac{4}{3}}\log\left((b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(3)*b^(4/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - b^(2/
3))/b^(2/3)) + b^(4/3)*log((b*tan(d*x + c))^(4/3) - (b*tan(d*x + c))^(2/3)*
b^(2/3) + b^(4/3)) - 2*b^(4/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b*d)
```

Fricas [A]
time = 0.39, size = 124, normalized size = 0.95

$$\frac{2\sqrt{3}\,(-b)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(b\tan(dx+c))^{\frac{2}{3}}(-b)^{\frac{1}{3}}+\sqrt{3}\,b}{3\,b}\right)-(-b)^{\frac{1}{3}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b\tan(dx+c)-(b\tan(dx+c))^{\frac{2}{3}}(-b)^{\frac{2}{3}}-(-b)^{\frac{1}{3}}b\right)+2\,(-b)^{\frac{1}{3}}\log\left((b\tan(dx+c))^{\frac{2}{3}}+(-b)^{\frac{2}{3}}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*tan(d*x + c))^(2/3)*(-b)
^(1/3) + sqrt(3)*b)/b) - (-b)^(1/3)*log((b*tan(d*x + c))^(1/3)*b*tan(d*x +
c) - (b*tan(d*x + c))^(2/3)*(-b)^(2/3) - (-b)^(1/3)*b) + 2*(-b)^(1/3)*log((
b*tan(d*x + c))^(2/3) + (-b)^(2/3)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\tan{(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(1/3), x)
```

Giac [A]
time = 0.46, size = 127, normalized size = 0.97

$$\frac{1}{4}\,b\left(\frac{2\,\sqrt{3}\,|b|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{b^2d}+\frac{|b|^{\frac{4}{3}}\log\left((b\tan\left(dx+c\right))^{\frac{1}{3}}\,b\tan\left(dx+c\right)-(b\tan\left(dx+c\right))^{\frac{2}{3}}\,|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}\right)}{b^2d}-\frac{2\,|b|^{\frac{4}{3}}\log\left((b\tan\left(dx+c\right))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{b^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] 1/4*b*(2*sqrt(3)*abs(b)^(4/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3)
- abs(b)^(2/3))/abs(b)^(2/3))/(b^2*d) + abs(b)^(4/3)*log((b*tan(d*x + c))^(
1/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*abs(b)^(2/3) + abs(b)^(4/3))/(
b^2*d) - 2*abs(b)^(4/3)*log((b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^2*d))
```

Mupad [B]
time = 2.63, size = 146, normalized size = 1.11

$$\frac{(-b)^{1/3}\,\ln\left(81\,(-b)^{16/3}\,(b\tan(c+d\,x))^{2/3}+81\,b^6\right)}{2\,d}-\frac{(-b)^{1/3}\,\ln\left(\frac{81\,b^6}{d^4}-\frac{81\,(-b)^{16/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(b\tan(c+d\,x))^{2/3}}{d^4}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}+\frac{(-b)^{1/3}\,\ln\left(\frac{81\,b^6}{d^4}+\frac{162\,(-b)^{16/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\,(b\tan(c+d\,x))^{2/3}}{d^4}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x))^(1/3),x)
```

```
[Out] ((-b)^(1/3)*log(81*(-b)^(16/3)*(b*tan(c + d*x))^(2/3) + 81*b^6))/(2*d) - ((
-b)^(1/3)*log((81*b^6)/d^4 - (81*(-b)^(16/3)*((3^(1/2)*1i)/2 + 1/2)*(b*tan(
c + d*x))^(2/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-b)^(1/3)*log((81*b
^6)/d^4 + (162*(-b)^(16/3)*((3^(1/2)*1i)/4 - 1/4)*(b*tan(c + d*x))^(2/3))/d
^4)*((3^(1/2)*1i)/4 - 1/4))/d
```

3.20 $\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx$

Optimal. Leaf size=131

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}\,b^{2/3}}\right)}{2\sqrt[3]{b}\,d}+\frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d}-\frac{\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\right.}{4\sqrt[3]{b}\,d}$$

```
[Out] 1/2*ln(b^(2/3)+(b*tan(d*x+c))^(2/3))/b^(1/3)/d-1/4*ln(b^(4/3)-b^(2/3)*(b*ta
n(d*x+c))^(2/3)+(b*tan(d*x+c))^(4/3))/b^(1/3)/d-1/2*arctan(1/3*(b^(2/3)-2*(
b*tan(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/b^(1/3)/d
```

Rubi [A]
time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3557, 335, 281, 206, 31, 648, 631, 210, 642}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{b^{2/3}-2(b\tan(c+dx))^{2/3}}{\sqrt{3}\,b^{2/3}}\right)}{2\sqrt[3]{b}\,d}+\frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d}-\frac{\log\left(-b^{2/3}(b\tan(c+dx))^{2/3}+b^{4/3}+(b\tan(c+dx))^{4/3}\right)}{4\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(b^(2/3) - 2*(b*Tan[c + d*x])^(2/3))/(Sqrt[3]*b^(2/3))
])/(b^(1/3)*d) + Log[b^(2/3) + (b*Tan[c + d*x])^(2/3)]/(2*b^(1/3)*d) - Log[
b^(4/3) - b^(2/3)*(b*Tan[c + d*x])^(2/3) + (b*Tan[c + d*x])^(4/3)]/(4*b^(1/
3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\tan(c+dx)}}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\,(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$
$$= \frac{(3b)\text{Subst}\left(\int \frac{x}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$
$$= \frac{(3b)\text{Subst}\left(\int \frac{1}{b^2+x^3}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}+x}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{2b^{2/3}-x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+\right.}{2\sqrt[3]{b}\,d}$$
$$= \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} - \frac{\text{Subst}\left(\int \frac{-b^{2/3}+2x}{b^{4/3}-b^{2/3}x+x^2}\,dx, x, (b\tan(c+dx))^{2/3}\right)}{4\sqrt[3]{b}\,d}$$
$$= \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} - \frac{\log\left(b^{4/3}-b^{2/3}(b\tan(c+dx))^{2/3}+(b\tan(c+dx\right.}{4\sqrt[3]{b}\,d}$$
$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2(b\tan(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2\sqrt[3]{b}\,d} + \frac{\log\left(b^{2/3}+(b\tan(c+dx))^{2/3}\right)}{2\sqrt[3]{b}\,d} - \frac{\log\left(b^{4/3}-b^{2}\right.}{}$$

Mathematica [A]
time = 0.16, size = 100, normalized size = 0.76

$$\frac{\left(2\sqrt{3}\,\text{ArcTan}\left(\frac{-1+2\tan^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)+2\log\left(1+\tan^{\frac{2}{3}}(c+dx)\right)-\log\left(1-\tan^{\frac{2}{3}}(c+dx)+\tan^{\frac{4}{3}}(c+dx)\right)\right)\sqrt[3]{\tan(c+dx)}}{4d\sqrt[3]{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-1/3),x]
```

```
[Out] ((2*Sqrt[3]*ArcTan[(-1 + 2*Tan[c + d*x]^(2/3))/Sqrt[3]] + 2*Log[1 + Tan[c +
 d*x]^(2/3)] - Log[1 - Tan[c + d*x]^(2/3) + Tan[c + d*x]^(4/3)])*Tan[c + d*
x]^(1/3))/(4*d*(b*Tan[c + d*x])^(1/3))
```

Maple [A]
time = 0.06, size = 108, normalized size = 0.82

method	result

derivativedivides	$\frac{3b\left(\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{2}{3}}}-\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b^2)^{\frac{1}{3}}(b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}\right.}{3}\right.}{6(b^2)^{\frac{2}{3}}}\right.}{d}$
default	$\frac{3b\left(\frac{\ln\left((b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{2}{3}}}-\frac{\ln\left((b\tan(dx+c))^{\frac{4}{3}}-(b^2)^{\frac{1}{3}}(b\tan(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\tan(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}\right.}{3}\right.}{6(b^2)^{\frac{2}{3}}}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(1/6/(b^2)^(2/3)*ln((b*tan(d*x+c))^(2/3)+(b^2)^(1/3))-1/12/(b^2)^(2/3
)*ln((b*tan(d*x+c))^(4/3)-(b^2)^(1/3)*(b*tan(d*x+c))^(2/3)+(b^2)^(2/3))+1/6
/(b^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*tan(d*x+c))^(2/3)
-1)))
```

Maxima [A]
time = 0.50, size = 99, normalized size = 0.76

$$\frac{2\sqrt{3}\,b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2(b\tan(dx+c))^{\frac{2}{3}}-b^{\frac{2}{3}}\right)}{3\,b^{\frac{2}{3}}}\right)-b^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{4}{3}}-(b\tan(dx+c))^{\frac{2}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)+2\,b^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(3)*b^(2/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - b^(2/
3))/b^(2/3)) - b^(2/3)*log((b*tan(d*x + c))^(4/3) - (b*tan(d*x + c))^(2/3)*
b^(2/3) + b^(4/3)) + 2*b^(2/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b*d)
```

Fricas [A]
time = 0.35, size = 299, normalized size = 2.28

$$\left[\frac{\sqrt{3}\,b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(\frac{2\sqrt{3}(b\tan(dx+c))^{\frac{1}{3}}b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\tan(dx+c)+2b\tan(dx+c)^2-\sqrt{3}\,b^{\frac{4}{3}}\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+(b\tan(dx+c))^{\frac{2}{3}}\left(\sqrt{3}\,b^{\frac{2}{3}}\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3b^{\frac{1}{3}}\right)-b}{\tan(dx+c)^2+1}\right)-b^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b\tan(dx+c)-(b\tan(dx+c))^{\frac{2}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)+2\,b^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{4\,bd},\ \frac{2\sqrt{3}\,b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2(b\tan(dx+c))^{\frac{2}{3}}b^{\frac{2}{3}}-b^{\frac{4}{3}}\right)}{3\,b^{\frac{4}{3}}}\right)-b^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{1}{3}}b\tan(dx+c)-(b\tan(dx+c))^{\frac{2}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)+2\,b^{\frac{2}{3}}\log\left((b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{4\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*sqrt(3)*(b*tan(d*x + c))^(1/3)*b*sq
rt(-1/b^(2/3))*tan(d*x + c) + 2*b*tan(d*x + c)^2 - sqrt(3)*b^(4/3)*sqrt(-1/
b^(2/3)) + (b*tan(d*x + c))^(2/3)*(sqrt(3)*b^(2/3)*sqrt(-1/b^(2/3)) - 3*b^(
1/3)) - b)/(tan(d*x + c)^2 + 1)) - b^(2/3)*log((b*tan(d*x + c))^(1/3)*b*tan
(d*x + c) - (b*tan(d*x + c))^(2/3)*b^(2/3) + b^(4/3)) + 2*b^(2/3)*log((b*ta
n(d*x + c))^(2/3) + b^(2/3)))/(b*d), 1/4*(2*sqrt(3)*b^(2/3)*arctan(1/3*sqrt
(3)*(2*(b*tan(d*x + c))^(2/3)*b^(2/3) - b^(4/3))/b^(4/3)) - b^(2/3)*log((b*
tan(d*x + c))^(1/3)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*b^(2/3) + b^(4/
3)) + 2*b^(2/3)*log((b*tan(d*x + c))^(2/3) + b^(2/3)))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b\tan\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-1/3), x)
```

Giac [A]
time = 0.45, size = 125, normalized size = 0.95

$$\frac{\sqrt{3}\,|b|^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(b\tan(dx+c))^{\frac{2}{3}}-|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{2\,bd}-\frac{|b|^{\frac{2}{3}}\log\left((b\tan\left(dx+c\right))^{\frac{1}{3}}\,b\tan\left(dx+c\right)-(b\tan\left(dx+c\right))^{\frac{2}{3}}\,|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}\right)}{4\,bd}+\frac{|b|^{\frac{2}{3}}\log\left((b\tan\left(dx+c\right))^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*tan(d*x + c))^(2/3) - abs
(b)^(2/3))/abs(b)^(2/3))/(b*d) - 1/4*abs(b)^(2/3)*log((b*tan(d*x + c))^(1/3
)*b*tan(d*x + c) - (b*tan(d*x + c))^(2/3)*abs(b)^(2/3) + abs(b)^(4/3))/(b*d
) + 1/2*abs(b)^(2/3)*log((b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b*d)
```

Mupad [B]
time = 2.73, size = 128, normalized size = 0.98

$$\frac{\ln\left((b\tan(c+d\,x))^{2/3}+b^{2/3}\right)}{2\,b^{1/3}\,d}+\frac{\ln\left(\frac{81\,b^{11/3}\,\left(-1+\sqrt{3}\,1i\right)}{d^3}+\frac{162\,b^3\,(b\tan(c+d\,x))^{2/3}}{d^3}\right)\,\left(-1+\sqrt{3}\,1i\right)}{4\,b^{1/3}\,d}-\frac{\ln\left(\frac{81\,b^{11/3}\,\left(1+\sqrt{3}\,1i\right)}{d^3}-\frac{162\,b^3\,(b\tan(c+d\,x))^{2/3}}{d^3}\right)\,\left(1+\sqrt{3}\,1i\right)}{4\,b^{1/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x))^(1/3),x)
```

```
[Out] log((b*tan(c + d*x))^(2/3) + b^(2/3))/(2*b^(1/3)*d) + (log((81*b^(11/3)*(3^
(1/2)*1i - 1))/d^3 + (162*b^3*(b*tan(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i - 1)
)/(4*b^(1/3)*d) - (log((81*b^(11/3)*(3^(1/2)*1i + 1))/d^3 - (162*b^3*(b*tan
(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i + 1))/(4*b^(1/3)*d)
```

3.21 $\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=224

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d}-\frac{\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d}+\frac{\text{ArcTan}\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d}-$$

```
[Out] arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d+1/2*arctan(-3^(1/2)+2*(b*tan
(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d+1/2*arctan(3^(1/2)+2*(b*tan(d*x+c))^(1/3)
/b^(1/3))/b^(2/3)/d-1/4*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c))^(1/3)+(b*
tan(d*x+c))^(2/3))*3^(1/2)/b^(2/3)/d+1/4*ln(b^(2/3)+b^(1/3)*3^(1/2)*(b*tan(
d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/b^(2/3)/d
```

Rubi [A]
time = 0.22, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3557, 335, 215, 648, 632, 210, 642, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d}-\frac{\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d}+\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2b^{2/3}d}-\frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d}+\frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-2/3),x]
```

```
[Out] ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)]/(b^(2/3)*d) - ArcTan[Sqrt[3] - (2*(b
*Tan[c + d*x])^(1/3))/b^(1/3)]/(2*b^(2/3)*d) + ArcTan[Sqrt[3] + (2*(b*Tan[c
 + d*x])^(1/3))/b^(1/3)]/(2*b^(2/3)*d) - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*b^(
1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*b^(2/3)*d) + (Sqr
t[3]*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x]
)^(2/3)])/(4*b^(2/3)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
```

```
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(b\tan(c+dx))^{2/3}}\,dx = \frac{b\text{Subst}\left(\int \frac{1}{x^{2/3}(b^2+x^2)}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= \frac{(3b)\text{Subst}\left(\int \frac{1}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{\sqrt{3}x}{2}}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{b^{2/3}d} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+\frac{\sqrt{3}x}{2}}{b^{2/3}+\sqrt{3}\sqrt[3]{b}x+x\right.}{b^{2}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\sqrt{3}\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{b}+2x}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+d\right.}{4b^{2/3}d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(\right.}{4b^{2/3}d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2b^{2/3}d} + \frac{\tan^{-1}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 38, normalized size = 0.17

$$\frac{3\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\tan^2(c+dx)\right)\sqrt[3]{b\tan(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-2/3),x]
[Out] (3*Hypergeometric2F1[1/6, 1, 7/6, -Tan[c + d*x]^2]*(b*Tan[c + d*x])^(1/3))/
(b*d)
```

Maple [A]
time = 0.10, size = 203, normalized size = 0.91

method	result
derivativedivides	$3b\left(-\frac{\sqrt{3}\left(b^2\right)^{\frac{1}{6}}\ln\left(-(b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\left(b^2\right)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{12b^2}+\frac{\left(b^2\right)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{\left(b^2\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b^2}+\right.$

default	$\frac{3b\left(-\frac{\sqrt{3}\,\left(b^2\right)^{\frac{1}{6}}\ln\left(-(b\tan(dx+c))^{\frac{2}{3}}+\sqrt{3}\,\left(b^2\right)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{12b^2}+\frac{\left(b^2\right)^{\frac{1}{6}}\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{\left(b^2\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b^2}+\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(-1/12/b^2*3^(1/2)*(b^2)^(1/6)*ln(-(b*tan(d*x+c))^(2/3)+3^(1/2)*(b^2)
^(1/6)*(b*tan(d*x+c))^(1/3)-(b^2)^(1/3))+1/6/b^2*(b^2)^(1/6)*arctan(2*(b*ta
n(d*x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))+1/12/b^2*3^(1/2)*(b^2)^(1/6)*ln((b*tan
(d*x+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/b^
2*(b^2)^(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))+1/3/b^2*(b
^2)^(1/6)*arctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))
```

Maxima [A]
time = 0.50, size = 170, normalized size = 0.76

$$\frac{\sqrt{3}\,b^{\frac{1}{3}}\log\left(\sqrt{3}\,(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)-\sqrt{3}\,b^{\frac{1}{3}}\log\left(-\sqrt{3}\,(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)+2\,b^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,b^{\frac{1}{3}}+2\,(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)+2\,b^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,b^{\frac{1}{3}}-2\,(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)+4\,b^{\frac{1}{3}}\arctan\left(\frac{(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(3)*b^(1/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d*
x + c))^(2/3) + b^(2/3)) - sqrt(3)*b^(1/3)*log(-sqrt(3)*(b*tan(d*x + c))^(1
/3)*b^(1/3) + (b*tan(d*x + c))^(2/3) + b^(2/3)) + 2*b^(1/3)*arctan((sqrt(3)
*b^(1/3) + 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 2*b^(1/3)*arctan(-(sqrt(3)*
b^(1/3) - 2*(b*tan(d*x + c))^(1/3))/b^(1/3)) + 4*b^(1/3)*arctan((b*tan(d*x
+ c))^(1/3)/b^(1/3)))/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(168) = 336$.
time = 0.37, size = 548, normalized size = 2.45

$$\frac{1}{4}\sqrt{3}\left(\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(b^2d^2\left(\frac{1}{b^4d^6}\right)^{\frac{1}{3}}+\sqrt{3}\,bd\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{1}{b^4d^6}\right)^{\frac{1}{6}}+\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}\right)-\frac{1}{4}\sqrt{3}\left(\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\log\left(b^2d^2\left(\frac{1}{b^4d^6}\right)^{\frac{1}{3}}-\sqrt{3}\,bd\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{1}{b^4d^6}\right)^{\frac{1}{6}}+\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}\right)-\left(\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\arctan\left(2\sqrt{b^2d^2\left(\frac{1}{b^4d^6}\right)^{\frac{1}{3}}+\sqrt{3}\,bd\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{1}{b^4d^6}\right)^{\frac{1}{6}}+\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}}\,b^3d^5\left(\frac{1}{b^4d^6}\right)^{\frac{5}{6}}-2\,b^3d^5\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{1}{b^4d^6}\right)^{\frac{5}{6}}-\sqrt{3}\right)-\left(\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\arctan\left(2\sqrt{b^2d^2\left(\frac{1}{b^4d^6}\right)^{\frac{1}{3}}-\sqrt{3}\,bd\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{1}{b^4d^6}\right)^{\frac{1}{6}}+\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}}\,b^3d^5\left(\frac{1}{b^4d^6}\right)^{\frac{5}{6}}-2\,b^3d^5\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{1}{b^4d^6}\right)^{\frac{5}{6}}+\sqrt{3}\right)-2\left(\frac{1}{b^4d^6}\right)^{\frac{1}{6}}\arctan\left(\sqrt{b^2d^2\left(\frac{1}{b^4d^6}\right)^{\frac{1}{3}}+\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{2}{3}}}\,b^3d^5\left(\frac{1}{b^4d^6}\right)^{\frac{5}{6}}-b^3d^5\left(\frac{b\sin(dx+c)}{\cos(dx+c)}\right)^{\frac{1}{3}}\left(\frac{1}{b^4d^6}\right)^{\frac{5}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*(1/(b^4*d^6))^(1/6)*log(b^2*d^2*(1/(b^4*d^6))^(1/3) + sqrt(3)*b
*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^4*d^6))^(1/6) + (b*sin(d*x + c
)/cos(d*x + c))^(2/3)) - 1/4*sqrt(3)*(1/(b^4*d^6))^(1/6)*log(b^2*d^2*(1/(b^
4*d^6))^(1/3) - sqrt(3)*b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^4*d^6
))^(1/6) + (b*sin(d*x + c)/cos(d*x + c))^(2/3)) - (1/(b^4*d^6))^(1/6)*arcta
```

```
n(2*sqrt(b^2*d^2*(1/(b^4*d^6))^(1/3) + sqrt(3)*b*d*(b*sin(d*x + c)/cos(d*x
+ c))^(1/3)*(1/(b^4*d^6))^(1/6) + (b*sin(d*x + c)/cos(d*x + c))^(2/3))*b^3*
d^5*(1/(b^4*d^6))^(5/6) - 2*b^3*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/
(b^4*d^6))^(5/6) - sqrt(3)) - (1/(b^4*d^6))^(1/6)*arctan(2*sqrt(b^2*d^2*(1/
(b^4*d^6))^(1/3) - sqrt(3)*b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^4*
d^6))^(1/6) + (b*sin(d*x + c)/cos(d*x + c))^(2/3))*b^3*d^5*(1/(b^4*d^6))^(5
/6) - 2*b^3*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^4*d^6))^(5/6) + s
qrt(3)) - 2*(1/(b^4*d^6))^(1/6)*arctan(sqrt(b^2*d^2*(1/(b^4*d^6))^(1/3) + (
b*sin(d*x + c)/cos(d*x + c))^(2/3))*b^3*d^5*(1/(b^4*d^6))^(5/6) - b^3*d^5*(
b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^4*d^6))^(5/6))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\tan(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(2/3),x)
[Out] Integral((b*tan(c + d*x))**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c))^(-2/3), x)
```

Mupad [B]
time = 2.70, size = 230, normalized size = 1.03

$$\frac{(-1)^{1/6}\,\mathrm{atan}\left(\frac{(-1)^{5/6}\,(b\tan(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{b^{2/3}\,d}-\frac{(-1)^{1/6}\,\ln\left((-1)^{1/6}\,b^{1/3}-2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{2/3}\,d}-\frac{(-1)^{1/6}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}-(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{2/3}\,d}+\frac{(-1)^{1/6}\,\ln\left((-1)^{1/6}\,b^{1/3}+2\,(b\tan(c+d\,x))^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{2/3}\,d}+\frac{(-1)^{1/6}\,\ln\left(2\,(b\tan(c+d\,x))^{1/3}-(-1)^{1/6}\,b^{1/3}+(-1)^{2/3}\,\sqrt{3}\,b^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{2/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x))^(2/3),x)
[Out] ((-1)^(1/6)*atan(((-1)^(5/6)*(b*tan(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/(b^(2/
3)*d) - ((-1)^(1/6)*log((-1)^(1/6)*b^(1/3) - 2*(b*tan(c + d*x))^(1/3) + (-1
)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(2/3)*d) - ((-1)^(1/6
)*log((-1)^(1/6)*b^(1/3) + 2*(b*tan(c + d*x))^(1/3) - (-1)^(2/3)*3^(1/2)*b^
(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(2/3)*d) + ((-1)^(1/6)*log((-1)^(1/6)*b
^(1/3) + 2*(b*tan(c + d*x))^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1
i)/4 + 1/4))/(b^(2/3)*d) + ((-1)^(1/6)*log(2*(b*tan(c + d*x))^(1/3) - (-1)^
(1/6)*b^(1/3) + (-1)^(2/3)*3^(1/2)*b^(1/3))*((3^(1/2)*1i)/4 - 1/4))/(b^(2/3
)*d)
```

3.22 $\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=245

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d}+\frac{\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d}-\frac{\text{ArcTan}\left(\sqrt{3}+\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d}-$$

```
[Out] -arctan((b*tan(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d-1/2*arctan(-3^(1/2)+2*(b*ta
n(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d-1/2*arctan(3^(1/2)+2*(b*tan(d*x+c))^(1/3
)/b^(1/3))/b^(4/3)/d-1/4*ln(b^(2/3)-b^(1/3)*3^(1/2)*(b*tan(d*x+c))^(1/3)+(b
*tan(d*x+c))^(2/3))*3^(1/2)/b^(4/3)/d+1/4*ln(b^(2/3)+b^(1/3)*3^(1/2)*(b*tan
(d*x+c))^(1/3)+(b*tan(d*x+c))^(2/3))*3^(1/2)/b^(4/3)/d-3/b/d/(b*tan(d*x+c))
^(1/3)
```

Rubi [A]
time = 0.30, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3555, 3557, 335, 301, 648, 632, 210, 642, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d}+\frac{\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d}-\frac{\text{ArcTan}\left(\frac{2\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}+\sqrt{3}\right)}{2b^{4/3}d}-\frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d}+\frac{\sqrt{3}\log\left(b^{2/3}+\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan(c+dx))^{2/3}\right)}{4b^{4/3}d}-\frac{3}{bd\sqrt[3]{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^(-4/3),x]
```

```
[Out] -(ArcTan[(b*Tan[c + d*x])^(1/3)/b^(1/3)]/(b^(4/3)*d)) + ArcTan[Sqrt[3] - (2
*(b*Tan[c + d*x])^(1/3))/b^(1/3)]/(2*b^(4/3)*d) - ArcTan[Sqrt[3] + (2*(b*Ta
n[c + d*x])^(1/3))/b^(1/3)]/(2*b^(4/3)*d) - (Sqrt[3]*Log[b^(2/3) - Sqrt[3]*
b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d*x])^(2/3)])/(4*b^(4/3)*d) + (
Sqrt[3]*Log[b^(2/3) + Sqrt[3]*b^(1/3)*(b*Tan[c + d*x])^(1/3) + (b*Tan[c + d
*x])^(2/3)])/(4*b^(4/3)*d) - 3/(b*d*(b*Tan[c + d*x])^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(b\tan(c+dx))^{4/3}}\,dx = -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}} - \frac{\int (b\tan(c+dx))^{2/3}\,dx}{b^2}$$
$$= -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{x^{2/3}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$
$$= -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}} - \frac{3\text{Subst}\left(\int \frac{x^4}{b^2+x^6}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{bd}$$
$$= -\frac{3}{bd\sqrt[3]{b\tan(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\frac{\sqrt{3}x}{2}}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\tan(c+dx)}\right)}{b^{4/3}d} - \text{Su}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\tan(c+dx)}} - \frac{\sqrt{3}\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[3]{b}+2}{b^{2/3}-\sqrt{3}\sqrt[3]{b}x}\right.}{4b^4}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{\sqrt{3}\log\left(b^{2/3}-\sqrt{3}\sqrt[3]{b}\sqrt[3]{b\tan(c+dx)}+(b\tan\right.}{4b^{4/3}d}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} + \frac{\tan^{-1}\left(\frac{1}{3}\left(3\sqrt{3}-\frac{6\sqrt[3]{b\tan(c+dx)}}{\sqrt[3]{b}}\right)\right)}{2b^{4/3}d} - \tan^{-1}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 38, normalized size = 0.16

$$-\frac{3\,{}_2F_1\left(-\frac{1}{6},1;\frac{5}{6};-\tan^2(c+dx)\right)}{bd\sqrt[3]{b\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/6, 1, 5/6, -Tan[c + d*x]^2])/(b*d*(b*Tan[c + d*x])
^(1/3))
```

Maple [A]
time = 0.07, size = 212, normalized size = 0.87

method	result
derivativedivides	$\frac{3b\left(-\frac{\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^2)^{\frac{1}{6}}}-\sqrt{3}\right)}{6(b^2)^{\frac{1}{6}}}-\frac{\sqrt{3}\,(b^2 \cdots}{\cdots}\right.}{\cdots}$
default	$\frac{3b\left(-\frac{\frac{\sqrt{3}\,(b^2)^{\frac{5}{6}}\ln\left((b\tan(dx+c))^{\frac{2}{3}}-\sqrt{3}\,(b^2)^{\frac{1}{6}}(b\tan(dx+c))^{\frac{1}{3}}+(b^2)^{\frac{1}{3}}\right)}{12b^2}+\frac{\arctan\left(\frac{2(b\tan(dx+c))^{\frac{1}{3}}}{(b^2)^{\frac{1}{6}}}-\sqrt{3}\right)}{6(b^2)^{\frac{1}{6}}}-\frac{\sqrt{3}\,(b^2 \cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/d*b*(-(1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x+c))^(2/3)-3^(1/2)*(b^2)
^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/(b^2)^(1/6)*arctan(2*(b*tan(d*
x+c))^(1/3)/(b^2)^(1/6)-3^(1/2))-1/12/b^2*3^(1/2)*(b^2)^(5/6)*ln((b*tan(d*x
+c))^(2/3)+3^(1/2)*(b^2)^(1/6)*(b*tan(d*x+c))^(1/3)+(b^2)^(1/3))+1/6/(b^2)^
(1/6)*arctan(2*(b*tan(d*x+c))^(1/3)/(b^2)^(1/6)+3^(1/2))+1/3/(b^2)^(1/6)*ar
ctan((b*tan(d*x+c))^(1/3)/(b^2)^(1/6)))/b^2-1/b^2/(b*tan(d*x+c))^(1/3))
```

Maxima [A]
time = 0.49, size = 182, normalized size = 0.74

$$\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\,(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\,(b\tan(dx+c))^{\frac{1}{3}}b^{\frac{1}{3}}+(b\tan(dx+c))^{\frac{2}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{1}{3}}}-\frac{2\arctan\left(\frac{\sqrt{3}\,b^{\frac{1}{3}}+2(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{2\arctan\left(-\frac{\sqrt{3}\,b^{\frac{1}{3}}-2(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{4\arctan\left(\frac{(b\tan(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{12}{(b\tan(dx+c))^{\frac{1}{3}}}}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1/3) + (b*tan(d*x + c))^
(2/3) + b^(2/3))/b^(1/3) - sqrt(3)*log(-sqrt(3)*(b*tan(d*x + c))^(1/3)*b^(1
/3) + (b*tan(d*x + c))^(2/3) + b^(2/3))/b^(1/3) - 2*arctan((sqrt(3)*b^(1/3)
 + 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 2*arctan(-(sqrt(3)*b^(1/3)
- 2*(b*tan(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 4*arctan((b*tan(d*x + c))^(1
/3)/b^(1/3))/b^(1/3) - 12/(b*tan(d*x + c))^(1/3))/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(187) = 374$.

time = 0.41, size = 701, normalized size = 2.86

$$\frac{12 \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{2}{3}} \cos\left(dx+c\right) \sin\left(dx+c\right) + 4 \left(b^{2} d \cos\left(dx+c\right)^{2} - b^{2} d\right) \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{1}{6}} \arctan\left(2 \sqrt{\sqrt{3} b^{7} d^{5} \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{1}{3}} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{5}{6}} + b^{6} d^{4} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{2}{3}} + \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{2}{3}}}\, b d \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{1}{6}} - 2 b d \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{1}{3}} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{1}{6}} - \sqrt{3}\right) + 4 \left(b^{2} d \cos\left(dx+c\right)^{2} - b^{2} d\right) \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{1}{6}} \arctan\left(2 \sqrt{-\sqrt{3} b^{7} d^{5} \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{1}{3}} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{5}{6}} + b^{6} d^{4} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{2}{3}} + \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{2}{3}}}\, b d \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{1}{6}} - 2 b d \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{1}{3}} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{1}{6}} + \sqrt{3}\right) + 8 \left(b^{2} d \cos\left(dx+c\right)^{2} - b^{2} d\right) \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{1}{6}} \arctan\left(\sqrt{b^{6} d^{4} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{2}{3}} + \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{2}{3}}}\, b d \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{1}{6}} - b d \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{1}{3}} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{1}{6}}\right) + \left(\sqrt{3} b^{2} d \cos\left(dx+c\right)^{2} - \sqrt{3} b^{2} d\right) \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{1}{6}} \log\left(\sqrt{3} b^{7} d^{5} \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{1}{3}} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{5}{6}} + b^{6} d^{4} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{2}{3}} + \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{2}{3}}\right) - \left(\sqrt{3} b^{2} d \cos\left(dx+c\right)^{2} - \sqrt{3} b^{2} d\right) \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{1}{6}} \log\left(-\sqrt{3} b^{7} d^{5} \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{1}{3}} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{5}{6}} + b^{6} d^{4} \left(\frac{1}{b^{8} d^{6}}\right)^{\frac{2}{3}} + \left(\frac{b \sin\left(dx+c\right)}{\cos\left(dx+c\right)}\right)^{\frac{2}{3}}\right)}{4 \left(b^{2} d \cos\left(dx+c\right)^{2} - b^{2} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] 1/4*(12*(b*sin(d*x + c)/cos(d*x + c))^(2/3)*cos(d*x + c)*sin(d*x + c) + 4*(
b^2*d*cos(d*x + c)^2 - b^2*d)*(1/(b^8*d^6))^(1/6)*arctan(2*sqrt(sqrt(3)*b^7
*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(5/6) + b^6*d^4*(1/(
b^8*d^6))^(2/3) + (b*sin(d*x + c)/cos(d*x + c))^(2/3))*b*d*(1/(b^8*d^6))^(1
/6) - 2*b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(1/6) - sqrt(
3)) + 4*(b^2*d*cos(d*x + c)^2 - b^2*d)*(1/(b^8*d^6))^(1/6)*arctan(2*sqrt(-s
qrt(3)*b^7*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(5/6) + b^
6*d^4*(1/(b^8*d^6))^(2/3) + (b*sin(d*x + c)/cos(d*x + c))^(2/3))*b*d*(1/(b^
8*d^6))^(1/6) - 2*b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(1/
6) + sqrt(3)) + 8*(b^2*d*cos(d*x + c)^2 - b^2*d)*(1/(b^8*d^6))^(1/6)*arctan
(sqrt(b^6*d^4*(1/(b^8*d^6))^(2/3) + (b*sin(d*x + c)/cos(d*x + c))^(2/3))*b*
d*(1/(b^8*d^6))^(1/6) - b*d*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6
))^(1/6)) + (sqrt(3)*b^2*d*cos(d*x + c)^2 - sqrt(3)*b^2*d)*(1/(b^8*d^6))^(1
/6)*log(sqrt(3)*b^7*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(
5/6) + b^6*d^4*(1/(b^8*d^6))^(2/3) + (b*sin(d*x + c)/cos(d*x + c))^(2/3)) -
 (sqrt(3)*b^2*d*cos(d*x + c)^2 - sqrt(3)*b^2*d)*(1/(b^8*d^6))^(1/6)*log(-sq
rt(3)*b^7*d^5*(b*sin(d*x + c)/cos(d*x + c))^(1/3)*(1/(b^8*d^6))^(5/6) + b^6
*d^4*(1/(b^8*d^6))^(2/3) + (b*sin(d*x + c)/cos(d*x + c))^(2/3)))/(b^2*d*cos
(d*x + c)^2 - b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \tan\left(c + dx\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*tan(c + d*x))**(-4/3), x)
```

Giac [A]
time = 0.57, size = 227, normalized size = 0.93

$$\frac{1}{4} b \left(\frac{\sqrt{3} |b|^{\frac{5}{3}} \log\left(\sqrt{3} \left(b \tan\left(dx+c\right)\right)^{\frac{1}{3}} |b|^{\frac{1}{3}} + \left(b \tan\left(dx+c\right)\right)^{\frac{2}{3}} + |b|^{\frac{2}{3}}\right)}{b^{4} d} - \frac{\sqrt{3} |b|^{\frac{5}{3}} \log\left(-\sqrt{3} \left(b \tan\left(dx+c\right)\right)^{\frac{1}{3}} |b|^{\frac{1}{3}} + \left(b \tan\left(dx+c\right)\right)^{\frac{2}{3}} + |b|^{\frac{2}{3}}\right)}{b^{4} d} - \frac{2 |b|^{\frac{5}{3}} \arctan\left(\frac{\sqrt{3} |b|^{\frac{1}{3}} + 2 \left(b \tan\left(dx+c\right)\right)^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{b^{4} d} - \frac{2 |b|^{\frac{5}{3}} \arctan\left(-\frac{\sqrt{3} |b|^{\frac{1}{3}} - 2 \left(b \tan\left(dx+c\right)\right)^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{b^{4} d} - \frac{4 |b|^{\frac{5}{3}} \arctan\left(\frac{\left(b \tan\left(dx+c\right)\right)^{\frac{1}{3}}}{|b|^{\frac{1}{3}}}\right)}{b^{4} d} - \frac{12}{\left(b \tan\left(dx+c\right)\right)^{\frac{1}{3}} b^{2} d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] 1/4*b*(sqrt(3)*abs(b)^(5/3)*log(sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3)
 + (b*tan(d*x + c))^(2/3) + abs(b)^(2/3))/(b^4*d) - sqrt(3)*abs(b)^(5/3)*lo
g(-sqrt(3)*(b*tan(d*x + c))^(1/3)*abs(b)^(1/3) + (b*tan(d*x + c))^(2/3) + a
bs(b)^(2/3))/(b^4*d) - 2*abs(b)^(5/3)*arctan((sqrt(3)*abs(b)^(1/3) + 2*(b*t
an(d*x + c))^(1/3))/abs(b)^(1/3))/(b^4*d) - 2*abs(b)^(5/3)*arctan(-(sqrt(3)
*abs(b)^(1/3) - 2*(b*tan(d*x + c))^(1/3))/abs(b)^(1/3))/(b^4*d) - 4*abs(b)^
(5/3)*arctan((b*tan(d*x + c))^(1/3)/abs(b)^(1/3))/(b^4*d) - 12/((b*tan(d*x
+ c))^(1/3)*b^2*d))
```

Mupad [B]
time = 2.55, size = 278, normalized size = 1.13

$$-\frac{3}{b\,d\,{\left(b\,\tan\left(c+d\,x\right)\right)}^{1/3}}-\frac{{\left(-1\right)}^{1/6}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,{\left(b\,\tan\left(c+d\,x\right)\right)}^{1/3}}{b^{1/3}}\right)\,1{}\mathrm{i}}{b^{4/3}\,d}-\frac{{\left(-1\right)}^{1/6}\,\ln\left(972\,b^{12}\,d^6-972\,{\left(-1\right)}^{1/6}\,b^{35/3}\,d^6\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(b\,\tan\left(c+d\,x\right)\right)}^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,b^{4/3}\,d}-\frac{{\left(-1\right)}^{1/6}\,\ln\left(972\,b^{12}\,d^6-972\,{\left(-1\right)}^{1/6}\,b^{35/3}\,d^6\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(b\,\tan\left(c+d\,x\right)\right)}^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,b^{4/3}\,d}+\frac{{\left(-1\right)}^{1/6}\,\ln\left(972\,b^{12}\,d^6+1944\,{\left(-1\right)}^{1/6}\,b^{35/3}\,d^6\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)\,{\left(b\,\tan\left(c+d\,x\right)\right)}^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}{b^{4/3}\,d}+\frac{{\left(-1\right)}^{1/6}\,\ln\left(972\,b^{12}\,d^6+1944\,{\left(-1\right)}^{1/6}\,b^{35/3}\,d^6\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)\,{\left(b\,\tan\left(c+d\,x\right)\right)}^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}{b^{4/3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x))^(4/3),x)
```

```
[Out] ((-1)^(1/6)*log(972*b^12*d^6 + 1944*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/4
 - 1/4)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/4 - 1/4))/(b^(4/3)*d) - ((-1)
^(1/6)*atan(((-1)^(2/3)*(b*tan(c + d*x))^(1/3))/b^(1/3))*1i)/(b^(4/3)*d) -
((-1)^(1/6)*log(972*b^12*d^6 - 972*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/2
- 1/2)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*b^(4/3)*d) - ((-1
)^(1/6)*log(972*b^12*d^6 - 972*(-1)^(1/6)*b^(35/3)*d^6*((3^(1/2)*1i)/2 + 1/
2)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(4/3)*d) - 3/(b*d*(
b*tan(c + d*x))^(1/3)) + ((-1)^(1/6)*log(972*b^12*d^6 + 1944*(-1)^(1/6)*b^(
35/3)*d^6*((3^(1/2)*1i)/4 + 1/4)*(b*tan(c + d*x))^(1/3))*((3^(1/2)*1i)/4 +
1/4))/(b^(4/3)*d)
```

3.23 $\int (b\tan(c+dx))^n\,dx$

Optimal. Leaf size=50

$$\frac{{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(c+dx)\right)(b\tan(c+dx))^{1+n}}{bd(1+n)}$$

```
[Out] hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(d*x+c)^2)*(b*tan(d*x+c))^(1+n)/b/
d/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3557, 371}

$$\frac{(b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(c+dx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[c + d*x]^2]*(b*Tan[c + d*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\tan(c+dx))^n\,dx &= \frac{b\text{Subst}\left(\int \frac{x^n}{b^2+x^2}\,dx,x,b\tan(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(c+dx)\right)(b\tan(c+dx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 53, normalized size = 1.06

$$\frac{{}_2F_1\left(1,\frac{1+n}{2};1+\frac{1+n}{2};-\tan^2(c+dx)\right)\tan(c+dx)(b\tan(c+dx))^n}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x])^n)/(d*(1 + n))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (b\tan(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c))^n,x)
```

```
[Out] int((b*tan(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))**n,x)
```

```
[Out] Integral((b*tan(c + d*x))**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\tan(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x))^n,x)
```

```
[Out] int((b*tan(c + d*x))^n, x)
```

3.24 $\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=98

$$-\frac{b^2\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d}-\frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}+\frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d}$$

```
[Out] -b^2*cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d-1/2*b^2*(b*tan(d*x+
c)^2)^(1/2)*tan(d*x+c)/d+1/4*b^2*(b*tan(d*x+c)^2)^(1/2)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 3556}

$$-\frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}+\frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d}-\frac{b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\,\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] -((b^2*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d) - (b^2*Tan
[c + d*x]*Sqrt[b*Tan[c + d*x]^2])/(2*d) + (b^2*Tan[c + d*x]^3*Sqrt[b*Tan[c
+ d*x]^2])/(4*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^2(c+dx)\right)^{5/2}\,dx = \left(b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int \tan^5(c+dx)\,dx$$
$$= \frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d} - \left(b^2\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int \tan^3(c$$
$$= -\frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^2(c+dx)}}{4d} + \Big(b^2\cot($$
$$= -\frac{b^2\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d} - \frac{b^2\tan(c+dx)\sqrt{b\tan^2(c+}}{2d}$$

Mathematica [A]
time = 0.40, size = 56, normalized size = 0.57

$$-\frac{\cot(c+dx)\left(-1+2\cot^2(c+dx)+4\cot^4(c+dx)\log(\cos(c+dx))\right)\left(b\tan^2(c+dx)\right)^{5/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^2)^(5/2),x]
```

```
[Out] -1/4*(Cot[c + d*x]*(-1 + 2*Cot[c + d*x]^2 + 4*Cot[c + d*x]^4*Log[Cos[c + d*
x]])*(b*Tan[c + d*x]^2)^(5/2))/d
```

Maple [A]
time = 0.11, size = 58, normalized size = 0.59

method	result
derivativedivides	$\frac{\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}\left(\tan^4(dx+c)-2\left(\tan^2(dx+c)\right)+2\ln\left(1+\tan^2(dx+c)\right)\right)}{4d\tan(dx+c)^5}$
default	$\frac{\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}\left(\tan^4(dx+c)-2\left(\tan^2(dx+c)\right)+2\ln\left(1+\tan^2(dx+c)\right)\right)}{4d\tan(dx+c)^5}$
risch	$\frac{b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1} - \frac{2b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d} - \frac{4ib^2\sqrt{-\frac{b\left(e^{2i(dx-}\right.}{\left(e^{2i(dx+}\right.}}}{\left(e^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*(b*tan(d*x+c)^2)^(5/2)*(tan(d*x+c)^4-2*tan(d*x+c)^2+2*ln(1+tan(d*x+c)
^2))/tan(d*x+c)^5
```

Maxima [A]
time = 0.51, size = 47, normalized size = 0.48

$$\frac{b^{\frac{5}{2}}\tan\left(dx+c\right)^{4}-2\,b^{\frac{5}{2}}\tan\left(dx+c\right)^{2}+2\,b^{\frac{5}{2}}\log\left(\tan\left(dx+c\right)^{2}+1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b^(5/2)*tan(d*x + c)^4 - 2*b^(5/2)*tan(d*x + c)^2 + 2*b^(5/2)*log(tan(
d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.35, size = 74, normalized size = 0.76

$$\frac{\left(b^{2}\tan\left(dx+c\right)^{4}-2\,b^{2}\tan\left(dx+c\right)^{2}-2\,b^{2}\log\left(\frac{1}{\tan(dx+c)^{2}+1}\right)-3\,b^{2}\right)\sqrt{b\tan\left(dx+c\right)^{2}}}{4\,d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*tan(d*x + c)^4 - 2*b^2*tan(d*x + c)^2 - 2*b^2*log(1/(tan(d*x + c)^
2 + 1)) - 3*b^2)*sqrt(b*tan(d*x + c)^2)/(d*tan(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan^{2}\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 696 vs. 2(88) = 176.
time = 1.16, size = 696, normalized size = 7.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*sgn(tan(d*x +
```

```
c))*tan(d*x)^4*tan(c)^4 + 3*b^2*sgn(tan(d*x + c))*tan(d*x)^4*tan(c)^4 - 8*
b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*sgn(tan(d*x + c))*tan
(d*x)^3*tan(c)^3 + 2*b^2*sgn(tan(d*x + c))*tan(d*x)^4*tan(c)^2 - 8*b^2*sgn(
tan(d*x + c))*tan(d*x)^3*tan(c)^3 + 2*b^2*sgn(tan(d*x + c))*tan(d*x)^2*tan(
c)^4 + 12*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*sgn(tan(d*x
 + c))*tan(d*x)^2*tan(c)^2 - b^2*sgn(tan(d*x + c))*tan(d*x)^4 - 8*b^2*sgn(t
an(d*x + c))*tan(d*x)^3*tan(c) + 4*b^2*sgn(tan(d*x + c))*tan(d*x)^2*tan(c)^
2 - 8*b^2*sgn(tan(d*x + c))*tan(d*x)*tan(c)^3 - b^2*sgn(tan(d*x + c))*tan(c
)^4 - 8*b^2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*t
an(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*sgn(tan(d*x +
 c))*tan(d*x)*tan(c) + 2*b^2*sgn(tan(d*x + c))*tan(d*x)^2 - 8*b^2*sgn(tan(d
*x + c))*tan(d*x)*tan(c) + 2*b^2*sgn(tan(d*x + c))*tan(c)^2 + 2*b^2*log(4*(
tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^
2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*sgn(tan(d*x + c)) + 3*b^2*sgn(ta
n(d*x + c)))*sqrt(b)/(d*tan(d*x)^4*tan(c)^4 - 4*d*tan(d*x)^3*tan(c)^3 + 6*d
*tan(d*x)^2*tan(c)^2 - 4*d*tan(d*x)*tan(c) + d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan(c+d\,x)^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^2)^(5/2),x)
```

```
[Out] int((b*tan(c + d*x)^2)^(5/2), x)
```

3.25 $\int \left(b\tan^2(c+dx)\right)^{3/2}\,dx$

Optimal. Leaf size=61

$$\frac{b\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d}+\frac{b\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}$$

```
[Out] b*cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d+1/2*b*(b*tan(d*x+c)^2)
^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$\frac{b\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}+\frac{b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\,\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (b*Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d + (b*Tan[c + d*
x]*Sqrt[b*Tan[c + d*x]^2])/(2*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^2(c+dx)\right)^{3/2} dx = \left(b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int \tan^3(c+dx)\,dx$$

$$= \frac{b\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d} - \left(b\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int \tan(c+d$$

$$= \frac{b\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d} + \frac{b\tan(c+dx)\sqrt{b\tan^2(c+dx)}}{2d}$$

Mathematica [A]
time = 0.12, size = 47, normalized size = 0.77

$$\frac{\cot^3(c+dx)\left(b\tan^2(c+dx)\right)^{3/2}\left(2\log(\cos(c+dx))+\tan^2(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^2)^(3/2),x]
```

```
[Out] (Cot[c + d*x]^3*(b*Tan[c + d*x]^2)^(3/2)*(2*Log[Cos[c + d*x]] + Tan[c + d*x
]^2))/(2*d)
```

Maple [A]
time = 0.08, size = 48, normalized size = 0.79

method	result
derivativedivides	$-\frac{\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}\left(-\left(\tan^2(dx+c)\right)+\ln\left(1+\tan^2(dx+c)\right)\right)}{2d\tan(dx+c)^3}$
default	$-\frac{\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}\left(-\left(\tan^2(dx+c)\right)+\ln\left(1+\tan^2(dx+c)\right)\right)}{2d\tan(dx+c)^3}$
risch	$-\frac{b\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,x}{e^{2i(dx+c)}-1} + \frac{2b\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,(dx+c)}{\left(e^{2i(dx+c)}-1\right)d} + \frac{2ib\sqrt{-\frac{b\left(e^{2i(dx+\right.}}{\left(e^{2i(dx+c\right.}}}}{\left(e^{2i(dx+c)}-1\right)(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(b*tan(d*x+c)^2)^(3/2)*(-tan(d*x+c)^2+ln(1+tan(d*x+c)^2))/tan(d*x+c)
^3
```

Maxima [A]
time = 0.50, size = 34, normalized size = 0.56

$$\frac{b^{\frac{3}{2}}\tan\left(dx+c\right)^2 - b^{\frac{3}{2}}\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^(3/2)*tan(d*x + c)^2 - b^(3/2)*log(tan(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.37, size = 52, normalized size = 0.85

$$\frac{\left(b\tan\left(dx+c\right)^{2}+b\log\left(\frac{1}{\tan\left(dx+c\right)^{2}+1}\right)+b\right)\sqrt{b\tan\left(dx+c\right)^{2}}}{2\,d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*tan(d*x + c)^2 + b*log(1/(tan(d*x + c)^2 + 1)) + b)*sqrt(b*tan(d*x +
 c)^2)/(d*tan(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan^{2}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(55) = 110.
time = 0.74, size = 256, normalized size = 4.20

$$\frac{\left(\log\left(\frac{4\left(\tan\left(dx\right)^{4}\tan\left(c\right)^{2}-2\tan\left(dx\right)^{3}\tan\left(c\right)+\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+\tan\left(dx\right)^{2}-2\tan\left(dx\right)\tan\left(c\right)+1\right)}{\tan\left(c\right)^{2}+1}\right)\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-2\log\left(\frac{4\left(\tan\left(dx\right)^{4}\tan\left(c\right)^{2}-2\tan\left(dx\right)^{3}\tan\left(c\right)+\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+\tan\left(dx\right)^{2}-2\tan\left(dx\right)\tan\left(c\right)+1\right)}{\tan\left(c\right)^{2}+1}\right)\tan\left(dx\right)\tan\left(c\right)+\tan\left(dx\right)^{2}+\tan\left(c\right)^{2}+\log\left(\frac{4\left(\tan\left(dx\right)^{4}\tan\left(c\right)^{2}-2\tan\left(dx\right)^{3}\tan\left(c\right)+\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+\tan\left(dx\right)^{2}-2\tan\left(dx\right)\tan\left(c\right)+1\right)}{\tan\left(c\right)^{2}+1}\right)+1\right)b^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}{2\left(d\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-2\,d\tan\left(dx\right)\tan\left(c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2
+ tan(d*x)^2*tan(c)^2 - 2*log(4*(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
+ tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(c)^2 + 1))
*tan(d*x)*tan(c) + tan(d*x)^2 + tan(c)^2 + log(4*(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
)/(tan(c)^2 + 1)) + 1)*b^(3/2)*sgn(tan(d*x + c))/(d*tan(d*x)^2*tan(c)^2 - 2
*d*tan(d*x)*tan(c) + d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\tan(c+d\,x)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^2)^(3/2),x)
[Out] int((b*tan(c + d*x)^2)^(3/2), x)
```

3.26 $\int \sqrt{b \tan^2(c+dx)}\, dx$

Optimal. Leaf size=32

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d}$$

```
[Out] -cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^2)^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3739, 3556}

$$-\frac{\cot(c+dx)\sqrt{b\tan^2(c+dx)}\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[c + d*x]^2],x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\tan^2(c+dx)}\, dx &= \left(\cot(c+dx)\sqrt{b\tan^2(c+dx)}\right)\int \tan(c+dx)\, dx\\ &= -\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 1.00

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\sqrt{b\tan^2(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^2],x]
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*Sqrt[b*Tan[c + d*x]^2])/d)
```

Maple [A]
time = 0.08, size = 37, normalized size = 1.16

method	result
derivativedivides	$\frac{\sqrt{b\left(\tan^2\left(dx+c\right)\right)}\,\ln(1+\tan^2(dx+c))}{2d\tan(dx+c)}$
default	$\frac{\sqrt{b\left(\tan^2\left(dx+c\right)\right)}\,\ln(1+\tan^2(dx+c))}{2d\tan(dx+c)}$
risch	$\frac{\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(e^{2i(dx+c)}+1\right)x}{e^{2i(dx+c)}-1}-\frac{2\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,\left(e^{2i(dx+c)}+1\right)(dx+c)}{\left(e^{2i(dx+c)}-1\right)d}-\frac{i\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}{\left(e^{2\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/d*(b*tan(d*x+c)^2)^(1/2)/tan(d*x+c)*ln(1+tan(d*x+c)^2)
```

Maxima [A]
time = 0.52, size = 19, normalized size = 0.59

$$\frac{\sqrt{b}\,\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(b)*log(tan(d*x + c)^2 + 1)/d
```

Fricas [A]
time = 0.35, size = 38, normalized size = 1.19

$$-\frac{\sqrt{b\tan\left(dx+c\right)^2}\,\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*tan(d*x + c)^2)*log(1/(tan(d*x + c)^2 + 1))/(d*tan(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(c + d*x)**2), x)
```

Giac [A]
time = 0.45, size = 23, normalized size = 0.72

$$-\frac{\sqrt{b}\,\log\left(\left|\cos\left(dx+c\right)\right|\right)\operatorname{sgn}(\tan\left(dx+c\right))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b)*log(abs(cos(d*x + c)))*sgn(tan(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{b\tan\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^2)^(1/2),x)
```

```
[Out] int((b*tan(c + d*x)^2)^(1/2), x)
```

3.27 $\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx$

Optimal. Leaf size=31

$$\frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{b\tan^2(c+dx)}}$$

```
[Out] ln(sin(d*x+c))*tan(d*x+c)/d/(b*tan(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{3739, 3556\}$

$$\frac{\tan(c+dx)\log(\sin(c+dx))}{d\sqrt{b\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^2],x]
```

```
[Out] (Log[Sin[c + d*x]]*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*x]^2])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx &= \frac{\tan(c+dx)\int \cot(c+dx)\,dx}{\sqrt{b\tan^2(c+dx)}}\\ &= \frac{\log(\sin(c+dx))\tan(c+dx)}{d\sqrt{b\tan^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 39, normalized size = 1.26

$$\frac{(\log(\cos(c+dx))+\log(\tan(c+dx)))\tan(c+dx)}{d\sqrt{b\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^2],x]
```

```
[Out] ((Log[Cos[c + d*x]] + Log[Tan[c + d*x]])*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*
x]^2])
```

Maple [A]
time = 0.09, size = 47, normalized size = 1.52

method	result
derivativedivides	$\frac{\tan(dx+c)(2\ln(\tan(dx+c))-\ln(1+\tan^2(dx+c)))}{2d\sqrt{b(\tan^2(dx+c))}}$
default	$\frac{\tan(dx+c)(2\ln(\tan(dx+c))-\ln(1+\tan^2(dx+c)))}{2d\sqrt{b(\tan^2(dx+c))}}$
risch	$\frac{(e^{2i(dx+c)}-1)x}{\sqrt{-\frac{b(e^{2i(dx+c)}-1)^2}{(e^{2i(dx+c)}+1)^2}}\,(e^{2i(dx+c)}+1)} - \frac{2(e^{2i(dx+c)}-1)(dx+c)}{\sqrt{-\frac{b(e^{2i(dx+c)}-1)^2}{(e^{2i(dx+c)}+1)^2}}\,(e^{2i(dx+c)}+1)d} - \frac{i(e^{2i(dx+c)}-1)\ln(e^{2i(dx+}}{\sqrt{-\frac{b(e^{2i(dx+c)}-1)^2}{(e^{2i(dx+c)}+1)^2}}\,(e^{2i(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*tan(d*x+c)*(2*ln(tan(d*x+c))-ln(1+tan(d*x+c)^2))/(b*tan(d*x+c)^2)^(1/
2)
```

Maxima [A]
time = 0.50, size = 33, normalized size = 1.06

$$-\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{\sqrt{b}}-\frac{2\log(\tan(dx+c))}{\sqrt{b}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(tan(d*x + c)^2 + 1)/sqrt(b) - 2*log(tan(d*x + c))/sqrt(b))/d
```

Fricas [A]
time = 0.36, size = 50, normalized size = 1.61

$$\frac{\sqrt{b\tan(dx+c)^2}\,\log\left(\frac{\tan(dx+c)^2}{\tan(dx+c)^2+1}\right)}{2\,bd\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(b*tan(d*x + c)^2)*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))/(b*d*ta
n(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan^2(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(c + d*x)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(29) = 58.
time = 0.45, size = 81, normalized size = 2.61

$$\frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{\sqrt{b}\,\mathrm{sgn}(\tan(dx+c))} - \frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{\sqrt{b}\,\mathrm{sgn}(\tan(dx+c))}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(sqrt(b)*sgn(tan(d*x
 + c))) - 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/(sqrt(b)*s
gn(tan(d*x + c))))/d
```

Mupad [B]
time = 2.45, size = 34, normalized size = 1.10

$$\frac{\mathrm{atan}\left(\frac{\sqrt{-b}\,\tan(c+d\,x)}{\sqrt{b\,{\tan\left(c+d\,x\right)}^2}}\right)}{\sqrt{-b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^2)^(1/2),x)
```

```
[Out] atan(((-b)^(1/2)*tan(c + d*x))/(b*tan(c + d*x)^2)^(1/2))/((-b)^(1/2)*d)
```

3.28 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}}-\frac{\log(\sin(c+dx))\tan(c+dx)}{bd\sqrt{b\tan^2(c+dx)}}$$

```
[Out] -1/2*cot(d*x+c)/b/d/(b*tan(d*x+c)^2)^(1/2)-ln(sin(d*x+c))*tan(d*x+c)/b/d/(b
*tan(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {3739, 3554, 3556}

$$-\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}}-\frac{\tan(c+dx)\log(\sin(c+dx))}{bd\sqrt{b\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^2)^(-3/2),x]
```

```
[Out] -1/2*Cot[c + d*x]/(b*d*Sqrt[b*Tan[c + d*x]^2]) - (Log[Sin[c + d*x]]*Tan[c +
 d*x])/(b*d*Sqrt[b*Tan[c + d*x]^2])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(b\tan^2(c+dx)\right)^{3/2}}\,dx &= \frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{b\sqrt{b\tan^2(c+dx)}} \\
&= -\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}} - \frac{\tan(c+dx)\int \cot(c+dx)\,dx}{b\sqrt{b\tan^2(c+dx)}} \\
&= -\frac{\cot(c+dx)}{2bd\sqrt{b\tan^2(c+dx)}} - \frac{\log(\sin(c+dx))\tan(c+dx)}{bd\sqrt{b\tan^2(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.41, size = 56, normalized size = 0.85

$$
-\frac{\left(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx))\right)\tan^3(c+dx)}{2d\left(b\tan^2(c+dx)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^2)^(-3/2),x]
```

```
[Out] -1/2*((Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Tan[c + d*x]])*Tan[c +
d*x]^3)/(d*(b*Tan[c + d*x]^2)^(3/2))
```

Maple [A]
time = 0.10, size = 64, normalized size = 0.97

method	result
derivativedivides	$-\frac{\tan(dx+c)\left(2\ln(\tan(dx+c))\left(\tan^2(dx+c)\right)-\ln\left(1+\tan^2(dx+c)\right)\left(\tan^2(dx+c)\right)+1\right)}{2d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}$
default	$-\frac{\tan(dx+c)\left(2\ln(\tan(dx+c))\left(\tan^2(dx+c)\right)-\ln\left(1+\tan^2(dx+c)\right)\left(\tan^2(dx+c)\right)+1\right)}{2d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{3}{2}}}$
risch	$-\frac{\left(e^{2i(dx+c)}-1\right)x}{b\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}}+\frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{b\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d}-\frac{2ie^{2i(}}{b\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*tan(d*x+c)*(2*ln(tan(d*x+c))*tan(d*x+c)^2-ln(1+tan(d*x+c)^2)*tan(d*x
+c)^2+1)/(b*tan(d*x+c)^2)^(3/2)
```

Maxima [A]
time = 0.49, size = 46, normalized size = 0.70

$$
\frac{\frac{\log\left(\tan(dx+c)^2+1\right)}{b^{\frac{3}{2}}} - \frac{2\,\log(\tan(dx+c))}{b^{\frac{3}{2}}} - \frac{1}{b^{\frac{3}{2}}\,\tan(dx+c)^2}}{2\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(tan(d*x + c)^2 + 1)/b^(3/2) - 2*log(tan(d*x + c))/b^(3/2) - 1/(b^(
3/2)*tan(d*x + c)^2))/d
```

Fricas [A]
time = 0.35, size = 69, normalized size = 1.05

$$-\frac{\sqrt{b\tan\left(dx+c\right)^{2}}\left(\log\left(\frac{\tan(dx+c)^{2}}{\tan(dx+c)^{2}+1}\right)\tan\left(dx+c\right)^{2}+\tan\left(dx+c\right)^{2}+1\right)}{2\,b^{2}d\tan\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(b*tan(d*x + c)^2)*(log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d
*x + c)^2 + tan(d*x + c)^2 + 1)/(b^2*d*tan(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\tan^{2}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**2)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**2)**(-3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(60) = 120.
time = 0.52, size = 176, normalized size = 2.67

$$-\frac{\frac{4\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{\sqrt{b}\operatorname{sgn}(\tan(dx+c))}-\frac{8\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{\sqrt{b}\operatorname{sgn}(\tan(dx+c))}-\frac{\left(\sqrt{b}+\frac{4\sqrt{b}(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{b(\cos(dx+c)-1)\operatorname{sgn}(\tan(dx+c))}-\frac{\cos(dx+c)-1}{\sqrt{b}(\cos(dx+c)+1)\operatorname{sgn}(\tan(dx+c))}}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(sqrt(b)*sgn(tan(
d*x + c))) - 8*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/(sqrt(b
)*sgn(tan(d*x + c))) - (sqrt(b) + 4*sqrt(b)*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))*(cos(d*x + c) + 1)/(b*(cos(d*x + c) - 1)*sgn(tan(d*x + c))) - (cos(
d*x + c) - 1)/(sqrt(b)*(cos(d*x + c) + 1)*sgn(tan(d*x + c))))/(b*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\tan\left(c+d\,x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^2)^(3/2),x)
[Out] int(1/(b*tan(c + d*x)^2)^(3/2), x)
```

3.29 $\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}}-\frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}}+\frac{\log(\sin(c+dx))\tan(c+dx)}{b^2d\sqrt{b\tan^2(c+dx)}}$$

```
[Out] 1/2*cot(d*x+c)/b^2/d/(b*tan(d*x+c)^2)^(1/2)-1/4*cot(d*x+c)^3/b^2/d/(b*tan(d
*x+c)^2)^(1/2)+ln(sin(d*x+c))*tan(d*x+c)/b^2/d/(b*tan(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$-\frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}}+\frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}}+\frac{\tan(c+dx)\log(\sin(c+dx))}{b^2d\sqrt{b\tan^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^2)^(-5/2),x]
```

```
[Out] Cot[c + d*x]/(2*b^2*d*Sqrt[b*Tan[c + d*x]^2]) - Cot[c + d*x]^3/(4*b^2*d*Sqr
t[b*Tan[c + d*x]^2]) + (Log[Sin[c + d*x]]*Tan[c + d*x])/(b^2*d*Sqrt[b*Tan[c
 + d*x]^2])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(b\tan^2(c+dx)\right)^{5/2}}\,dx &= \frac{\tan(c+dx)\int \cot^5(c+dx)\,dx}{b^2\sqrt{b\tan^2(c+dx)}}\\
&= -\frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}} - \frac{\tan(c+dx)\int \cot^3(c+dx)\,dx}{b^2\sqrt{b\tan^2(c+dx)}}\\
&= \frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}} + \frac{\tan(c+dx)\int \cot(c+dx)\,dx}{b^2\sqrt{b\tan^2(c+dx)}}\\
&= \frac{\cot(c+dx)}{2b^2d\sqrt{b\tan^2(c+dx)}} - \frac{\cot^3(c+dx)}{4b^2d\sqrt{b\tan^2(c+dx)}} + \frac{\log(\sin(c+dx))\tan(c+dx)}{b^2d\sqrt{b\tan^2(c+dx)}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.29, size = 68, normalized size = 0.70

$$
\frac{\left(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx))\right)\tan^5(c+dx)}{4d\left(b\tan^2(c+dx)\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^2)^(-5/2),x]
```

```
[Out] ((2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d
*x]])*Tan[c + d*x]^5)/(4*d*(b*Tan[c + d*x]^2)^(5/2))
```

Maple [**A**]
time = 0.10, size = 74, normalized size = 0.76

method	result
derivativedivides	$\frac{\tan(dx+c)\left(4\ln(\tan(dx+c))\left(\tan^4(dx+c)\right)-2\ln\left(1+\tan^2(dx+c)\right)\left(\tan^4(dx+c)\right)+2\left(\tan^2(dx+c)\right)-1\right)}{4d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}$
default	$\frac{\tan(dx+c)\left(4\ln(\tan(dx+c))\left(\tan^4(dx+c)\right)-2\ln\left(1+\tan^2(dx+c)\right)\left(\tan^4(dx+c)\right)+2\left(\tan^2(dx+c)\right)-1\right)}{4d\left(b\left(\tan^2(dx+c)\right)\right)^{\frac{5}{2}}}$
risch	$\frac{\left(e^{2i(dx+c)}-1\right)x}{b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}} - \frac{2\left(e^{2i(dx+c)}-1\right)(dx+c)}{b^2\left(e^{2i(dx+c)}+1\right)\sqrt{-\frac{b\left(e^{2i(dx+c)}-1\right)^2}{\left(e^{2i(dx+c)}+1\right)^2}}\,d} + \frac{4i\left(e^{6i(dx+c)}-e^{4i}\right.}{b^2\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*tan(d*x+c)*(4*ln(tan(d*x+c))*tan(d*x+c)^4-2*ln(1+tan(d*x+c)^2)*tan(d*
x+c)^4+2*tan(d*x+c)^2-1)/(b*tan(d*x+c)^2)^(5/2)
```

Maxima [A]
time = 0.50, size = 66, normalized size = 0.68

$$-\frac{\frac{2\,\log\left(\tan\left(dx+c\right)^{2}+1\right)}{b^{\frac{5}{2}}}-\frac{4\,\log\left(\tan\left(dx+c\right)\right)}{b^{\frac{5}{2}}}-\frac{2\,\sqrt{b}\,\tan\left(dx+c\right)^{2}-\sqrt{b}}{b^{3}\,\tan\left(dx+c\right)^{4}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*log(tan(d*x + c)^2 + 1)/b^(5/2) - 4*log(tan(d*x + c))/b^(5/2) - (2*
sqrt(b)*tan(d*x + c)^2 - sqrt(b))/(b^3*tan(d*x + c)^4))/d
```

Fricas [A]
time = 0.35, size = 82, normalized size = 0.85

$$\frac{\left(2\,\log\left(\frac{\tan\left(dx+c\right)^{2}}{\tan\left(dx+c\right)^{2}+1}\right)\tan\left(dx+c\right)^{4}+3\,\tan\left(dx+c\right)^{4}+2\,\tan\left(dx+c\right)^{2}-1\right)\sqrt{b\tan\left(dx+c\right)^{2}}}{4\,b^{3}d\tan\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(tan(d*x + c)^2/(tan(d*x + c)^2 + 1))*tan(d*x + c)^4 + 3*tan(d*x
+ c)^4 + 2*tan(d*x + c)^2 - 1)*sqrt(b*tan(d*x + c)^2)/(b^3*d*tan(d*x + c)^5
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan^{2}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**2)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**2)**(-5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(87) = 174.
time = 0.56, size = 235, normalized size = 2.42

$$\frac{\frac{32\,\log\left(\frac{\left|-\cos\left(dx+c\right)+1\right|}{\left|\cos\left(dx+c\right)+1\right|}\right)}{b^{\frac{5}{2}}\,\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}-\frac{64\,\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}+1\right|\right)}{b^{\frac{5}{2}}\,\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}-\frac{\left(\sqrt{b}+\frac{12\,\sqrt{b}\,\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+\frac{48\,\sqrt{b}\,\left(\cos\left(dx+c\right)-1\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}\right)\left(\cos\left(dx+c\right)+1\right)^{2}}{b^{3}\left(\cos\left(dx+c\right)-1\right)^{2}\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}-\frac{\frac{12\,b^{\frac{7}{2}}\left(\cos\left(dx+c\right)-1\right)\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}{\cos\left(dx+c\right)+1}+\frac{b^{\frac{7}{2}}\left(\cos\left(dx+c\right)-1\right)^{2}\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}{\left(\cos\left(dx+c\right)+1\right)^{2}}}{b^{6}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(32*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(b^(5/2)*sgn(tan
(d*x + c))) - 64*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/(b^(5
/2)*sgn(tan(d*x + c))) - (sqrt(b) + 12*sqrt(b)*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 48*sqrt(b)*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x
+ c) + 1)^2/(b^3*(cos(d*x + c) - 1)^2*sgn(tan(d*x + c))) - (12*b^(7/2)*(cos
(d*x + c) - 1)*sgn(tan(d*x + c))/(cos(d*x + c) + 1) + b^(7/2)*(cos(d*x + c)
 - 1)^2*sgn(tan(d*x + c))/(cos(d*x + c) + 1)^2)/b^6)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\tan \left(c+d\,x\right)^2\right)^{5/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^2)^(5/2),x)
```

```
[Out] int(1/(b*tan(c + d*x)^2)^(5/2), x)
```

3.30 $\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=364

$$-\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d}-\frac{b^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}+\frac{b^2\text{ArcTan}\left(1+\right.}{}$$

```
[Out] -2*b^2*cot(d*x+c)*(b*tan(d*x+c)^3)^(1/2)/d+1/2*b^2*arctan(-1+2^(1/2)*tan(d*
x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/2*b^2*arcta
n(1+2^(1/2)*tan(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(
3/2)-1/4*b^2*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^3)^(1/
2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/4*b^2*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x
+c))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+2/5*b^2*(b*tan(d*x+c
)^3)^(1/2)*tan(d*x+c)/d-2/9*b^2*(b*tan(d*x+c)^3)^(1/2)*tan(d*x+c)^3/d+2/13*
b^2*(b*tan(d*x+c)^3)^(1/2)*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.11, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{b^2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}+\frac{b^2\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}-\frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d}+\frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d}+\frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d}-\frac{b^2\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}+\frac{b^2\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}-\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^3)^(5/2),x]
```

```
[Out] (-2*b^2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/d - (b^2*ArcTan[1 - Sqrt[2]*Sq
rt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) +
(b^2*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2
]*d*Tan[c + d*x]^(3/2)) - (b^2*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c +
 d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) + (b^2*Log[
1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*S
qrt[2]*d*Tan[c + d*x]^(3/2)) + (2*b^2*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/
(5*d) - (2*b^2*Tan[c + d*x]^3*Sqrt[b*Tan[c + d*x]^3])/(9*d) + (2*b^2*Tan[c
+ d*x]^5*Sqrt[b*Tan[c + d*x]^3])/(13*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^3(c+dx)\right)^{5/2}\,dx = \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{15}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} - \frac{\left(b^2\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{11}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + \frac{2b^2\tan^5(c+dx)\sqrt{b\tan^3(c+dx)}}{13d} + \left(b^2\right.$$

$$= \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - \frac{2b^2\tan^3(c+dx)\sqrt{b\tan^3(c+dx)}}{9d} + 2b^2\tan$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - 2b^2\tan$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - 2b^2\tan$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - 2b^2\tan$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - 2b^2\tan$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{2b^2\tan(c+dx)\sqrt{b\tan^3(c+dx)}}{5d} - 2b^2\tan$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{b^2\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+\right.}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b^2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{b^2\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.81, size = 199, normalized size = 0.55

$$\frac{b\left(b\tan^3(c+dx)\right)^{3/2}\left(-1170\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)+1170\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)-585\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+585\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-4680\sqrt{\tan(c+dx)}+936\tan^{\frac{5}{2}}(c+dx)-520\tan^{\frac{9}{2}}(c+dx)+360\tan^{\frac{13}{2}}(c+dx)\right)}{2340d\tan^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^3)^(5/2),x]
```

```
[Out] (b*(b*Tan[c + d*x]^3)^(3/2)*(-1170*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c +
d*x]]] + 1170*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]] - 585*Sqrt[2]*
Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] + 585*Sqrt[2]*Log[1 + Sq
rt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] - 4680*Sqrt[Tan[c + d*x]] + 936*Ta
n[c + d*x]^(5/2) - 520*Tan[c + d*x]^(9/2) + 360*Tan[c + d*x]^(13/2)))/(2340
*d*Tan[c + d*x]^(9/2))
```

Maple [A]
time = 0.09, size = 266, normalized size = 0.73

method	result
derivativedivides	$\frac{\left(b\left(\tan^3(dx+c)\right)\right)^{\frac{5}{2}}\left(360\left(b\tan(dx+c)\right)^{\frac{13}{2}}-520b^2\left(b\tan(dx+c)\right)^{\frac{9}{2}}+936b^4\left(b\tan(dx+c)\right)^{\frac{5}{2}}+585b^6\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(-\frac{b\tan(\ldots}{\left(b^2\right)^{\ldots}}\right.\right.}{\ldots}$
default	$\frac{\left(b\left(\tan^3(dx+c)\right)\right)^{\frac{5}{2}}\left(360\left(b\tan(dx+c)\right)^{\frac{13}{2}}-520b^2\left(b\tan(dx+c)\right)^{\frac{9}{2}}+936b^4\left(b\tan(dx+c)\right)^{\frac{5}{2}}+585b^6\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(-\frac{b\tan(\ldots}{\left(b^2\right)^{\ldots}}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2340/d*(b*tan(d*x+c)^3)^(5/2)*(360*(b*tan(d*x+c))^(13/2)-520*b^2*(b*tan(d
*x+c))^(9/2)+936*b^4*(b*tan(d*x+c))^(5/2)+585*b^6*(b^2)^(1/4)*2^(1/2)*ln(-(
b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/((b^2)^(
1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2)))+1170*b^6*(b^2)
^(1/4)*2^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4
))+1170*b^6*(b^2)^(1/4)*2^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^
(1/4))/(b^2)^(1/4))-4680*b^6*(b*tan(d*x+c))^(1/2))/tan(d*x+c)^5/(b*tan(d*x+
c))^(5/2)/b^4
```

Maxima [A]
time = 0.51, size = 178, normalized size = 0.49

$$\frac{360\,b^{\frac{5}{2}}\tan\left(dx+c\right)^{\frac{13}{2}}-520\,b^{\frac{5}{2}}\tan\left(dx+c\right)^{\frac{9}{2}}+936\,b^{\frac{5}{2}}\tan\left(dx+c\right)^{\frac{5}{2}}+585\left(2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+\sqrt{2}\sqrt{b}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-\sqrt{2}\sqrt{b}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)\right)b^2-4680\,b^{\frac{5}{2}}\sqrt{\tan\left(dx+c\right)}}{2340\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(360*b^(5/2)*tan(d*x + c)^(13/2) - 520*b^(5/2)*tan(d*x + c)^(9/2) +
936*b^(5/2)*tan(d*x + c)^(5/2) + 585*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*
(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(
```

```
sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(d*x
 + c)) + tan(d*x + c) + 1) - sqrt(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(d*x + c)
) + tan(d*x + c) + 1))*b^2 - 4680*b^(5/2)*sqrt(tan(d*x + c)))/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^3\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**3)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**(5/2), x)
```

Giac [**A**]
time = 0.60, size = 291, normalized size = 0.80

$$\frac{1}{2340}\left(\frac{1170\sqrt{2}\,b\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{1170\sqrt{2}\,b\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{585\sqrt{2}\,b\sqrt{|b|}\log\left(b\tan\left(dx+c\right)+\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{|b|}+|b|\right)}{d}-\frac{585\sqrt{2}\,b\sqrt{|b|}\log\left(b\tan\left(dx+c\right)-\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{|b|}+|b|\right)}{d}+\frac{8\left(45\sqrt{b\tan\left(dx+c\right)}\,b^{66}d^{12}\tan\left(dx+c\right)^{6}-65\sqrt{b\tan\left(dx+c\right)}\,b^{66}d^{12}\tan\left(dx+c\right)^{4}+117\sqrt{b\tan\left(dx+c\right)}\,b^{66}d^{12}\tan\left(dx+c\right)^{2}-585\sqrt{b\tan\left(dx+c\right)}\,b^{66}d^{12}\right)}{b^{65}d^{13}}\right)b\,\mathrm{sgn}\left(\tan\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2340*(1170*sqrt(2)*b*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)
) + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 1170*sqrt(2)*b*sqrt(abs(b))*a
rctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs
(b)))/d + 585*sqrt(2)*b*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*ta
n(d*x + c))*sqrt(abs(b)) + abs(b))/d - 585*sqrt(2)*b*sqrt(abs(b))*log(b*tan
(d*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d + 8*(45*s
qrt(b*tan(d*x + c))*b^66*d^12*tan(d*x + c)^6 - 65*sqrt(b*tan(d*x + c))*b^66
*d^12*tan(d*x + c)^4 + 117*sqrt(b*tan(d*x + c))*b^66*d^12*tan(d*x + c)^2 -
585*sqrt(b*tan(d*x + c))*b^66*d^12)/(b^65*d^13))*b*sgn(tan(d*x + c))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^3\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^3)^(5/2),x)
[Out] int((b*tan(c + d*x)^3)^(5/2), x)
```

3.31 $\int (b\tan^3(c+dx))^{3/2}\,dx$

Optimal. Leaf size=286

$$-\frac{2b\sqrt{b\tan^3(c+dx)}}{3d}-\frac{b\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}+\frac{b\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+\right.}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c}$$

```
[Out] -2/3*b*(b*tan(d*x+c)^3)^(1/2)/d+1/2*b*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*(
b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/2*b*arctan(1+2^(1/2)*tan
(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/4*b*ln(1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(
d*x+c)^(3/2)-1/4*b*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^
3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+2/7*b*(b*tan(d*x+c)^3)^(1/2)*tan(d*x+c)
^2/d
```

Rubi [A]

time = 0.09, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{b\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}+\frac{b\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}-\frac{2b\sqrt{b\tan^3(c+dx)}}{3d}+\frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d}+\frac{b\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}-\frac{b\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^3)^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Tan[c + d*x]^3])/(3*d) - (b*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*
x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) + (b*ArcTan[1 +
 Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x
]^(3/2)) + (b*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan
[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (b*Log[1 + Sqrt[2]*Sqrt[Ta
n[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*
x]^(3/2)) + (2*b*Tan[c + d*x]^2*Sqrt[b*Tan[c + d*x]^3])/(7*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^3(c+dx)\right)^{3/2}\,dx = \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{9}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} - \frac{\left(b\sqrt{b\tan^3(c+dx)}\right)\int \tan^{\frac{5}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\tan^3(c+d\cdots\right.}{\tan\cdots}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\tan^3(c+d\cdots\right.}{\cdots}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(2b\sqrt{b\tan^3(c+\cdots\right.}{\cdots}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} - \frac{\left(b\sqrt{b\tan^3(c+d\cdots\right.}{\cdots}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{2b\tan^2(c+dx)\sqrt{b\tan^3(c+dx)}}{7d} + \frac{\left(b\sqrt{b\tan^3(c+d\cdots\right.}{\cdots}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} + \frac{b\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\tan\cdots}}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{2b\sqrt{b\tan^3(c+dx)}}{3d} - \frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 54, normalized size = 0.19

$$\frac{2b\sqrt{b\tan^3(c+dx)}\left(-7+7\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(c+dx)\right)+3\tan^2(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^3)^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Tan[c + d*x]^3]*(-7 + 7*Hypergeometric2F1[3/4, 1, 7/4, -Tan[c +
 d*x]^2] + 3*Tan[c + d*x]^2))/(21*d)
```

Maple [A]
time = 0.06, size = 236, normalized size = 0.83

method	result
derivativedivides	$\frac{(b(\tan^3(dx+c)))^{\frac{3}{2}}\left(24(b\tan(dx+c))^{\frac{7}{2}}(b^2)^{\frac{1}{4}}+21b^4\sqrt{2}\ \ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right.\right.}{84d}$
default	$\frac{(b(\tan^3(dx+c)))^{\frac{3}{2}}\left(24(b\tan(dx+c))^{\frac{7}{2}}(b^2)^{\frac{1}{4}}+21b^4\sqrt{2}\ \ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\sqrt{2}+\sqrt{b^2}}\right.\right.}{84d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/84/d*(b*tan(d*x+c)^3)^(3/2)*(24*(b*tan(d*x+c))^(7/2)*(b^2)^(1/4)+21*b^4*2
^(1/2)*ln(-((b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/
2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+42
*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))
+42*b^4*2^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/
4))-56*b^2*(b*tan(d*x+c))^(3/2)*(b^2)^(1/4))/tan(d*x+c)^3/(b*tan(d*x+c))^(3
/2)/b^2/(b^2)^(1/4)
```

Maxima [A]
time = 0.52, size = 140, normalized size = 0.49

$$\frac{24\,b^{\frac{3}{2}}\tan(dx+c)^{\frac{7}{2}}-56\,b^{\frac{3}{2}}\tan(dx+c)^{\frac{3}{2}}+21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)\right)b^{\frac{3}{2}}}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/84*(24*b^(3/2)*tan(d*x + c)^(7/2) - 56*b^(3/2)*tan(d*x + c)^(3/2) + 21*(2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(sqrt(2)*
sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(tan(d*x
+ c)) + tan(d*x + c) + 1))*b^(3/2))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{3}\left(c+dx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**3)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**(3/2), x)
```

Giac [**A**]
time = 0.53, size = 253, normalized size = 0.88

$$\frac{1}{84}\,b\left(\frac{42\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{|b|}}\right)}{bd}+\frac{42\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan\left(dx+c\right)}\right)}{2\sqrt{|b|}}\right)}{bd}-\frac{21\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(dx+c\right)+\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{|b|}+|b|\right)}{bd}+\frac{21\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(dx+c\right)-\sqrt{2}\sqrt{b\tan\left(dx+c\right)}\sqrt{|b|}+|b|\right)}{bd}+\frac{8\left(3\sqrt{b\tan\left(dx+c\right)}\,b^{21}d^{6}\tan\left(dx+c\right)^{3}-7\sqrt{b\tan\left(dx+c\right)}\,b^{21}d^{6}\tan\left(dx+c\right)\right)}{b^{21}d^{7}}\right)\mathrm{sgn}(\tan\left(dx+c\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/84*b*(42*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b*d) + 42*sqrt(2)*abs(b)^(3/2)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b))
)/(b*d) - 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d
*x + c))*sqrt(abs(b)) + abs(b))/(b*d) + 21*sqrt(2)*abs(b)^(3/2)*log(b*tan(d
*x + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b*d) + 8*(3*
sqrt(b*tan(d*x + c))*b^21*d^6*tan(d*x + c)^3 - 7*sqrt(b*tan(d*x + c))*b^21*
d^6*tan(d*x + c))/(b^21*d^7))*sgn(tan(d*x + c))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^3\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^3)^(3/2),x)
```

```
[Out] int((b*tan(c + d*x)^3)^(3/2), x)
```

3.32 $\int \sqrt{b\tan^3(c+dx)}\, dx$

Optimal. Leaf size=255

$$\frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d}+\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}-\frac{\text{ArcTan}\left(1+\sqrt{2}\sqrt{\text{ta}}\right.}{\sqrt{2}\,d}$$

```
[Out] 2*cot(d*x+c)*(b*tan(d*x+c)^3)^(1/2)/d-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2
))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)-1/2*arctan(1+2^(1/2)*t
an(d*x+c)^(1/2))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)+1/4*ln(1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^3)^(1/2)/d*2^(1/2)/tan(
d*x+c)^(3/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*(b*tan(d*x+c)^3)
^(1/2)/d*2^(1/2)/tan(d*x+c)^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}-\frac{\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)\sqrt{b\tan^3(c+dx)}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}+\frac{\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}-\frac{\sqrt{b\tan^3(c+dx)}\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}+\frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[c + d*x]^3],x]
```

```
[Out] (2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^3])/d + (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c
+ d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (ArcTan[1
 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Sqrt[b*Tan[c + d*x]^3])/(Sqrt[2]*d*Tan[c + d
*x]^(3/2)) + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan
[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]^(3/2)) - (Log[1 + Sqrt[2]*Sqrt[Tan[
c + d*x]] + Tan[c + d*x]]*Sqrt[b*Tan[c + d*x]^3])/(2*Sqrt[2]*d*Tan[c + d*x]
^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{b\tan^3(c+dx)}\,dx &= \frac{\sqrt{b\tan^3(c+dx)}\int \tan^{\frac{3}{2}}(c+dx)\,dx}{\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\sqrt{b\tan^3(c+dx)}\int \frac{1}{\sqrt{\tan(c+dx)}}\,dx}{\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\sqrt{b\tan^3(c+dx)}\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(\cdots\right.}{d\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\left(2\sqrt{b\tan^3(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{\tan\cdots}\right.}{d\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\sqrt{b\tan^3(c+dx)}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+\cdots}\right.}{d\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} - \frac{\sqrt{b\tan^3(c+dx)}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\text{ta}\cdots}\right.}{2d\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\sqrt{b\cdots}}{2\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}\\
&= \frac{2\cot(c+dx)\sqrt{b\tan^3(c+dx)}}{d} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+d\cdots}}{\sqrt{2}\,d\tan^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 161, normalized size = 0.63

$$\frac{\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)+8\sqrt{\tan(c+dx)}\right)\sqrt{b\tan^3(c+dx)}}{4d\tan^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^3],x]
```

```
[Out] ((2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]] - 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*Sqrt[Tan[c + d*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + T
an[c + d*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]] +
 8*Sqrt[Tan[c + d*x]])*Sqrt[b*Tan[c + d*x]^3])/(4*d*Tan[c + d*x]^(3/2))
```

Maple [A]
time = 0.08, size = 208, normalized size = 0.82

method	result
derivativedivides	$-\frac{\sqrt{b\left(\tan^{3}\left(dx+c\right)\right)}\left(\left(b^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(-\frac{b\tan(dx+c)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\,\sqrt{2}+\sqrt{b^{2}}}{\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^{2}}}\right)+2\left(b^{2}\right)^{\frac{1}{4}}\cdots\right.}{4d\,\mathrm{t}\cdots}$
default	$-\frac{\sqrt{b\left(\tan^{3}\left(dx+c\right)\right)}\left(\left(b^{2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(-\frac{b\tan(dx+c)+\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\,\sqrt{2}+\sqrt{b^{2}}}{\left(b^{2}\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^{2}}}\right)+2\left(b^{2}\right)^{\frac{1}{4}}\cdots\right.}{4d\,\mathrm{t}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(b*tan(d*x+c)^3)^(1/2)*((b^2)^(1/4)*2^(1/2)*ln(-(b*tan(d*x+c)+(b^2)^
(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/((b^2)^(1/4)*(b*tan(d*x+c))
^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2)))+2*(b^2)^(1/4)*2^(1/2)*arctan((2^(
1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*(b^2)^(1/4)*2^(1/2)*a
rctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))-8*(b*tan(d*x+
c))^(1/2))/tan(d*x+c)/(b*tan(d*x+c))^(1/2)
```

Maxima [A]
time = 0.50, size = 133, normalized size = 0.52

$$-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\sqrt{b}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+\sqrt{2}\sqrt{b}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-\sqrt{2}\sqrt{b}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan\left(dx+c\right)+1\right)-8\sqrt{b}\sqrt{\tan\left(dx+c\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))
) + 2*sqrt(2)*sqrt(b)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c))))
 + sqrt(2)*sqrt(b)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqr
t(2)*sqrt(b)*log(-sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - 8*sqrt(b
)*sqrt(tan(d*x + c)))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan^3{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(c + d*x)**3), x)
```

Giac [A]
time = 0.48, size = 195, normalized size = 0.76

$$-\frac{1}{4}\left(\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{2\sqrt{2}\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{d}+\frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{|b|}+|b|\right)}{d}-\frac{\sqrt{2}\sqrt{|b|}\log\left(b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{|b|}+|b|\right)}{d}-\frac{8\sqrt{b\tan(dx+c)}}{d}\right)\mathrm{sgn}(\tan(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) + 2*s
qrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + 2*sqrt(2)*sqrt(abs(b))*arctan(-1/2*s
qrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/d + sq
rt(2)*sqrt(abs(b))*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(a
bs(b)) + abs(b))/d - sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) - sqrt(2)*sqrt
(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/d - 8*sqrt(b*tan(d*x + c))/d)*sgn(t
an(d*x + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b\,{\tan\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^3)^(1/2),x)
```

```
[Out] int((b*tan(c + d*x)^3)^(1/2), x)
```

3.33 $\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx$

Optimal. Leaf size=255

$$-\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}}+\frac{\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}\,d\sqrt{b\tan^3(c+dx)}}-\frac{\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\sqrt{b\tan^3(c+dx)}}$$

```
[Out] -2*tan(d*x+c)/d/(b*tan(d*x+c)^3)^(1/2)-1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/
2))*tan(d*x+c)^(3/2)/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)-1/2*arctan(1+2^(1/2)*
tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)-1/4*ln(
1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/d*2^(1/2)/(b*tan(d*
x+c)^3)^(1/2)+1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2
)/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,d\sqrt{b\tan^3(c+dx)}}-\frac{\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,d\sqrt{b\tan^3(c+dx)}}-\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}}-\frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\sqrt{b\tan^3(c+dx)}}+\frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,d\sqrt{b\tan^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^3],x]
```

```
[Out] (-2*Tan[c + d*x])/(d*Sqrt[b*Tan[c + d*x]^3]) + (ArcTan[1 - Sqrt[2]*Sqrt[Tan
[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*d*Sqrt[b*Tan[c + d*x]^3]) - (ArcTa
n[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*d*Sqrt[b*Tan
[c + d*x]^3]) - (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c +
 d*x]^(3/2))/(2*Sqrt[2]*d*Sqrt[b*Tan[c + d*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[T
an[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*d*Sqrt[b*Tan[c
+ d*x]^3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^3(c+dx)}}\,dx = \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{3}{2}}(c+dx)}\,dx}{\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\int \sqrt{\tan(c+dx)}\,dx}{\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\left(2\tan^{\frac{3}{2}}(c+dx)\right)\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(c+dx)}\right)}{d\sqrt{b\tan^3(c+dx)}} - \mathrm{tan}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{\tan(c+dx)}\right)}{2d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} - \frac{\log\left(1-\sqrt{2}\sqrt{\tan(c+dx)}+\tan(c+dx)\right)\tan^{\frac{3}{2}}(c+dx}{2\sqrt{2}\,d\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}\,d\sqrt{b\tan^3(c+dx)}} - \tan^{-1}($$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order

3 in optimal.
time = 0.04, size = 43, normalized size = 0.17

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(c+dx)\right)\tan(c+dx)}{d\sqrt{b\tan^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^3],x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*Sqrt[
b*Tan[c + d*x]^3])
```

Maple [A]
time = 0.08, size = 211, normalized size = 0.83

method	result
derivativedivides	$-\frac{\tan(dx+c)\left(\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\sqrt{2}\ \cdots\right.}{\cdots}$
default	$-\frac{\tan(dx+c)\left(\sqrt{2}\,\sqrt{b\tan(dx+c)}\,\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+2\sqrt{2}\ \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*tan(d*x+c)*(2^(1/2)*(b*tan(d*x+c))^(1/2)*ln(-((b^2)^(1/4)*(b*tan(d*x
+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)^(1/4)*(b*t
an(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+2*2^(1/2)*(b*tan(d*x+c))^(1/2)*arcta
n((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+2*2^(1/2)*(b*tan(
d*x+c))^(1/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4)
)+8*(b^2)^(1/4))/(b*tan(d*x+c)^3)^(1/2)/(b^2)^(1/4)
```

Maxima [A]
time = 0.50, size = 126, normalized size = 0.49

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)}{\sqrt{b}}+\frac{8}{\sqrt{b}\sqrt{\tan(dx+c)}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) + 2*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2)*log(
```

```
sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)*sqrt(
tan(d*x + c)) + tan(d*x + c) + 1))/sqrt(b) + 8/(sqrt(b)*sqrt(tan(d*x + c)))
)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan^3{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(c + d*x)**3), x)
```

Giac [**A**]
time = 0.56, size = 251, normalized size = 0.98

$$-\frac{1}{4}b^2\left(\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}+2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d\operatorname{sgn}(\tan(dx+c))}+\frac{2\sqrt{2}|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|b|}-2\sqrt{b\tan(dx+c)}\right)}{2\sqrt{|b|}}\right)}{b^4d\operatorname{sgn}(\tan(dx+c))}-\frac{\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan(dx+c)+\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{|b|}+|b|\right)}{b^4d\operatorname{sgn}(\tan(dx+c))}+\frac{\sqrt{2}|b|^{\frac{3}{2}}\log\left(b\tan(dx+c)-\sqrt{2}\sqrt{b\tan(dx+c)}\sqrt{|b|}+|b|\right)}{b^4d\operatorname{sgn}(\tan(dx+c))}+\frac{8}{\sqrt{b\tan(dx+c)}\,b^2d\operatorname{sgn}(\tan(dx+c))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b^2*(2*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) +
 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^4*d*sgn(tan(d*x + c))) + 2*sqrt(2
)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d*x
 + c)))/sqrt(abs(b)))/(b^4*d*sgn(tan(d*x + c))) - sqrt(2)*abs(b)^(3/2)*log(
b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^4*d
*sgn(tan(d*x + c))) + sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x + c) - sqrt(2)*sqr
t(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^4*d*sgn(tan(d*x + c))) + 8/(sqr
t(b*tan(d*x + c))*b^2*d*sgn(tan(d*x + c))))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\,{\mathrm{tan}\left(c+d\,x\right)}^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^3)^(1/2),x)
[Out] int(1/(b*tan(c + d*x)^3)^(1/2), x)
```

3.34 $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=298

$$\frac{2}{3bd\sqrt{b\tan^3(c+dx)}}-\frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}}-\frac{\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)\tan^{\frac{3}{2}}(c+dx)}{\sqrt{2}\,bd\sqrt{b\tan^3(c+dx)}}+\frac{\text{ArcTan}\left(1\right.}{}$$

```
[Out] 2/3/b/d/(b*tan(d*x+c)^3)^(1/2)-2/7*cot(d*x+c)^2/b/d/(b*tan(d*x+c)^3)^(1/2)+
1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)/b/d*2^(1/2)/(b*tan
(d*x+c)^3)^(1/2)+1/2*arctan(1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d*x+c)^(3/2)/b/
d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)-1/4*ln(1-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+
c))*tan(d*x+c)^(3/2)/b/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)+1/4*ln(1+2^(1/2)*ta
n(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/b/d*2^(1/2)/(b*tan(d*x+c)^3)^(1
/2)
```

Rubi [A]
time = 0.10, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,bd\sqrt{b\tan^3(c+dx)}}+\frac{\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,bd\sqrt{b\tan^3(c+dx)}}+\frac{2}{3bd\sqrt{b\tan^3(c+dx)}}-\frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)-\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,bd\sqrt{b\tan^3(c+dx)}}+\frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)+\sqrt{2}\,\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,bd\sqrt{b\tan^3(c+dx)}}-\frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^3)^(-3/2),x]
```

```
[Out] 2/(3*b*d*Sqrt[b*Tan[c + d*x]^3]) - (2*Cot[c + d*x]^2)/(7*b*d*Sqrt[b*Tan[c +
 d*x]^3]) - (ArcTan[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sq
rt[2]*b*d*Sqrt[b*Tan[c + d*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]
*Tan[c + d*x]^(3/2))/(Sqrt[2]*b*d*Sqrt[b*Tan[c + d*x]^3]) - (Log[1 - Sqrt[2
]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b*d*Sqr
t[b*Tan[c + d*x]^3]) + (Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*
Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b*d*Sqrt[b*Tan[c + d*x]^3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(b\tan^3(c+dx))^{3/2}}\,dx = \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{9}{2}}(c+dx)}\,dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{5}{2}}(c+dx)}\,dx}{b\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\sqrt{\tan(c+dx)}}\,d}{b\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\Big(\int \frac{1}{\sqrt{x}\,(1+x^2}}{bd\sqrt{b\tan^3(c+}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\left(2\tan^{\frac{3}{2}}(c+dx)\right)\text{Subst}\Big(\int \frac{1}{1+x^4}}{bd\sqrt{b\tan^3(c+}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\Big(\int \frac{1-x^2}{1+x^4}\,dx, x}{bd\sqrt{b\tan^3(c+d}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\text{Subst}\Big(\int \frac{1}{1-\sqrt{2}\,x+x}}{2bd\sqrt{b\tan^3(c}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} - \frac{\log\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}+t\right.}{2\sqrt{2}\,bd\sqrt{b\tan^3}}$$

$$= \frac{2}{3bd\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^2(c+dx)}{7bd\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{-1}\left(1-\sqrt{2}\,\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,bd\sqrt{b\tan^3(c+d}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.08, size = 45, normalized size = 0.15

$$-\frac{2\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};-\tan^2(c+dx)\right)\tan(c+dx)}{7d\,(b\tan^3(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^3)^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-7/4, 1, -3/4, -Tan[c + d*x]^2]*Tan[c + d*x])/(7*d*(b
*Tan[c + d*x]^3)^(3/2))
```

Maple [A]
time = 0.08, size = 236, normalized size = 0.79

method	result
derivativedivides	$\frac{\tan(dx+c)\left(21\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\left(b\tan(dx+c)\right)^{\frac{7}{2}}\ln\left(-\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\,\sqrt{2}+\sqrt{b^2}}{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}\right)+42\left(b^2\right)^{\frac{1}{4}}}{}$
default	$\frac{\tan(dx+c)\left(21\left(b^2\right)^{\frac{1}{4}}\sqrt{2}\,\left(b\tan(dx+c)\right)^{\frac{7}{2}}\ln\left(-\frac{b\tan(dx+c)+\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\,\sqrt{2}+\sqrt{b^2}}{\left(b^2\right)^{\frac{1}{4}}\sqrt{b\tan\left(dx+c\right)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}\right)+42\left(b^2\right)^{\frac{1}{4}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/84/d*tan(d*x+c)/b^4*(21*(b^2)^(1/4)*2^(1/2)*(b*tan(d*x+c))^(7/2)*ln(-(b*t
an(d*x+c)+(b^2)^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2))/((b^2)^(1/4
)*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2)))+42*(b^2)^(1/4)*2^
(1/2)*(b*tan(d*x+c))^(7/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4)
)/(b^2)^(1/4))+42*(b^2)^(1/4)*2^(1/2)*(b*tan(d*x+c))^(7/2)*arctan((2^(1/2)*
(b*tan(d*x+c))^(1/2)-(b^2)^(1/4))/(b^2)^(1/4))+56*b^4*tan(d*x+c)^2-24*b^4)/
(b*tan(d*x+c)^3)^(3/2)
```

Maxima [A]
time = 0.50, size = 163, normalized size = 0.55

$$\frac{\frac{21\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan\left(dx+c\right)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan\left(dx+c\right)}\right)\right)+\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan(dx+c)+1\right)-\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan\left(dx+c\right)}+\tan(dx+c)+1\right)\right)}{b^{\frac{3}{2}}}+\frac{8\left(21\sqrt{\tan\left(dx+c\right)}+\frac{7}{\tan(dx+c)^{\frac{3}{2}}}-\frac{3}{\tan(dx+c)^{\frac{7}{2}}}\right)}{b^{\frac{3}{2}}}-\frac{168\sqrt{\tan\left(dx+c\right)}}{b^{\frac{3}{2}}}}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/84*(21*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c)))) +
2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) + sqrt(2)*l
og(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) - sqrt(2)*log(-sqrt(2)*sq
rt(tan(d*x + c)) + tan(d*x + c) + 1))/b^(3/2) + 8*(21*sqrt(tan(d*x + c)) +
7/tan(d*x + c)^(3/2) - 3/tan(d*x + c)^(7/2))/b^(3/2) - 168*sqrt(tan(d*x + c
))/b^(3/2))/d
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan^{3}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**3)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**(-3/2), x)
```

Giac [A]
time = 0.66, size = 279, normalized size = 0.94

$$\frac{1}{84}\,b^{4}\left(\frac{42\,\sqrt{2}\,\sqrt{|b|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}+2\,\sqrt{b\tan\left(dx+c\right)}\right)}{2\,\sqrt{|b|}}\right)}{b^{6}d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}+\frac{42\,\sqrt{2}\,\sqrt{|b|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}-2\,\sqrt{b\tan\left(dx+c\right)}\right)}{2\,\sqrt{|b|}}\right)}{b^{6}d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}+\frac{21\,\sqrt{2}\,\sqrt{|b|}\log\left(b\tan\left(dx+c\right)+\sqrt{2}\,\sqrt{b\tan\left(dx+c\right)}\,\sqrt{|b|}+|b|\right)}{b^{6}d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}-\frac{21\,\sqrt{2}\,\sqrt{|b|}\log\left(b\tan\left(dx+c\right)-\sqrt{2}\,\sqrt{b\tan\left(dx+c\right)}\,\sqrt{|b|}+|b|\right)}{b^{6}d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}+\frac{8\left(7\,b^{2}\tan\left(dx+c\right)^{2}-3\,b^{2}\right)}{\sqrt{b\tan\left(dx+c\right)}\,b^{7}d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)\tan\left(dx+c\right)^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/84*b^4*(42*sqrt(2)*sqrt(abs(b))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b))
+ 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^6*d*sgn(tan(d*x + c))) + 42*sqrt
(2)*sqrt(abs(b))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(b*tan(d
*x + c)))/sqrt(abs(b)))/(b^6*d*sgn(tan(d*x + c))) + 21*sqrt(2)*sqrt(abs(b))
*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(
b^6*d*sgn(tan(d*x + c))) - 21*sqrt(2)*sqrt(abs(b))*log(b*tan(d*x + c) - sqr
t(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^6*d*sgn(tan(d*x + c)))
+ 8*(7*b^2*tan(d*x + c)^2 - 3*b^2)/(sqrt(b*tan(d*x + c))*b^7*d*sgn(tan(d*x
+ c))*tan(d*x + c)^3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,{\tan\left(c+d\,x\right)}^3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^3)^(3/2),x)
```

```
[Out] int(1/(b*tan(c + d*x)^3)^(3/2), x)
```

3.35 $\int \frac{1}{\left(b\tan^3(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=364

$$-\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}}+\frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}}-\frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}}+\frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}}-\frac{\text{ArcTan}(}{}$$

```
[Out] -2/5*cot(d*x+c)/b^2/d/(b*tan(d*x+c)^3)^(1/2)+2/9*cot(d*x+c)^3/b^2/d/(b*tan(
d*x+c)^3)^(1/2)-2/13*cot(d*x+c)^5/b^2/d/(b*tan(d*x+c)^3)^(1/2)+2*tan(d*x+c)
/b^2/d/(b*tan(d*x+c)^3)^(1/2)+1/2*arctan(-1+2^(1/2)*tan(d*x+c)^(1/2))*tan(d
*x+c)^(3/2)/b^2/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)+1/2*arctan(1+2^(1/2)*tan(d
*x+c)^(1/2))*tan(d*x+c)^(3/2)/b^2/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)+1/4*ln(1
-2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(3/2)/b^2/d*2^(1/2)/(b*tan
(d*x+c)^3)^(1/2)-1/4*ln(1+2^(1/2)*tan(d*x+c)^(1/2)+tan(d*x+c))*tan(d*x+c)^(
3/2)/b^2/d*2^(1/2)/(b*tan(d*x+c)^3)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(c+dx)}\right)}{\sqrt{2}\,b^2d\sqrt{b\tan^3(c+dx)}}+\frac{\tan^{\frac{3}{2}}(c+dx)\text{ArcTan}\left(\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{\sqrt{2}\,b^2d\sqrt{b\tan^3(c+dx)}}+\frac{2\tan(c+dx)}{b^2d\sqrt{b\tan^3(c+dx)}}+\frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)-\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,b^2d\sqrt{b\tan^3(c+dx)}}-\frac{\tan^{\frac{3}{2}}(c+dx)\log\left(\tan(c+dx)+\sqrt{2}\sqrt{\tan(c+dx)}+1\right)}{2\sqrt{2}\,b^2d\sqrt{b\tan^3(c+dx)}}-\frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}}+\frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}}-\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^3)^(-5/2),x]
```

```
[Out] (-2*Cot[c + d*x])/(5*b^2*d*Sqrt[b*Tan[c + d*x]^3]) + (2*Cot[c + d*x]^3)/(9*
b^2*d*Sqrt[b*Tan[c + d*x]^3]) - (2*Cot[c + d*x]^5)/(13*b^2*d*Sqrt[b*Tan[c +
 d*x]^3]) + (2*Tan[c + d*x])/(b^2*d*Sqrt[b*Tan[c + d*x]^3]) - (ArcTan[1 - S
qrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(Sqrt[2]*b^2*d*Sqrt[b*Tan[c
+ d*x]^3]) + (ArcTan[1 + Sqrt[2]*Sqrt[Tan[c + d*x]]]*Tan[c + d*x]^(3/2))/(S
qrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3]) + (Log[1 - Sqrt[2]*Sqrt[Tan[c + d*x]]
+ Tan[c + d*x]]*Tan[c + d*x]^(3/2))/(2*Sqrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3]
) - (Log[1 + Sqrt[2]*Sqrt[Tan[c + d*x]] + Tan[c + d*x]]*Tan[c + d*x]^(3/2))
/(2*Sqrt[2]*b^2*d*Sqrt[b*Tan[c + d*x]^3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(b\tan^3(c+dx))^{5/2}}\,dx = \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{15}{2}}(c+dx)}\,dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} - \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{11}{2}}(c+dx)}\,dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$= \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{\tan^{\frac{3}{2}}(c+dx)\int \frac{1}{\tan^{\frac{7}{2}}(c+dx)}\,dx}{b^2\sqrt{b\tan^3(c+dx)}}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} - \text{ta}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{}{b^2}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{}{b^2}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{}{b^2}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{}{b^2}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{}{b^2}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{}{b^2}$$

$$= -\frac{2\cot(c+dx)}{5b^2d\sqrt{b\tan^3(c+dx)}} + \frac{2\cot^3(c+dx)}{9b^2d\sqrt{b\tan^3(c+dx)}} - \frac{2\cot^5(c+dx)}{13b^2d\sqrt{b\tan^3(c+dx)}} + \frac{}{b^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.06, size = 45, normalized size = 0.12

$$-\frac{2\,{}_2F_1\left(-\frac{13}{4},1;-\frac{9}{4};-\tan^2(c+dx)\right)\tan(c+dx)}{13d\left(b\tan^3(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^3)^(-5/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-13/4, 1, -9/4, -Tan[c + d*x]^2]*Tan[c + d*x])/(13*d*
(b*Tan[c + d*x]^3)^(5/2))
```

Maple [A]
time = 0.06, size = 272, normalized size = 0.75

method	result
derivativedivides	$\frac{\tan(dx+c)\left(585\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\text{t}\cdots}{\cdots}$
default	$\frac{\tan(dx+c)\left(585\sqrt{2}\,(b\tan(dx+c))^{\frac{13}{2}}\ln\left(-\frac{(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}-b\tan(dx+c)-\sqrt{b^2}}{b\tan(dx+c)+(b^2)^{\frac{1}{4}}\sqrt{b\tan(dx+c)}\,\sqrt{2}+\sqrt{b^2}}\right)+1170\sqrt{2}\,(b\text{t}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2340/d*tan(d*x+c)/b^6*(585*2^(1/2)*(b*tan(d*x+c))^(13/2)*ln(-((b^2)^(1/4)
*(b*tan(d*x+c))^(1/2)*2^(1/2)-b*tan(d*x+c)-(b^2)^(1/2))/(b*tan(d*x+c)+(b^2)
^(1/4)*(b*tan(d*x+c))^(1/2)*2^(1/2)+(b^2)^(1/2)))+1170*2^(1/2)*(b*tan(d*x+c
))^(13/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)+(b^2)^(1/4))/(b^2)^(1/4))+11
70*2^(1/2)*(b*tan(d*x+c))^(13/2)*arctan((2^(1/2)*(b*tan(d*x+c))^(1/2)-(b^2)
^(1/4))/(b^2)^(1/4))+4680*(b^2)^(1/4)*tan(d*x+c)^6*b^6-936*b^6*(b^2)^(1/4)*
tan(d*x+c)^4+520*b^6*(b^2)^(1/4)*tan(d*x+c)^2-360*b^6*(b^2)^(1/4))/(b*tan(d
*x+c)^3)^(5/2)/(b^2)^(1/4)
```

Maxima [A]
time = 0.52, size = 172, normalized size = 0.47

$$\frac{\frac{585\left(2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(dx+c)}\right)\right)+2\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(dx+c)}\right)\right)-\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)+\sqrt{2}\log\left(-\sqrt{2}\sqrt{\tan(dx+c)}+\tan(dx+c)+1\right)\right)}{b^{\frac{5}{2}}}+\frac{8\left(\frac{585\sqrt{b}}{\sqrt{\tan(dx+c)}}-\frac{117\sqrt{b}}{\tan(dx+c)^{\frac{5}{2}}}+\frac{65\sqrt{b}}{\tan(dx+c)^{\frac{9}{2}}}-\frac{45\sqrt{b}}{\tan(dx+c)^{\frac{13}{2}}}\right)}{b^3}}{2340\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2340*(585*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(d*x + c))))
 + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(d*x + c)))) - sqrt(2
```

```
)*log(sqrt(2)*sqrt(tan(d*x + c)) + tan(d*x + c) + 1) + sqrt(2)*log(-sqrt(2)
*sqrt(tan(d*x + c)) + tan(d*x + c) + 1))/b^(5/2) + 8*(585*sqrt(b)/sqrt(tan(
d*x + c)) - 117*sqrt(b)/tan(d*x + c)^(5/2) + 65*sqrt(b)/tan(d*x + c)^(9/2)
- 45*sqrt(b)/tan(d*x + c)^(13/2))/b^3)/d
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tan^{3}\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**3)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**3)**(-5/2), x)
```

Giac [**A**]
time = 0.88, size = 305, normalized size = 0.84

$$\frac{1}{2340}\,b^{6}\left(\frac{1170\,\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}+2\,\sqrt{b\tan\left(dx+c\right)}\right)}{2\,\sqrt{|b|}}\right)}{b^{10}d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}+\frac{1170\,\sqrt{2}\,|b|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|b|}-2\,\sqrt{b\tan\left(dx+c\right)}\right)}{2\,\sqrt{|b|}}\right)}{b^{10}d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}-\frac{585\,\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(dx+c\right)+\sqrt{2}\,\sqrt{b\tan\left(dx+c\right)}\,\sqrt{|b|}+|b|\right)}{b^{10}d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}+\frac{585\,\sqrt{2}\,|b|^{\frac{3}{2}}\log\left(b\tan\left(dx+c\right)-\sqrt{2}\,\sqrt{b\tan\left(dx+c\right)}\,\sqrt{|b|}+|b|\right)}{b^{10}d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)}+\frac{8\left(585\,b^{6}\tan\left(dx+c\right)^{6}-117\,b^{6}\tan\left(dx+c\right)^{4}+65\,b^{6}\tan\left(dx+c\right)^{2}-45\,b^{6}\right)}{\sqrt{b\tan\left(dx+c\right)}\,b^{14}d\mathrm{sgn}\left(\tan\left(dx+c\right)\right)\tan\left(dx+c\right)^{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^3)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2340*b^6*(1170*sqrt(2)*abs(b)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(
b)) + 2*sqrt(b*tan(d*x + c)))/sqrt(abs(b)))/(b^10*d*sgn(tan(d*x + c))) + 11
70*sqrt(2)*abs(b)^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(b)) - 2*sqrt(
b*tan(d*x + c)))/sqrt(abs(b)))/(b^10*d*sgn(tan(d*x + c))) - 585*sqrt(2)*abs
(b)^(3/2)*log(b*tan(d*x + c) + sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) +
abs(b))/(b^10*d*sgn(tan(d*x + c))) + 585*sqrt(2)*abs(b)^(3/2)*log(b*tan(d*x
 + c) - sqrt(2)*sqrt(b*tan(d*x + c))*sqrt(abs(b)) + abs(b))/(b^10*d*sgn(tan
(d*x + c))) + 8*(585*b^6*tan(d*x + c)^6 - 117*b^6*tan(d*x + c)^4 + 65*b^6*t
an(d*x + c)^2 - 45*b^6)/(sqrt(b*tan(d*x + c))*b^14*d*sgn(tan(d*x + c))*tan(
d*x + c)^6))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\tan\left(c+d\,x\right)^3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^3)^(5/2),x)
[Out] int(1/(b*tan(c + d*x)^3)^(5/2), x)
```

3.36 $\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=182

$$\frac{b^2\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - b^2x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} - \frac{b^2\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b^2\tan^3}{}$$

```
[Out] b^2*cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-b^2*x*cot(d*x+c)^2*(tan(d*x+c)^4*b)
^(1/2)-1/3*b^2*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)/d+1/5*b^2*(tan(d*x+c)^4*b)
^(1/2)*tan(d*x+c)^3/d-1/7*b^2*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)^5/d+1/9*b^2
*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$-\frac{b^2\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d} - \frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - b^2x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} + \frac{b^2\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^4)^(5/2),x]
```

```
[Out] (b^2*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - b^2*x*Cot[c + d*x]^2*Sqrt[b*T
an[c + d*x]^4] - (b^2*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/(3*d) + (b^2*Tan
[c + d*x]^3*Sqrt[b*Tan[c + d*x]^4])/(5*d) - (b^2*Tan[c + d*x]^5*Sqrt[b*Tan[
c + d*x]^4])/(7*d) + (b^2*Tan[c + d*x]^7*Sqrt[b*Tan[c + d*x]^4])/(9*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^4(c+dx)\right)^{5/2}\,dx = \left(b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int \tan^{10}(c+dx)\,dx$$
$$= \frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d} - \left(b^2\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int \tan^8$$
$$= -\frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} + \frac{b^2\tan^7(c+dx)\sqrt{b\tan^4(c+dx)}}{9d} + \Big(b^2 c$$
$$= \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \frac{b^2\tan^5(c+dx)\sqrt{b\tan^4(c+dx)}}{7d} + \frac{b^2\tan^7}{}$$
$$= -\frac{b^2\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b^2\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \frac{b^2\tan}{}$$
$$= \frac{b^2\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - \frac{b^2\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b^2\tan^3(c}{}$$
$$= \frac{b^2\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - b^2x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} - \frac{b^2\tan(c}{}$$

Mathematica [A]
time = 0.78, size = 86, normalized size = 0.47

$$\frac{\cot(c+dx)\left(35-45\cot^2(c+dx)+63\cot^4(c+dx)-105\cot^6(c+dx)+315\cot^8(c+dx)-315\text{ArcTan}(\tan(c+dx))\cot^9(c+dx)\right)\left(b\tan^4(c+dx)\right)^{5/2}}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^4)^(5/2),x]
```

```
[Out] (Cot[c + d*x]*(35 - 45*Cot[c + d*x]^2 + 63*Cot[c + d*x]^4 - 105*Cot[c + d*x
]^6 + 315*Cot[c + d*x]^8 - 315*ArcTan[Tan[c + d*x]]*Cot[c + d*x]^9)*(b*Tan[
c + d*x]^4)^(5/2))/(315*d)
```

Maple [A]
time = 0.09, size = 84, normalized size = 0.46

method	result
derivativedivides	$-\frac{\left(b\left(\tan^4(dx+c)\right)\right)^{\frac{5}{2}}\left(-35\left(\tan^9(dx+c)\right)+45\left(\tan^7(dx+c)\right)-63\left(\tan^5(dx+c)\right)+105\left(\tan^3(dx+c)\right)+315\arctan(\tan(dx+c\right.}{315d\tan(dx+c)^{10}}$
default	$-\frac{\left(b\left(\tan^4(dx+c)\right)\right)^{\frac{5}{2}}\left(-35\left(\tan^9(dx+c)\right)+45\left(\tan^7(dx+c)\right)-63\left(\tan^5(dx+c)\right)+105\left(\tan^3(dx+c)\right)+315\arctan(\tan(dx+c\right.}{315d\tan(dx+c)^{10}}$

risch	$\frac{b^2\left(e^{2i(dx+c)}+1\right)^2\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\,x}{\left(e^{2i(dx+c)}-1\right)^2} - \frac{2ib^2\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\left(1575\,e^{16i(dx+c)}+6300\,e^{14i(dx+c)}+21000\,e^{12i(dx}\right.}{315\left(e^{2i}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315/d*(b*tan(d*x+c)^4)^(5/2)*(-35*tan(d*x+c)^9+45*tan(d*x+c)^7-63*tan(d*
x+c)^5+105*tan(d*x+c)^3+315*arctan(tan(d*x+c))-315*tan(d*x+c))/tan(d*x+c)^1
0
```

Maxima [A]
time = 0.49, size = 79, normalized size = 0.43

$$\frac{35\,b^{\frac{5}{2}}\tan\left(dx+c\right)^9-45\,b^{\frac{5}{2}}\tan\left(dx+c\right)^7+63\,b^{\frac{5}{2}}\tan\left(dx+c\right)^5-105\,b^{\frac{5}{2}}\tan\left(dx+c\right)^3-315\,(dx+c)b^{\frac{5}{2}}+315\,b^{\frac{5}{2}}\tan\left(dx+c\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*b^(5/2)*tan(d*x + c)^9 - 45*b^(5/2)*tan(d*x + c)^7 + 63*b^(5/2)*t
an(d*x + c)^5 - 105*b^(5/2)*tan(d*x + c)^3 - 315*(d*x + c)*b^(5/2) + 315*b^
(5/2)*tan(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 96, normalized size = 0.53

$$\frac{\left(35\,b^2\tan\left(dx+c\right)^9-45\,b^2\tan\left(dx+c\right)^7+63\,b^2\tan\left(dx+c\right)^5-105\,b^2\tan\left(dx+c\right)^3-315\,b^2dx+315\,b^2\tan\left(dx+c\right)\right)\sqrt{b\tan\left(dx+c\right)^4}}{315\,d\tan\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^2*tan(d*x + c)^9 - 45*b^2*tan(d*x + c)^7 + 63*b^2*tan(d*x + c)^
5 - 105*b^2*tan(d*x + c)^3 - 315*b^2*d*x + 315*b^2*tan(d*x + c))*sqrt(b*tan
(d*x + c)^4)/(d*tan(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan^4\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)**4*b)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**4)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 960 vs. 2(162) = 324.
time = 5.60, size = 960, normalized size = 5.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(5/2),x, algorithm="giac")
```

```
[Out] -1/315*(315*b^2*d*x*tan(d*x)^9*tan(c)^9 - 2835*b^2*d*x*tan(d*x)^8*tan(c)^8
+ 315*b^2*tan(d*x)^9*tan(c)^8 + 315*b^2*tan(d*x)^8*tan(c)^9 + 11340*b^2*d*x
*tan(d*x)^7*tan(c)^7 - 105*b^2*tan(d*x)^9*tan(c)^6 - 2835*b^2*tan(d*x)^8*ta
n(c)^7 - 2835*b^2*tan(d*x)^7*tan(c)^8 - 105*b^2*tan(d*x)^6*tan(c)^9 - 26460
*b^2*d*x*tan(d*x)^6*tan(c)^6 + 63*b^2*tan(d*x)^9*tan(c)^4 + 945*b^2*tan(d*x
)^8*tan(c)^5 + 11340*b^2*tan(d*x)^7*tan(c)^6 + 11340*b^2*tan(d*x)^6*tan(c)^
7 + 945*b^2*tan(d*x)^5*tan(c)^8 + 63*b^2*tan(d*x)^4*tan(c)^9 + 39690*b^2*d*
x*tan(d*x)^5*tan(c)^5 - 45*b^2*tan(d*x)^9*tan(c)^2 - 567*b^2*tan(d*x)^8*tan
(c)^3 - 3780*b^2*tan(d*x)^7*tan(c)^4 - 26460*b^2*tan(d*x)^6*tan(c)^5 - 2646
0*b^2*tan(d*x)^5*tan(c)^6 - 3780*b^2*tan(d*x)^4*tan(c)^7 - 567*b^2*tan(d*x)
^3*tan(c)^8 - 45*b^2*tan(d*x)^2*tan(c)^9 - 39690*b^2*d*x*tan(d*x)^4*tan(c)^
4 + 35*b^2*tan(d*x)^9 + 405*b^2*tan(d*x)^8*tan(c) + 2268*b^2*tan(d*x)^7*tan
(c)^2 + 8820*b^2*tan(d*x)^6*tan(c)^3 + 39690*b^2*tan(d*x)^5*tan(c)^4 + 3969
0*b^2*tan(d*x)^4*tan(c)^5 + 8820*b^2*tan(d*x)^3*tan(c)^6 + 2268*b^2*tan(d*x
)^2*tan(c)^7 + 405*b^2*tan(d*x)*tan(c)^8 + 35*b^2*tan(c)^9 + 26460*b^2*d*x*
tan(d*x)^3*tan(c)^3 - 45*b^2*tan(d*x)^7 - 567*b^2*tan(d*x)^6*tan(c) - 3780*
b^2*tan(d*x)^5*tan(c)^2 - 26460*b^2*tan(d*x)^4*tan(c)^3 - 26460*b^2*tan(d*x
)^3*tan(c)^4 - 3780*b^2*tan(d*x)^2*tan(c)^5 - 567*b^2*tan(d*x)*tan(c)^6 - 4
5*b^2*tan(c)^7 - 11340*b^2*d*x*tan(d*x)^2*tan(c)^2 + 63*b^2*tan(d*x)^5 + 94
5*b^2*tan(d*x)^4*tan(c) + 11340*b^2*tan(d*x)^3*tan(c)^2 + 11340*b^2*tan(d*x
)^2*tan(c)^3 + 945*b^2*tan(d*x)*tan(c)^4 + 63*b^2*tan(c)^5 + 2835*b^2*d*x*t
an(d*x)*tan(c) - 105*b^2*tan(d*x)^3 - 2835*b^2*tan(d*x)^2*tan(c) - 2835*b^2
*tan(d*x)*tan(c)^2 - 105*b^2*tan(c)^3 - 315*b^2*d*x + 315*b^2*tan(d*x) + 31
5*b^2*tan(c))*sqrt(b)/(d*tan(d*x)^9*tan(c)^9 - 9*d*tan(d*x)^8*tan(c)^8 + 36
*d*tan(d*x)^7*tan(c)^7 - 84*d*tan(d*x)^6*tan(c)^6 + 126*d*tan(d*x)^5*tan(c)
^5 - 126*d*tan(d*x)^4*tan(c)^4 + 84*d*tan(d*x)^3*tan(c)^3 - 36*d*tan(d*x)^2
*tan(c)^2 + 9*d*tan(d*x)*tan(c) - d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan(c+d\,x)^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^4)^(5/2),x)
[Out] int((b*tan(c + d*x)^4)^(5/2), x)
```

3.37 $\int (b\tan^4(c+dx))^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - bx\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} - \frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b\tan^3(c$$

```
[Out] b*cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-b*x*cot(d*x+c)^2*(tan(d*x+c)^4*b)^(1/
2)-1/3*b*(tan(d*x+c)^4*b)^(1/2)*tan(d*x+c)/d+1/5*b*(tan(d*x+c)^4*b)^(1/2)*t
an(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$-\frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - bx\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} + \frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^4)^(3/2),x]
```

```
[Out] (b*Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - b*x*Cot[c + d*x]^2*Sqrt[b*Tan[c
 + d*x]^4] - (b*Tan[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/(3*d) + (b*Tan[c + d*x
]^3*Sqrt[b*Tan[c + d*x]^4])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^4(c+dx)\right)^{3/2}\,dx = \left(b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int \tan^6(c+dx)\,dx$$
$$= \frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} - \left(b\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int \tan^4(c$$
$$= -\frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b\tan^3(c+dx)\sqrt{b\tan^4(c+dx)}}{5d} + \Big(b\cot^2(c$$
$$= \frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - \frac{b\tan(c+dx)\sqrt{b\tan^4(c+dx)}}{3d} + \frac{b\tan^3(c+dx}{}$$
$$= \frac{b\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - bx\cot^2(c+dx)\sqrt{b\tan^4(c+dx)} - \frac{b\tan(c+d}{}$$

Mathematica [A]
time = 0.81, size = 66, normalized size = 0.60

$$\frac{\cot(c+dx)\left(3-5\cot^2(c+dx)+15\cot^4(c+dx)-15\text{ArcTan}(\tan(c+dx))\cot^5(c+dx)\right)\left(b\tan^4(c+dx)\right)^{3/2}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^4)^(3/2),x]
```

```
[Out] (Cot[c + d*x]*(3 - 5*Cot[c + d*x]^2 + 15*Cot[c + d*x]^4 - 15*ArcTan[Tan[c +
 d*x]]*Cot[c + d*x]^5)*(b*Tan[c + d*x]^4)^(3/2))/(15*d)
```

Maple [A]
time = 0.06, size = 64, normalized size = 0.58

method	result
derivativedivides	$-\frac{\left(b\left(\tan^4(dx+c)\right)\right)^{\frac{3}{2}}\left(-3\left(\tan^5(dx+c)\right)+5\left(\tan^3(dx+c)\right)+15\arctan(\tan(dx+c))-15\tan(dx+c)\right)}{15d\tan(dx+c)^6}$
default	$-\frac{\left(b\left(\tan^4(dx+c)\right)\right)^{\frac{3}{2}}\left(-3\left(\tan^5(dx+c)\right)+5\left(\tan^3(dx+c)\right)+15\arctan(\tan(dx+c))-15\tan(dx+c)\right)}{15d\tan(dx+c)^6}$
risch	$\frac{b\left(e^{2i(dx+c)}+1\right)^2\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\,x}{\left(e^{2i(dx+c)}-1\right)^2} - \frac{2ib\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\left(45\,e^{8i(dx+c)}+90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}+70\,e^{2i(d}\right.}{15\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)^3 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/d*(b*tan(d*x+c)^4)^(3/2)*(-3*tan(d*x+c)^5+5*tan(d*x+c)^3+15*arctan(ta
n(d*x+c))-15*tan(d*x+c))/tan(d*x+c)^6
```

Maxima [A]
time = 0.50, size = 53, normalized size = 0.48

$$\frac{3\,b^{\frac{3}{2}}\tan\left(dx+c\right)^{5}-5\,b^{\frac{3}{2}}\tan\left(dx+c\right)^{3}-15\,(dx+c)b^{\frac{3}{2}}+15\,b^{\frac{3}{2}}\tan\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^(3/2)*tan(d*x + c)^5 - 5*b^(3/2)*tan(d*x + c)^3 - 15*(d*x + c)*b^
(3/2) + 15*b^(3/2)*tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 62, normalized size = 0.56

$$\frac{\left(3\,b\tan\left(dx+c\right)^{5}-5\,b\tan\left(dx+c\right)^{3}-15\,bdx+15\,b\tan\left(dx+c\right)\right)\sqrt{b\tan\left(dx+c\right)^{4}}}{15\,d\tan\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*tan(d*x + c)^5 - 5*b*tan(d*x + c)^3 - 15*b*d*x + 15*b*tan(d*x + c
))*sqrt(b*tan(d*x + c)^4)/(d*tan(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan^{4}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)**4*b)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**4)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(98) = 196.
time = 2.74, size = 992, normalized size = 9.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(3/2),x, algorithm="giac")
```

```
[Out] 1/60*(15*pi - 60*d*x*tan(d*x)^5*tan(c)^5 - 15*pi*sgn(2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^5*tan(c)^5 - 15*pi*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*ta
n(d*x)^5*tan(c)^5 + 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*ta
n(d*x)^5*tan(c)^5 + 300*d*x*tan(d*x)^4*tan(c)^4 + 75*pi*sgn(2*tan(d*x)^2*ta
n(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 7
5*pi*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan
(c)))*tan(d*x)^4*tan(c)^4 - 150*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c)
 - 1))*tan(d*x)^4*tan(c)^4 - 60*tan(d*x)^5*tan(c)^4 - 60*tan(d*x)^4*tan(c)^
5 - 600*d*x*tan(d*x)^3*tan(c)^3 - 150*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*
x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)^3*tan(c)^3 + 20*tan(d*x)^5*ta
n(c)^2 - 150*pi*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x)*tan(c) - 1)/(tan
(d*x) + tan(c)))*tan(d*x)^3*tan(c)^3 + 300*arctan((tan(d*x) + tan(c))/(tan(
d*x)*tan(c) - 1))*tan(d*x)^3*tan(c)^3 + 300*tan(d*x)^4*tan(c)^3 + 300*tan(d
*x)^3*tan(c)^4 + 20*tan(d*x)^2*tan(c)^5 + 600*d*x*tan(d*x)^2*tan(c)^2 + 150
*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*
tan(d*x)^2*tan(c)^2 - 12*tan(d*x)^5 - 100*tan(d*x)^4*tan(c) + 150*pi*tan(d*
x)^2*tan(c)^2 - 300*arctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d
*x)^2*tan(c)^2 - 300*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(
d*x)^2*tan(c)^2 - 600*tan(d*x)^3*tan(c)^2 - 600*tan(d*x)^2*tan(c)^3 - 100*t
an(d*x)*tan(c)^4 - 12*tan(c)^5 - 300*d*x*tan(d*x)*tan(c) - 75*pi*sgn(2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c
) + 20*tan(d*x)^3 - 75*pi*tan(d*x)*tan(c) + 150*arctan((tan(d*x)*tan(c) - 1
)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 150*arctan((tan(d*x) + tan(c))/(ta
n(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 300*tan(d*x)^2*tan(c) + 300*tan(d*x)*
tan(c)^2 + 20*tan(c)^3 + 60*d*x + 15*pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x
)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 30*arctan((tan(d*x)*tan(c) - 1)/(tan(
d*x) + tan(c))) - 30*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) - 60
*tan(d*x) - 60*tan(c))*b^(3/2)/(d*tan(d*x)^5*tan(c)^5 - 5*d*tan(d*x)^4*tan(
c)^4 + 10*d*tan(d*x)^3*tan(c)^3 - 10*d*tan(d*x)^2*tan(c)^2 + 5*d*tan(d*x)*t
an(c) - d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\tan(c+d\,x)^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^4)^(3/2),x)
```

```
[Out] int((b*tan(c + d*x)^4)^(3/2), x)
```

3.38 $\int \sqrt{b\tan^4(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}-x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

```
[Out] cot(d*x+c)*(tan(d*x+c)^4*b)^(1/2)/d-x*cot(d*x+c)^2*(tan(d*x+c)^4*b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$\frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d}-x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[c + d*x]^4],x]
```

```
[Out] (Cot[c + d*x]*Sqrt[b*Tan[c + d*x]^4])/d - x*Cot[c + d*x]^2*Sqrt[b*Tan[c + d
*x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{b\tan^4(c+dx)}\,dx = \left(\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int \tan^2(c+dx)\,dx$$
$$= \frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - \left(\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}\right)\int 1\,dx$$
$$= \frac{\cot(c+dx)\sqrt{b\tan^4(c+dx)}}{d} - x\cot^2(c+dx)\sqrt{b\tan^4(c+dx)}$$

Mathematica [A]
time = 0.11, size = 41, normalized size = 0.82

$$-\frac{\cot(c+dx)(-1+\text{ArcTan}(\tan(c+dx))\cot(c+dx))\sqrt{b\tan^4(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^4],x]
[Out] -((Cot[c + d*x]*(-1 + ArcTan[Tan[c + d*x]]*Cot[c + d*x])*Sqrt[b*Tan[c + d*x
]^4])/d)
```

Maple [A]
time = 0.06, size = 42, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{\sqrt{b\left(\tan^4(dx+c)\right)}\,(-\tan(dx+c)+\arctan(\tan(dx+c)))}{d\tan(dx+c)^2}$	42
default	$-\frac{\sqrt{b\left(\tan^4(dx+c)\right)}\,(-\tan(dx+c)+\arctan(\tan(dx+c)))}{d\tan(dx+c)^2}$	42
risch	$\frac{\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\,\left(e^{2i(dx+c)}+1\right)^2 x}{\left(e^{2i(dx+c)}-1\right)^2} - \frac{2i\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\,\left(e^{2i(dx+c)}+1\right)}{\left(e^{2i(dx+c)}-1\right)^2 d}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/d*(b*tan(d*x+c)^4)^(1/2)*(-tan(d*x+c)+arctan(tan(d*x+c)))/tan(d*x+c)^2
```

Maxima [A]
time = 0.49, size = 26, normalized size = 0.52

$$-\frac{(dx+c)\sqrt{b} - \sqrt{b}\,\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
```

```
[Out] -((d*x + c)*sqrt(b) - sqrt(b)*tan(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 37, normalized size = 0.74

$$-\frac{\sqrt{b\tan\left(dx+c\right)^4}\left(dx-\tan\left(dx+c\right)\right)}{d\tan\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*tan(d*x + c)^4)*(d*x - tan(d*x + c))/(d*tan(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\tan^4\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)**4*b)**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(c + d*x)**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(46) = 92.
time = 0.56, size = 229, normalized size = 4.58

$$\frac{\left(\pi-4\,dx\tan\left(dx\right)\tan\left(c\right)-\pi\mathrm{sgn}\left(2\,\tan\left(dx\right)^2\tan\left(c\right)+2\,\tan\left(dx\right)\tan\left(c\right)^2-2\,\tan\left(dx\right)-2\,\tan\left(c\right)\right)\tan\left(dx\right)\tan\left(c\right)-\pi\tan\left(dx\right)\tan\left(c\right)+2\,\arctan\left(\frac{\tan(dx)\tan(c)-1}{\tan(dx)+\tan(c)}\right)\tan\left(dx\right)\tan\left(c\right)+2\,\arctan\left(\frac{\tan(dx)+\tan(c)}{\tan(dx)\tan(c)-1}\right)\tan\left(dx\right)\tan\left(c\right)+4\,dx+\pi\mathrm{sgn}\left(2\,\tan\left(dx\right)^2\tan\left(c\right)+2\,\tan\left(dx\right)\tan\left(c\right)^2-2\,\tan\left(dx\right)-2\,\tan\left(c\right)\right)-2\,\arctan\left(\frac{\tan(dx)\tan(c)-1}{\tan(dx)+\tan(c)}\right)-2\,\arctan\left(\frac{\tan(dx)+\tan(c)}{\tan(dx)\tan(c)-1}\right)-4\,\tan\left(dx\right)-4\,\tan\left(c\right)\right)\sqrt{b}}{4\left(d\tan\left(dx\right)\tan\left(c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(pi - 4*d*x*tan(d*x)*tan(c) - pi*sgn(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 - 2*tan(d*x) - 2*tan(c))*tan(d*x)*tan(c) - pi*tan(d*x)*tan(c) + 2*a
rctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c)))*tan(d*x)*tan(c) + 2*arctan
((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)*tan(c) + 4*d*x + pi*sg
n(2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 - 2*tan(d*x) - 2*tan(c)) - 2*ar
ctan((tan(d*x)*tan(c) - 1)/(tan(d*x) + tan(c))) - 2*arctan((tan(d*x) + tan(
c))/(tan(d*x)*tan(c) - 1)) - 4*tan(d*x) - 4*tan(c))*sqrt(b)/(d*tan(d*x)*tan
(c) - d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b \tan(c + d x)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^4)^(1/2),x)
[Out] int((b*tan(c + d*x)^4)^(1/2), x)
```

3.39 $\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}-\frac{x\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)}}$$

```
[Out] -tan(d*x+c)/d/(tan(d*x+c)^4*b)^(1/2)-x*tan(d*x+c)^2/(tan(d*x+c)^4*b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$-\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}-\frac{x\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^4],x]
```

```
[Out] -(Tan[c + d*x]/(d*Sqrt[b*Tan[c + d*x]^4])) - (x*Tan[c + d*x]^2)/Sqrt[b*Tan[
c + d*x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{b\tan^4(c+dx)}}\,dx &= \frac{\tan^2(c+dx)\int \cot^2(c+dx)\,dx}{\sqrt{b\tan^4(c+dx)}}\\
&= -\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}-\frac{\tan^2(c+dx)\int 1\,dx}{\sqrt{b\tan^4(c+dx)}}\\
&= -\frac{\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}-\frac{x\tan^2(c+dx)}{\sqrt{b\tan^4(c+dx)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 43, normalized size = 0.84

$$
-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)\tan(c+dx)}{d\sqrt{b\tan^4(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^4],x]
```

```
[Out] -((Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*Sqrt[b
*Tan[c + d*x]^4]))
```

Maple [A]
time = 0.07, size = 40, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{\tan(dx+c)(\arctan(\tan(dx+c))\tan(dx+c)+1)}{d\sqrt{b\left(\tan^4(dx+c)\right)}}$	40
default	$-\frac{\tan(dx+c)(\arctan(\tan(dx+c))\tan(dx+c)+1)}{d\sqrt{b\left(\tan^4(dx+c)\right)}}$	40
risch	$\frac{\left(e^{2i(dx+c)}-1\right)^2 x}{\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\left(e^{2i(dx+c)}+1\right)^2}+\frac{2i\left(e^{2i(dx+c)}-1\right)}{\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\left(e^{2i(dx+c)}+1\right)^2 d}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*tan(d*x+c)*(arctan(tan(d*x+c))*tan(d*x+c)+1)/(b*tan(d*x+c)^4)^(1/2)
```

Maxima [A]
time = 0.49, size = 27, normalized size = 0.53

$$-\frac{\frac{dx+c}{\sqrt{b}}+\frac{1}{\sqrt{b}\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="maxima")
```

```
[Out] -((d*x + c)/sqrt(b) + 1/(sqrt(b)*tan(d*x + c)))/d
```

Fricas [A]
time = 0.37, size = 39, normalized size = 0.76

$$-\frac{\sqrt{b\tan\left(dx+c\right)^{4}}\left(dx\tan\left(dx+c\right)+1\right)}{bd\tan\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*tan(d*x + c)^4)*(d*x*tan(d*x + c) + 1)/(b*d*tan(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\tan^{4}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)**4*b)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(c + d*x)**4), x)
```

Giac [A]
time = 0.54, size = 45, normalized size = 0.88

$$-\frac{\frac{2\,(dx+c)}{\sqrt{b}}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{b}}+\frac{1}{\sqrt{b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)/sqrt(b) - tan(1/2*d*x + 1/2*c)/sqrt(b) + 1/(sqrt(b)*tan(1
/2*d*x + 1/2*c)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\tan\left(c+d\,x\right)^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^4)^(1/2),x)
[Out] int(1/(b*tan(c + d*x)^4)^(1/2), x)
```

3.40 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{3/2}}\,dx$

Optimal. Leaf size=119

$$\frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}}-\frac{x\tan^2(c+dx)}{b\sqrt{b\tan^4(c+dx)}}$$

```
[Out] 1/3*cot(d*x+c)/b/d/(tan(d*x+c)^4*b)^(1/2)-1/5*cot(d*x+c)^3/b/d/(tan(d*x+c)^
4*b)^(1/2)-tan(d*x+c)/b/d/(tan(d*x+c)^4*b)^(1/2)-x*tan(d*x+c)^2/b/(tan(d*x+
c)^4*b)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$-\frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}}-\frac{x\tan^2(c+dx)}{b\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}}+\frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^4)^(-3/2),x]
```

```
[Out] Cot[c + d*x]/(3*b*d*Sqrt[b*Tan[c + d*x]^4]) - Cot[c + d*x]^3/(5*b*d*Sqrt[b*
Tan[c + d*x]^4]) - Tan[c + d*x]/(b*d*Sqrt[b*Tan[c + d*x]^4]) - (x*Tan[c + d
*x]^2)/(b*Sqrt[b*Tan[c + d*x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(b\tan^4(c+dx))^{3/2}}\,dx = \frac{\tan^2(c+dx)\int\cot^6(c+dx)\,dx}{b\sqrt{b\tan^4(c+dx)}}$$
$$= -\frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}} - \frac{\tan^2(c+dx)\int\cot^4(c+dx)\,dx}{b\sqrt{b\tan^4(c+dx)}}$$
$$= \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}} + \frac{\tan^2(c+dx)\int\cot^2(c+dx)\,dx}{b\sqrt{b\tan^4(c+dx)}}$$
$$= \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}} - \frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}} - \frac{\tan^2(c+}{b\sqrt{b\tan}}$$
$$= \frac{\cot(c+dx)}{3bd\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5bd\sqrt{b\tan^4(c+dx)}} - \frac{\tan(c+dx)}{bd\sqrt{b\tan^4(c+dx)}} - \frac{x\tan^2(}{b\sqrt{b\tan^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 45, normalized size = 0.38

$$-\frac{{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)\tan(c+dx)}{5d\left(b\tan^4(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^4)^(-3/2),x]
```

```
[Out] -1/5*(Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*(b
*Tan[c + d*x]^4)^(3/2))
```

Maple [A]
time = 0.06, size = 63, normalized size = 0.53

method	result	size
derivativedivides	$-\frac{\tan(dx+c)\left(15\arctan(\tan(dx+c))\left(\tan^5(dx+c)\right)+15\left(\tan^4(dx+c)\right)-5\left(\tan^2(dx+c)\right)+3\right)}{15d\left(b\left(\tan^4(dx+c)\right)\right)^{\frac{3}{2}}}$	63
default	$-\frac{\tan(dx+c)\left(15\arctan(\tan(dx+c))\left(\tan^5(dx+c)\right)+15\left(\tan^4(dx+c)\right)-5\left(\tan^2(dx+c)\right)+3\right)}{15d\left(b\left(\tan^4(dx+c)\right)\right)^{\frac{3}{2}}}$	63
risch	$\frac{\left(e^{2i(dx+c)}-1\right)^2 x}{b\left(e^{2i(dx+c)}+1\right)^2\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}}+\frac{2i\left(45\,e^{8i(dx+c)}-90\,e^{6i(dx+c)}+140\,e^{4i(dx+c)}-70\,e^{2i(dx+c)}+23\right)}{15b\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)^2\sqrt{\frac{b\left(e^{2i(dx+c)}-1\right)^4}{\left(e^{2i(dx+c)}+1\right)^4}}\,d}$	174

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/d*tan(d*x+c)*(15*arctan(tan(d*x+c))*tan(d*x+c)^5+15*tan(d*x+c)^4-5*ta
n(d*x+c)^2+3)/(b*tan(d*x+c)^4)^(3/2)
```

Maxima [A]
time = 0.49, size = 50, normalized size = 0.42

$$-\frac{\frac{15\,(dx+c)}{b^{\frac{3}{2}}}+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{b^{\frac{3}{2}}\,\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(d*x + c)/b^(3/2) + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/(b
^(3/2)*tan(d*x + c)^5))/d
```

Fricas [A]
time = 0.35, size = 62, normalized size = 0.52

$$-\frac{\left(15\,dx\tan\left(dx+c\right)^5+15\,\tan\left(dx+c\right)^4-5\,\tan\left(dx+c\right)^2+3\right)\sqrt{b\tan\left(dx+c\right)^4}}{15\,b^2d\tan\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(15*d*x*tan(d*x + c)^5 + 15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)*sq
rt(b*tan(d*x + c)^4)/(b^2*d*tan(d*x + c)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\tan^4\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)**4*b)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**4)**(-3/2), x)
```

Giac [A]
time = 0.67, size = 124, normalized size = 1.04

$$-\frac{\frac{480\,(dx+c)}{\sqrt{b}}-\frac{3\,b^{\frac{9}{2}}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,b^{\frac{9}{2}}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+330\,b^{\frac{9}{2}}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{b^5}+\frac{330\,\sqrt{b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,\sqrt{b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,\sqrt{b}}{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(3/2),x, algorithm="giac")
```

```
[Out] -1/480*(480*(d*x + c)/sqrt(b) - (3*b^(9/2)*tan(1/2*d*x + 1/2*c)^5 - 35*b^(9
/2)*tan(1/2*d*x + 1/2*c)^3 + 330*b^(9/2)*tan(1/2*d*x + 1/2*c))/b^5 + (330*s
qrt(b)*tan(1/2*d*x + 1/2*c)^4 - 35*sqrt(b)*tan(1/2*d*x + 1/2*c)^2 + 3*sqrt(
b))/(b*tan(1/2*d*x + 1/2*c)^5))/(b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\tan\left(c+d\,x\right)^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^4)^(3/2),x)
```

```
[Out] int(1/(b*tan(c + d*x)^4)^(3/2), x)
```

3.41 $\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}}+\frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\tan(c}{b^2d\sqrt{b\tan}}$$

```
[Out] 1/3*cot(d*x+c)/b^2/d/(tan(d*x+c)^4*b)^(1/2)-1/5*cot(d*x+c)^3/b^2/d/(tan(d*x
+c)^4*b)^(1/2)+1/7*cot(d*x+c)^5/b^2/d/(tan(d*x+c)^4*b)^(1/2)-1/9*cot(d*x+c)
^7/b^2/d/(tan(d*x+c)^4*b)^(1/2)-tan(d*x+c)/b^2/d/(tan(d*x+c)^4*b)^(1/2)-x*t
an(d*x+c)^2/b^2/(tan(d*x+c)^4*b)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 8}

$$-\frac{\tan(c+dx)}{b^2d\sqrt{b\tan^4(c+dx)}}-\frac{x\tan^2(c+dx)}{b^2\sqrt{b\tan^4(c+dx)}}-\frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}}+\frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}}-\frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}}+\frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^4)^(-5/2),x]
```

```
[Out] Cot[c + d*x]/(3*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Cot[c + d*x]^3/(5*b^2*d*Sqr
t[b*Tan[c + d*x]^4]) + Cot[c + d*x]^5/(7*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Co
t[c + d*x]^7/(9*b^2*d*Sqrt[b*Tan[c + d*x]^4]) - Tan[c + d*x]/(b^2*d*Sqrt[b*
Tan[c + d*x]^4]) - (x*Tan[c + d*x]^2)/(b^2*Sqrt[b*Tan[c + d*x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(b\tan^4(c+dx)\right)^{5/2}}\,dx = \frac{\tan^2(c+dx)\int \cot^{10}(c+dx)\,dx}{b^2\sqrt{b\tan^4(c+dx)}}$$
$$= -\frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\tan^2(c+dx)\int \cot^8(c+dx)\,dx}{b^2\sqrt{b\tan^4(c+dx)}}$$
$$= \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\tan^2(c+dx)\int \cot^6(c+dx)\,dx}{b^2\sqrt{b\tan^4(c+dx)}}$$
$$= -\frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^7(c+dx)}{9b^2d\sqrt{b\tan^4(c+dx)}} - \tan$$
$$= \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{}{9b^2d}$$
$$= \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{}{9b^2d}$$
$$= \frac{\cot(c+dx)}{3b^2d\sqrt{b\tan^4(c+dx)}} - \frac{\cot^3(c+dx)}{5b^2d\sqrt{b\tan^4(c+dx)}} + \frac{\cot^5(c+dx)}{7b^2d\sqrt{b\tan^4(c+dx)}} - \frac{}{9b^2d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 45, normalized size = 0.25

$$-\frac{{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};-\tan^2(c+dx)\right)\tan(c+dx)}{9d\left(b\tan^4(c+dx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^4)^(-5/2),x]
```

```
[Out] -1/9*(Hypergeometric2F1[-9/2, 1, -7/2, -Tan[c + d*x]^2]*Tan[c + d*x])/(d*(b
*Tan[c + d*x]^4)^(5/2))
```

Maple [A]
time = 0.09, size = 83, normalized size = 0.45

method	result
derivativedivides	$-\frac{\tan(dx+c)(315\arctan(\tan(dx+c))(\tan^9(dx+c))+315(\tan^8(dx+c))-105(\tan^6(dx+c))+63(\tan^4(dx+c))-45(\tan^2(}{315d(b(\tan^4(dx+c)))^{\frac{5}{2}}}$
default	$-\frac{\tan(dx+c)(315\arctan(\tan(dx+c))(\tan^9(dx+c))+315(\tan^8(dx+c))-105(\tan^6(dx+c))+63(\tan^4(dx+c))-45(\tan^2(}{315d(b(\tan^4(dx+c)))^{\frac{5}{2}}}$
risch	$\frac{(e^{2i(dx+c)}-1)^2 x}{b^2(e^{2i(dx+c)}+1)^2\sqrt{\frac{b(e^{2i(dx+c)}-1)^4}{(e^{2i(dx+c)}+1)^4}}}+\frac{2i(1575\,e^{16i(dx+c)}-6300\,e^{14i(dx+c)}+21000\,e^{12i(dx+c)}-31500\,e^{10i(dx+c)}+}{315b^2(e^{2i(dx+c)}-1)^7(e^{2i(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315/d*tan(d*x+c)*(315*arctan(tan(d*x+c))*tan(d*x+c)^9+315*tan(d*x+c)^8-1
05*tan(d*x+c)^6+63*tan(d*x+c)^4-45*tan(d*x+c)^2+35)/(b*tan(d*x+c)^4)^(5/2)
```

Maxima [A]
time = 0.50, size = 70, normalized size = 0.38

$$-\frac{\frac{315\,(dx+c)}{b^{\frac{5}{2}}}+\frac{315\,\tan(dx+c)^8-105\,\tan(dx+c)^6+63\,\tan(dx+c)^4-45\,\tan(dx+c)^2+35}{b^{\frac{5}{2}}\tan(dx+c)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/315*(315*(d*x + c)/b^(5/2) + (315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 +
63*tan(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)/(b^(5/2)*tan(d*x + c)^9))/d
```

Fricas [A]
time = 0.37, size = 82, normalized size = 0.45

$$-\frac{(315\,dx\tan{(dx+c)}^9+315\,\tan{(dx+c)}^8-105\,\tan{(dx+c)}^6+63\,\tan{(dx+c)}^4-45\,\tan{(dx+c)}^2+35)\sqrt{b\tan{(dx+c)}^4}}{315\,b^3d\tan{(dx+c)}^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/315*(315*d*x*tan(d*x + c)^9 + 315*tan(d*x + c)^8 - 105*tan(d*x + c)^6 +
63*tan(d*x + c)^4 - 45*tan(d*x + c)^2 + 35)*sqrt(b*tan(d*x + c)^4)/(b^3*d*t
an(d*x + c)^11)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\tan^4{(c+dx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)**4*b)**(5/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**4)**(-5/2), x)
```

Giac [A]
time = 1.23, size = 185, normalized size = 1.01

$$-\frac{\frac{161280\,(dx+c)}{b^{\frac{5}{2}}}+\frac{121590\,\sqrt{b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-18480\,\sqrt{b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+3528\,\sqrt{b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-495\,\sqrt{b}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+35\,\sqrt{b}}{b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}}-\frac{35\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-495\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+3528\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-18480\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+121590\,b^{\frac{49}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{b^{27}}}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(tan(d*x+c)^4*b)^(5/2),x, algorithm="giac")
```

```
[Out] -1/161280*(161280*(d*x + c)/b^(5/2) + (121590*sqrt(b)*tan(1/2*d*x + 1/2*c)^
8 - 18480*sqrt(b)*tan(1/2*d*x + 1/2*c)^6 + 3528*sqrt(b)*tan(1/2*d*x + 1/2*c
)^4 - 495*sqrt(b)*tan(1/2*d*x + 1/2*c)^2 + 35*sqrt(b))/(b^3*tan(1/2*d*x + 1
/2*c)^9) - (35*b^(49/2)*tan(1/2*d*x + 1/2*c)^9 - 495*b^(49/2)*tan(1/2*d*x +
 1/2*c)^7 + 3528*b^(49/2)*tan(1/2*d*x + 1/2*c)^5 - 18480*b^(49/2)*tan(1/2*d
*x + 1/2*c)^3 + 121590*b^(49/2)*tan(1/2*d*x + 1/2*c))/b^27)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\tan\left(c+d\,x\right)}^4\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^4)^(5/2),x)
```

```
[Out] int(1/(b*tan(c + d*x)^4)^(5/2), x)
```

3.42 $\int (b\tan^p(c+dx))^n\,dx$

Optimal. Leaf size=59

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^n}{d(1+np)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(d
*x+c)^p)^n/d/(n*p+1)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3740, 3557, 371}

$$\frac{\tan(c+dx)\,(b\tan^p(c+dx))^n\;{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(c+dx)\right)}{d(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^p)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^p)^n)/(d*(1 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (b\tan^p(c+dx))^n\,dx = \left(\tan^{-np}(c+dx)\,(b\tan^p(c+dx))^n\right)\int \tan^{np}(c+dx)\,dx$$
$$= \frac{\left(\tan^{-np}(c+dx)\,(b\tan^p(c+dx))^n\right)\text{Subst}\left(\int \frac{x^{np}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1\left(1, \frac{1}{2}(1+np); \frac{1}{2}(3+np); -\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^n}{d(1+np)}$$

Mathematica [A]
time = 0.06, size = 57, normalized size = 0.97

$$\frac{{}_2F_1\left(1, \frac{1}{2}(1+np); \frac{1}{2}(3+np); -\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^n}{d+dnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^n,x]
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^p)^n)/(d + d*n*p)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int (b(\tan^p(dx+c)))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^p)^n,x)
[Out] int((b*tan(d*x+c)^p)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^p)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c)^p)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan^p(c + dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**p)**n,x)
[Out] Integral((b*tan(c + d*x)**p)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^p)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\tan(c + d\,x)^p)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^p)^n,x)
[Out] int((b*tan(c + d*x)^p)^n, x)
```

3.43 $\int (b\tan^2(c+dx))^n\, dx$

Optimal. Leaf size=59

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+2n);\frac{1}{2}(3+2n);-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^2(c+dx)\right)^n}{d(1+2n)}$$

```
[Out] hypergeom([1, 1/2+n],[3/2+n],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(d*x+c)^2)^n/d
/(1+2*n)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3739, 3557, 371}

$$\frac{\tan(c+dx)\left(b\tan^2(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(2n+1);\frac{1}{2}(2n+3);-\tan^2(c+dx)\right)}{d(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^2)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 2*n)/2, (3 + 2*n)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^2)^n)/(d*(1 + 2*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^2(c+dx)\right)^n dx = \left(\tan^{-2n}(c+dx)\left(b\tan^2(c+dx)\right)^n\right)\int \tan^{2n}(c+dx)\,dx$$
$$= \frac{\left(\tan^{-2n}(c+dx)\left(b\tan^2(c+dx)\right)^n\right)\operatorname{Subst}\left(\int \frac{x^{2n}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1\left(1, \frac{1}{2}(1+2n); \frac{1}{2}(3+2n); -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^2(c+dx)\right)^n}{d(1+2n)}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.83

$$\frac{{}_2F_1\left(1, \frac{1}{2}+n; \frac{3}{2}+n; -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^2(c+dx)\right)^n}{d+2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^2)^n,x]
[Out] (Hypergeometric2F1[1, 1/2 + n, 3/2 + n, -Tan[c + d*x]^2]*Tan[c + d*x]*(b*Ta
n[c + d*x]^2)^n)/(d + 2*d*n)
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(b\left(\tan^2(dx+c)\right)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^2)^n,x)
[Out] int((b*tan(d*x+c)^2)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^2)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c)^2)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^2\left(c+dx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**2)**n,x)
[Out] Integral((b*tan(c + d*x)**2)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^2)^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^2)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\tan(c+d\,x)^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^2)^n,x)
[Out] int((b*tan(c + d*x)^2)^n, x)
```

3.44 $\int \left(b \tan^3(c+dx)\right)^n \, dx$

Optimal. Leaf size=57

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+3n);\frac{3(1+n)}{2};-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^3(c+dx)\right)^n}{d(1+3n)}$$

```
[Out] hypergeom([1, 1/2+3/2*n],[3/2+3/2*n],-tan(d*x+c)^2)*tan(d*x+c)*(b*tan(d*x+c
)^3)^n/d/(1+3*n)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3739, 3557, 371}

$$\frac{\tan(c+dx)\left(b\tan^3(c+dx)\right)^n {}_2F_1\left(1,\frac{1}{2}(3n+1);\frac{3(n+1)}{2};-\tan^2(c+dx)\right)}{d(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^3)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, -Tan[c + d*x]^2]*Tan[c +
d*x]*(b*Tan[c + d*x]^3)^n)/(d*(1 + 3*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^3(c+dx)\right)^n\,dx = \left(\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\right)\int \tan^{3n}(c+dx)\,dx$$
$$= \frac{\left(\tan^{-3n}(c+dx)\left(b\tan^3(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{3n}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1\left(1, \frac{1}{2}(1+3n); \frac{3(1+n)}{2}; -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^3(c+dx)\right)^n}{d(1+3n)}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.96

$$\frac{{}_2F_1\left(1, \frac{1}{2}(1+3n); \frac{3(1+n)}{2}; -\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^3(c+dx)\right)^n}{d+3dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^3)^n,x]
[Out] (Hypergeometric2F1[1, (1 + 3*n)/2, (3*(1 + n))/2, -Tan[c + d*x]^2]*Tan[c +
d*x]*(b*Tan[c + d*x]^3)^n)/(d + 3*d*n)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(b\left(\tan^3(dx+c)\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^3)^n,x)
[Out] int((b*tan(d*x+c)^3)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^3)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c)^3)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^{3}\left(c+dx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**3)**n,x)
[Out] Integral((b*tan(c + d*x)**3)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^3)^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^3)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^3\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^3)^n,x)
[Out] int((b*tan(c + d*x)^3)^n, x)
```

3.45 $\int \left(b\tan^4(c+dx)\right)^n\,dx$

Optimal. Leaf size=59

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+4n);\frac{1}{2}(3+4n);-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^4(c+dx)\right)^n}{d(1+4n)}$$

```
[Out] hypergeom([1, 1/2+2*n],[3/2+2*n],-tan(d*x+c)^2)*tan(d*x+c)*(tan(d*x+c)^4*b)
^n/d/(1+4*n)
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3739, 3557, 371}

$$\frac{\tan(c+dx)\left(b\tan^4(c+dx)\right)^n\,{}_2F_1\left(1,\frac{1}{2}(4n+1);\frac{1}{2}(4n+3);-\tan^2(c+dx)\right)}{d(4n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^4)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + 4*n)/2, (3 + 4*n)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(b*Tan[c + d*x]^4)^n)/(d*(1 + 4*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(b\tan^4(c+dx)\right)^n dx = \left(\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n\right)\int \tan^{4n}(c+dx)\,dx$$
$$= \frac{\left(\tan^{-4n}(c+dx)\left(b\tan^4(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{4n}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{2}(1+4n);\frac{1}{2}(3+4n);-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^4(c+dx)\right)^n}{d(1+4n)}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 0.90

$$\frac{{}_2F_1\left(1,\frac{1}{2}+2n;\frac{3}{2}+2n;-\tan^2(c+dx)\right)\tan(c+dx)\left(b\tan^4(c+dx)\right)^n}{d+4dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^4)^n,x]
[Out] (Hypergeometric2F1[1, 1/2 + 2*n, 3/2 + 2*n, -Tan[c + d*x]^2]*Tan[c + d*x]*(
b*Tan[c + d*x]^4)^n)/(d + 4*d*n)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(b\left(\tan^4(dx+c)\right)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^4)^n,x)
[Out] int((b*tan(d*x+c)^4)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^4)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c)^4)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan^4\left(c+dx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)**4*b)**n,x)
[Out] Integral((b*tan(c + d*x)**4)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(d*x+c)^4*b)^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^4)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^4\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^4)^n,x)
[Out] int((b*tan(c + d*x)^4)^n, x)
```

3.46 $\int (b\tan^p(c+dx))^{5/2}\,dx$

Optimal. Leaf size=71

$$\frac{2b^2\,{}_2F_1\left(1,\frac{1}{4}(2+5p);\frac{1}{4}(6+5p);-\tan^2(c+dx)\right)\tan^{1+2p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+5p)}$$

```
[Out] 2*b^2*hypergeom([1, 1/2+5/4*p],[3/2+5/4*p],-tan(d*x+c)^2)*(b*tan(d*x+c)^p)^
(1/2)*tan(d*x+c)^(1+2*p)/d/(2+5*p)
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2b^2\tan^{2p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,{}_2F_1\left(1,\frac{1}{4}(5p+2);\frac{1}{4}(5p+6);-\tan^2(c+dx)\right)}{d(5p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^p)^(5/2),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[1, (2 + 5*p)/4, (6 + 5*p)/4, -Tan[c + d*x]^2]*Tan[
c + d*x]^(1 + 2*p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 5*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (b\tan^p(c+dx))^{5/2}\,dx = \left(b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\int \tan^{\frac{5p}{2}}(c+dx)\,dx$$
$$= \frac{\left(b^2\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\text{Subst}\left(\int \frac{x^{5p/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{2b^2\,{}_2F_1\left(1,\frac{1}{4}(2+5p);\frac{1}{4}(6+5p);-\tan^2(c+dx)\right)\tan^{1+2p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+5p)}$$

Mathematica [**A**]
time = 0.11, size = 62, normalized size = 0.87

$$\frac{2\,{}_2F_1\left(1,\frac{1}{4}(2+5p);\frac{1}{4}(6+5p);-\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^{5/2}}{d(2+5p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^(5/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + 5*p)/4, (6 + 5*p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]*(b*Tan[c + d*x]^p)^(5/2))/(d*(2 + 5*p))
```

Maple [**F**]
time = 0.43, size = 0, normalized size = 0.00

$$\int (b(\tan^p(dx+c)))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^p)^(5/2),x)
```

```
[Out] int((b*tan(d*x+c)^p)^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan^p(c + dx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**p)**(5/2),x)
[Out] Integral((b*tan(c + d*x)**p)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^p)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{tan}\left(c+d\,x\right)}^p\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^p)^(5/2),x)
[Out] int((b*tan(c + d*x)^p)^(5/2), x)
```

3.47 $\int (b\tan^p(c+dx))^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{2b\,{}_2F_1\left(1,\frac{1}{4}(2+3p);\frac{3(2+p)}{4};-\tan^2(c+dx)\right)\tan^{1+p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+3p)}$$

```
[Out] 2*b*hypergeom([1, 1/2+3/4*p],[3/2+3/4*p],-tan(d*x+c)^2)*(b*tan(d*x+c)^p)^(1
/2)*tan(d*x+c)^(1+p)/d/(2+3*p)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2b\tan^{p+1}(c+dx)\sqrt{b\tan^p(c+dx)}\,{}_2F_1\left(1,\frac{1}{4}(3p+2);\frac{3(p+2)}{4};-\tan^2(c+dx)\right)}{d(3p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^p)^(3/2),x]
```

```
[Out] (2*b*Hypergeometric2F1[1, (2 + 3*p)/4, (3*(2 + p))/4, -Tan[c + d*x]^2]*Tan[
c + d*x]^(1 + p)*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + 3*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (b\tan^p(c+dx))^{3/2}\,dx = \left(b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\int \tan^{\frac{3p}{2}}(c+dx)\,dx$$
$$= \frac{\left(b\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\operatorname{Subst}\left(\int \frac{x^{3p/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{2b\,{}_2F_1\left(1, \frac{1}{4}(2+3p); \frac{3(2+p)}{4}; -\tan^2(c+dx)\right)\tan^{1+p}(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+3p)}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.92

$$\frac{2\,{}_2F_1\left(1, \frac{1}{4}(2+3p); \frac{3(2+p)}{4}; -\tan^2(c+dx)\right)\tan(c+dx)\,(b\tan^p(c+dx))^{3/2}}{d(2+3p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^(3/2),x]
[Out] (2*Hypergeometric2F1[1, (2 + 3*p)/4, (3*(2 + p))/4, -Tan[c + d*x]^2]*Tan[c
+ d*x]*(b*Tan[c + d*x]^p)^(3/2))/(d*(2 + 3*p))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int (b(\tan^p(dx+c)))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^p)^(3/2),x)
[Out] int((b*tan(d*x+c)^p)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^p)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan^p(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**p)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**p)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\tan(c+d\,x)^p)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^p)^(3/2),x)
```

```
[Out] int((b*tan(c + d*x)^p)^(3/2), x)
```

3.48 $\int \sqrt{b\tan^p(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{2\,{}_2F_1\left(1,\frac{2+p}{4};\frac{6+p}{4};-\tan^2(c+dx)\right)\tan(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+p)}$$

```
[Out] 2*hypergeom([1, 1/2+1/4*p],[3/2+1/4*p],-tan(d*x+c)^2)*(b*tan(d*x+c)^p)^(1/2
)*tan(d*x+c)/d/(2+p)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2\tan(c+dx)\sqrt{b\tan^p(c+dx)}\,{}_2F_1\left(1,\frac{p+2}{4};\frac{p+6}{4};-\tan^2(c+dx)\right)}{d(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[c + d*x]^p],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 + p)/4, (6 + p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]
*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{b\tan^p(c+dx)}\,dx = \left(\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\int \tan^{\frac{p}{2}}(c+dx)\,dx$$
$$= \frac{\left(\tan^{-\frac{p}{2}}(c+dx)\sqrt{b\tan^p(c+dx)}\right)\mathrm{Subst}\left(\int \frac{x^{p/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{2\,{}_2F_1\left(1, \frac{2+p}{4}; \frac{6+p}{4}; -\tan^2(c+dx)\right)\tan(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+p)}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 1.00

$$\frac{2\,{}_2F_1\left(1, \frac{2+p}{4}; \frac{6+p}{4}; -\tan^2(c+dx)\right)\tan(c+dx)\sqrt{b\tan^p(c+dx)}}{d(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[c + d*x]^p],x]
[Out] (2*Hypergeometric2F1[1, (2 + p)/4, (6 + p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]
*Sqrt[b*Tan[c + d*x]^p])/(d*(2 + p))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \sqrt{b\left(\tan^p\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^p)^(1/2),x)
[Out] int((b*tan(d*x+c)^p)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(d*x + c)^p), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan^{p}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**p)**(1/2),x)
[Out] Integral(sqrt(b*tan(c + d*x)**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(d*x + c)^p), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,{\tan\left(c+d\,x\right)}^p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^p)^(1/2),x)
[Out] int((b*tan(c + d*x)^p)^(1/2), x)
```

3.49 $\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\,{}_2F_1\left(1,\frac{2-p}{4};\frac{6-p}{4};-\tan^2(c+dx)\right)\tan(c+dx)}{d(2-p)\sqrt{b\tan^p(c+dx)}}$$

```
[Out] 2*hypergeom([1, 1/2-1/4*p],[3/2-1/4*p],-tan(d*x+c)^2)*tan(d*x+c)/d/(2-p)/(b
*tan(d*x+c)^p)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2\tan(c+dx)\,{}_2F_1\left(1,\frac{2-p}{4};\frac{6-p}{4};-\tan^2(c+dx)\right)}{d(2-p)\sqrt{b\tan^p(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Tan[c + d*x]^p],x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - p)/4, (6 - p)/4, -Tan[c + d*x]^2]*Tan[c + d*x]
)/(d*(2 - p)*Sqrt[b*Tan[c + d*x]^p])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\tan^p(c+dx)}}\,dx = \frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{p}{2}}(c+dx)\,dx}{\sqrt{b\tan^p(c+dx)}}$$
$$= \frac{\tan^{\frac{p}{2}}(c+dx)\text{Subst}\left(\int \frac{x^{-p/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d\sqrt{b\tan^p(c+dx)}}$$
$$= \frac{2\,{}_2F_1\left(1, \frac{2-p}{4}; \frac{6-p}{4}; -\tan^2(c+dx)\right)\tan(c+dx)}{d(2-p)\sqrt{b\tan^p(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 60, normalized size = 0.97

$$-\frac{2\,{}_2F_1\left(1, \frac{2-p}{4}; \frac{6-p}{4}; -\tan^2(c+dx)\right)\tan(c+dx)}{d(-2+p)\sqrt{b\tan^p(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Tan[c + d*x]^p],x]
[Out] (-2*Hypergeometric2F1[1, (2 - p)/4, (6 - p)/4, -Tan[c + d*x]^2]*Tan[c + d*x
])/(d*(-2 + p)*Sqrt[b*Tan[c + d*x]^p])
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\left(\tan^p(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^p)^(1/2),x)
[Out] int(1/(b*tan(d*x+c)^p)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*tan(d*x + c)^p), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan^{p}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**p)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tan(c + d*x)**p), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*tan(d*x + c)^p), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\tan\left(c+d\,x\right)^p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^p)^(1/2),x)
```

```
[Out] int(1/(b*tan(c + d*x)^p)^(1/2), x)
```

3.50 $\int \frac{1}{(b \tan^p(c+dx))^{3/2}} \, dx$

Optimal. Leaf size=71

$$\frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-3p);\frac{3(2-p)}{4};-\tan^2(c+dx)\right)\tan^{1-p}(c+dx)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

[Out] 2*hypergeom([1, 1/2-3/4*p],[3/2-3/4*p],-tan(d*x+c)^2)*tan(d*x+c)^(1-p)/b/d/(2-3*p)/(b*tan(d*x+c)^p)^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2\tan^{1-p}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2-3p);\frac{3(2-p)}{4};-\tan^2(c+dx)\right)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Tan[c + d*x]^p)^(-3/2),x]

[Out] (2*Hypergeometric2F1[1, (2 - 3*p)/4, (3*(2 - p))/4, -Tan[c + d*x]^2]*Tan[c + d*x]^(1 - p))/(b*d*(2 - 3*p)*Sqrt[b*Tan[c + d*x]^p])

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
```

```
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(b\tan^p(c+dx))^{3/2}}\,dx = \frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{3p}{2}}(c+dx)\,dx}{b\sqrt{b\tan^p(c+dx)}}$$
$$= \frac{\tan^{\frac{p}{2}}(c+dx)\text{Subst}\left(\int \frac{x^{-3p/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{bd\sqrt{b\tan^p(c+dx)}}$$
$$= \frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-3p);\frac{3(2-p)}{4};-\tan^2(c+dx)\right)\tan^{1-p}(c+dx)}{bd(2-3p)\sqrt{b\tan^p(c+dx)}}$$

Mathematica [A]
time = 0.08, size = 60, normalized size = 0.85

$$-\frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-3p);-\frac{3}{4}(-2+p);-\tan^2(c+dx)\right)\tan(c+dx)}{d(-2+3p)\left(b\tan^p(c+dx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^(-3/2),x]
[Out] (-2*Hypergeometric2F1[1, (2 - 3*p)/4, (-3*(-2 + p))/4, -Tan[c + d*x]^2]*Tan
[c + d*x])/(d*(-2 + 3*p)*(b*Tan[c + d*x]^p)^(3/2))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\left(\tan^p(dx+c)\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^p)^(3/2),x)
[Out] int(1/(b*tan(d*x+c)^p)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(-3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\tan^p(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**p)**(3/2),x)
```

```
[Out] Integral((b*tan(c + d*x)**p)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\tan\left(c+d\,x\right)}^p\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^p)^(3/2),x)
```

```
[Out] int(1/(b*tan(c + d*x)^p)^(3/2), x)
```

3.51 $\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=71

$$\frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-5p);\frac{1}{4}(6-5p);-\tan^2(c+dx)\right)\tan^{1-2p}(c+dx)}{b^2d(2-5p)\sqrt{b\tan^p(c+dx)}}$$

```
[Out] 2*hypergeom([1, 1/2-5/4*p],[3/2-5/4*p],-tan(d*x+c)^2)*tan(d*x+c)^(1-2*p)/b^
2/d/(2-5*p)/(b*tan(d*x+c)^p)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{2\tan^{1-2p}(c+dx)\,{}_2F_1\left(1,\frac{1}{4}(2-5p);\frac{1}{4}(6-5p);-\tan^2(c+dx)\right)}{b^2d(2-5p)\sqrt{b\tan^p(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^p)^(-5/2),x]
```

```
[Out] (2*Hypergeometric2F1[1, (2 - 5*p)/4, (6 - 5*p)/4, -Tan[c + d*x]^2]*Tan[c +
d*x]^(1 - 2*p))/(b^2*d*(2 - 5*p)*Sqrt[b*Tan[c + d*x]^p])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(b\tan^p(c+dx))^{5/2}}\,dx = \frac{\tan^{\frac{p}{2}}(c+dx)\int \tan^{-\frac{5p}{2}}(c+dx)\,dx}{b^2\sqrt{b\tan^p(c+dx)}}$$
$$= \frac{\tan^{\frac{p}{2}}(c+dx)\text{Subst}\left(\int \frac{x^{-5p/2}}{1+x^2}\,dx, x, \tan(c+dx)\right)}{b^2 d\sqrt{b\tan^p(c+dx)}}$$
$$= \frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-5p);\frac{1}{4}(6-5p);-\tan^2(c+dx)\right)\tan^{1-2p}(c+dx)}{b^2 d(2-5p)\sqrt{b\tan^p(c+dx)}}$$

Mathematica [A]
time = 0.08, size = 62, normalized size = 0.87

$$-\frac{2\,{}_2F_1\left(1,\frac{1}{4}(2-5p);\frac{1}{4}(6-5p);-\tan^2(c+dx)\right)\tan(c+dx)}{d(-2+5p)\,(b\tan^p(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^(-5/2),x]
[Out] (-2*Hypergeometric2F1[1, (2 - 5*p)/4, (6 - 5*p)/4, -Tan[c + d*x]^2]*Tan[c +
 d*x])/(d*(-2 + 5*p)*(b*Tan[c + d*x]^p)^(5/2))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{(b(\tan^p(dx+c)))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(d*x+c)^p)^(5/2),x)
[Out] int(1/(b*tan(d*x+c)^p)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^p)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\tan^{p}(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)**p)**(5/2),x)
[Out] Integral((b*tan(c + d*x)**p)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*tan(d*x+c)^p)^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(d*x + c)^p)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\tan\left(c+d\,x\right)}^p\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*tan(c + d*x)^p)^(5/2),x)
[Out] int(1/(b*tan(c + d*x)^p)^(5/2), x)
```

3.52 $\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx$

Optimal. Leaf size=32

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

```
[Out] -cot(d*x+c)*ln(cos(d*x+c))*(b*tan(d*x+c)^p)^(1/p)/d
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3740, 3556}

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[c + d*x]^p)^p^(-1),x]
```

```
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*(b*Tan[c + d*x]^p)^p^(-1))/d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (b\tan^p(c+dx))^{\frac{1}{p}}\,dx &= \left(\cot(c+dx)\,(b\tan^p(c+dx))^{\frac{1}{p}}\right)\int \tan(c+dx)\,dx\\ &= -\frac{\cot(c+dx)\log(\cos(c+dx))\,(b\tan^p(c+dx))^{\frac{1}{p}}}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.00

$$-\frac{\cot(c+dx)\log(\cos(c+dx))\left(b\tan^p(c+dx)\right)^{\frac{1}{p}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[c + d*x]^p)^p^(-1),x]
[Out] -((Cot[c + d*x]*Log[Cos[c + d*x]]*(b*Tan[c + d*x]^p)^p^(-1))/d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.16, size = 12884, normalized size = 402.62

method	result	size
risch	Expression too large to display	12884

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(d*x+c)^p)^(1/p),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c)^p)^(1/p), x)
```

Fricas [A]
time = 0.38, size = 23, normalized size = 0.72

$$-\frac{b^{\left(\frac{1}{p}\right)}\log\left(\frac{1}{\tan(dx+c)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="fricas")
[Out] -1/2*b^(1/p)*log(1/(tan(d*x + c)^2 + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\tan^p\left(c+dx\right)\right)^{\frac{1}{p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)**p)**(1/p),x)
```

```
[Out] Integral((b*tan(c + d*x)**p)**(1/p), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(d*x+c)^p)^(1/p),x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c)^p)^(1/p), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \left(b\tan(c+d\,x)^p\right)^{1/p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(c + d*x)^p)^(1/p),x)
```

```
[Out] int((b*tan(c + d*x)^p)^(1/p), x)
```

3.53 $\int (a(b\tan(c+dx))^p)^n \, dx$

Optimal. Leaf size=61

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(c+dx)\right)\tan(c+dx)\left(a(b\tan(c+dx))^p\right)^n}{d(1+np)}$$

```
[Out] hypergeom([1, 1/2*n*p+1/2],[1/2*n*p+3/2],-tan(d*x+c)^2)*tan(d*x+c)*(a*(b*ta
n(d*x+c))^p)^n/d/(n*p+1)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\frac{\tan(c+dx)\,{}_2F_1\left(1,\frac{1}{2}(np+1);\frac{1}{2}(np+3);-\tan^2(c+dx)\right)\left(a(b\tan(c+dx))^p\right)^n}{d(np+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*Tan[c + d*x])^p)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(a*(b*Tan[c + d*x])^p)^n)/(d*(1 + n*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int (a(b\tan(c+dx))^p)^n\,dx &= \left((b\tan(c+dx))^{-np}\left(a(b\tan(c+dx))^p\right)^n\right)\int (b\tan(c+dx))^{np}\,dx\\
&= \frac{\left(b(b\tan(c+dx))^{-np}\left(a(b\tan(c+dx))^p\right)^n\right)\text{Subst}\left(\int \frac{x^{np}}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{d}\\
&= \frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(c+dx)\right)\tan(c+dx)\left(a(b\tan(c+dx))^p\right)^n}{d(1+np)}
\end{aligned}
$$

Mathematica [**A**]
time = 0.06, size = 59, normalized size = 0.97

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+np);\frac{1}{2}(3+np);-\tan^2(c+dx)\right)\tan(c+dx)\left(a(b\tan(c+dx))^p\right)^n}{d+dnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*Tan[c + d*x])^p)^n,x]
[Out] (Hypergeometric2F1[1, (1 + n*p)/2, (3 + n*p)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]*(a*(b*Tan[c + d*x])^p)^n)/(d + d*n*p)
```

Maple [**F**]
time = 0.22, size = 0, normalized size = 0.00

$$\int (a(b\tan(dx+c))^p)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*tan(d*x+c))^p)^n,x)
[Out] int((a*(b*tan(d*x+c))^p)^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="maxima")
[Out] integrate(((b*tan(d*x + c))^p*a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="fricas")
[Out] integral(((b*tan(d*x + c))^p*a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a(b\tan\left(c+dx\right))^p\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*tan(d*x+c))**p)**n,x)
[Out] Integral((a*(b*tan(c + d*x))**p)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*tan(d*x+c))^p)^n,x, algorithm="giac")
[Out] integrate(((b*tan(d*x + c))^p*a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a\,{\left(b\,\tan(c+d\,x)\right)}^p\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*tan(c + d*x))^p)^n,x)
[Out] int((a*(b*tan(c + d*x))^p)^n, x)
```

3.54 $\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=257

$$-\frac{21\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b}+\frac{21\sqrt{d}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b}+\frac{21\sqrt{d}\,\log\left(\sqrt{d}\right.}{}$$

```
[Out] -21/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+21/
64*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+21/128*
ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d^(1/2)/b*2^(1/
2)-21/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d^(1/
2)/b*2^(1/2)-7/16*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d-1/4*cos(b*x+a)^4*(d
*tan(b*x+a))^(7/2)/b/d^3
```

Rubi [A]

time = 0.13, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2671, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{21\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b}+\frac{21\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}\,b}-\frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3}+\frac{21\sqrt{d}\,\log\left(\sqrt{d}\,\tan(a+bx)-\sqrt{2}\,\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,b}-\frac{21\sqrt{d}\,\log\left(\sqrt{d}\,\tan(a+bx)+\sqrt{2}\,\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,b}-\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-21*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2
]*b) + (21*Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*
Sqrt[2]*b) + (21*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[
d*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (21*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a
+ b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (7*Cos[a + b*x]^2*
(d*Tan[a + b*x])^(3/2))/(16*b*d) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(7/2))/
(4*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{d\text{Subst}\left(\int \frac{x^{9/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} + \frac{(7d)\text{Subst}\left(\int \frac{x^{5/2}}{(d^2+x^2)^2}\,dx, x, d\tan(\right.}{8b}$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} +$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} +$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} -$$

$$= -\frac{7\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{7/2}}{4bd^3} +$$

$$= \frac{21\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{21\sqrt{}}{}$$

$$= -\frac{21\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} + \frac{21\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{}}{}\right.}{32\sqrt{}}$$

Mathematica [A]
time = 0.25, size = 122, normalized size = 0.47

$$-\frac{\left(21\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\csc(a+bx)\sqrt{\sin(2(a+bx))}+21\csc(a+bx)\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}+18\sin(2(a+bx))-2\sin(4(a+bx))\right)\sqrt{d\tan(a+bx)}}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/64*((21*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x]*Sqrt[Sin[2*(a +
 b*x)]] + 21*Csc[a + b*x]*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a +
 b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + 18*Sin[2*(a + b*x)] - 2*Sin[4*(a + b*x)])
*Sqrt[d*Tan[a + b*x]])/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 3.18, size = 542, normalized size = 2.11

method	result
default	$\frac{(-1+\cos(bx+a))\left(21i\,\text{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/b*(-1+cos(b*x+a))*(21*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)-21*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/
2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+8*cos(b*
x+a)^4*2^(1/2)+21*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+21*
EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/
2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-8*cos(b*x+a)^3*2^(1/2
)-22*cos(b*x+a)^2*2^(1/2)+22*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*
x+a)/cos(b*x+a))^(1/2)/sin(b*x+a)^3*2^(1/2)
```

Maxima [A]
time = 0.51, size = 225, normalized size = 0.88

$$\frac{21\,d^6\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{8\left(11\,(d\tan(bx+a))^{\frac{7}{2}}d^6+7\,(d\tan(bx+a))^{\frac{3}{2}}d^8\right)}{d^4\tan(bx+a)^4+2\,d^4\tan(bx+a)^2+d^4}}{128\,bd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/128*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a)
+ sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 8*(11*(d*tan(b
*x + a))^(7/2)*d^6 + 7*(d*tan(b*x + a))^(3/2)*d^8)/(d^4*tan(b*x + a)^4 + 2*
d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1916 vs. $2(197) = 394$.

time = 67.37, size = 1916, normalized size = 7.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/512*(84*sqrt(2)*b*(d^2/b^4)^(1/4)*arctan((sqrt(4*b^2*d^3*sqrt(d^2/b^4)*co
s(b*x + a)*sin(b*x + a) + d^4 - 2*(sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x
+ a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*si
n(b*x + a)/cos(b*x + a)))*(2*d^2*cos(b*x + a)*sin(b*x + a) + b^2*d*sqrt(d^2
/b^4) + (sqrt(2)*b^3*(d^2/b^4)^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*d*(d^2/b^4)
^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + (sqr
t(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2
/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(2*d^4*cos(b
*x + a)^2 - d^4)) + 84*sqrt(2)*b*(d^2/b^4)^(1/4)*arctan(-(sqrt(4*b^2*d^3*sq
rt(d^2/b^4)*cos(b*x + a)*sin(b*x + a) + d^4 + 2*(sqrt(2)*b^3*d^2*(d^2/b^4)^
(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a
)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(2*d^2*cos(b*x + a)*sin(b*x + a) +
b^2*d*sqrt(d^2/b^4) - (sqrt(2)*b^3*(d^2/b^4)^(3/4)*cos(b*x + a)^2 + sqrt(2)
*b*d*(d^2/b^4)^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x
 + a))) - (sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt
(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))
)/(2*d^4*cos(b*x + a)^2 - d^4)) + 84*sqrt(2)*b*(d^2/b^4)^(1/4)*arctan(-1/2*
(2*d^4*sin(b*x + a) - sqrt(4*b^2*d^3*sqrt(d^2/b^4)*cos(b*x + a)*sin(b*x + a
) + d^4 + 2*(sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) + sq
rt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a
)))*(sqrt(2)*b^3*(d^2/b^4)^(3/4)*cos(b*x + a) + sqrt(2)*b*d*(d^2/b^4)^(1/4)
*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + (sqrt(2)*b^3*d^2*(d^2/b^
4)^(3/4)*cos(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*sin(b*x + a))*sqrt(d*
sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^3*cos(b*x + a)^3 - b^2*d^3*cos(b*x +
a))*sqrt(d^2/b^4))/((2*d^4*cos(b*x + a)^2 - d^4)*sin(b*x + a))) + 84*sqrt(2
)*b*(d^2/b^4)^(1/4)*arctan(1/2*(2*d^4*sin(b*x + a) + sqrt(4*b^2*d^3*sqrt(d^
2/b^4)*cos(b*x + a)*sin(b*x + a) + d^4 - 2*(sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)
*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^2)*
sqrt(d*sin(b*x + a)/cos(b*x + a)))*(sqrt(2)*b^3*(d^2/b^4)^(3/4)*cos(b*x + a
) + sqrt(2)*b*d*(d^2/b^4)^(1/4)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x +
 a)) - (sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a) + sqrt(2)*b*d^3*(d^2/b
^4)^(1/4)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^3*cos(
b*x + a)^3 - b^2*d^3*cos(b*x + a))*sqrt(d^2/b^4))/((2*d^4*cos(b*x + a)^2 -
d^4)*sin(b*x + a))) - 21*sqrt(2)*b*(d^2/b^4)^(1/4)*log(343064484*b^2*d^3*sq
rt(d^2/b^4)*cos(b*x + a)*sin(b*x + a) + 85766121*d^4 + 171532242*(sqrt(2)*b
^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^
(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 21*sqrt(2)*b*(d^
2/b^4)^(1/4)*log(343064484*b^2*d^3*sqrt(d^2/b^4)*cos(b*x + a)*sin(b*x + a)
```

```
+ 85766121*d^4 - 171532242*(sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a)*si
n(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x +
 a)/cos(b*x + a))) - 21*sqrt(2)*b*(d^2/b^4)^(1/4)*log(85766121/4*b^2*d^3*sq
rt(d^2/b^4)*cos(b*x + a)*sin(b*x + a) + 85766121/16*d^4 + 85766121/8*(sqrt(
2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b
^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 21*sqrt(2)*b
*(d^2/b^4)^(1/4)*log(85766121/4*b^2*d^3*sqrt(d^2/b^4)*cos(b*x + a)*sin(b*x
+ a) + 85766121/16*d^4 - 85766121/8*(sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*
x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*
sin(b*x + a)/cos(b*x + a))) + 32*(4*cos(b*x + a)^3 - 11*cos(b*x + a))*sqrt(
d*sin(b*x + a)/cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\tan(a+bx)}\,\sin^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*sin(a + b*x)**4, x)
```

Giac [A]
time = 0.71, size = 245, normalized size = 0.95

$$\frac{\frac{42\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{42\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}-\frac{21\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{b}+\frac{21\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{b}-\frac{8\left(11\sqrt{d\tan(bx+a)}\,d^5\tan(bx+a)^3+7\sqrt{d\tan(bx+a)}\,d^5\tan(bx+a)\right)}{\left(d^2\tan(bx+a)^2+d^2\right)^2 b}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*(42*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 42*sqrt(2)*abs(d)^(3/2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b -
 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*
sqrt(abs(d)) + abs(d))/b + 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) - sqr
t(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*(11*sqrt(d*tan(b*x +
 a))*d^5*tan(b*x + a)^3 + 7*sqrt(d*tan(b*x + a))*d^5*tan(b*x + a))/((d^2*ta
n(b*x + a)^2 + d^2)^2*b))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\sin\left(a+b\,x\right)}^4\,\sqrt{d\,\tan\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2), x)
```

3.55 $\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=227

$$-\frac{3\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}+\frac{3\sqrt{d}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}+\frac{3\sqrt{d}\,\log\left(\sqrt{d}+\right.}{}$$

```
[Out] -3/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+3/8*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b*2^(1/2)+3/16*ln(d^(
1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d^(1/2)/b*2^(1/2)-3/1
6*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))*d^(1/2)/b*2^(
1/2)-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d
```

Rubi [A]
time = 0.12, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2671, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}+\frac{3\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}\,b}+\frac{3\sqrt{d}\,\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,b}-\frac{3\sqrt{d}\,\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,b}-\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-3*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*
b) + (3*Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt
[2]*b) + (3*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan
[a + b*x]]])/(8*Sqrt[2]*b) - (3*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x]
+ Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) - (Cos[a + b*x]^2*(d*Tan[a +
 b*x])^(3/2))/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{d\text{Subst}\left(\int \frac{x^{5/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} + \frac{(3d)\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+\right.}{4b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} + \frac{(3d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(}\right.}{2b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} - \frac{(3d)\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(}\right.}{4b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd} + \frac{\left(3\sqrt{d}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\right.}{8\sqrt{2}\,b}$$

$$= \frac{3\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}\,b} - \frac{3\sqrt{d}\,\text{l}}{}$$

$$= -\frac{3\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b} + \frac{3\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{4\sqrt{2}\,b}$$

Mathematica [A]
time = 0.18, size = 104, normalized size = 0.46

$$-\frac{\left(3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\csc(a+bx)+3\csc(a+bx)\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)+2\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}\sqrt{d\tan(a+bx)}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/8*((3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x] + 3*Csc[a + b*x]*
Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]] + 2*Sqrt[Sin[2*(a
 + b*x)]])*Sqrt[Sin[2*(a + b*x)]]*Sqrt[d*Tan[a + b*x]])/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.32, size = 516, normalized size = 2.27

method	result

default	$-\frac{(-1+\cos(bx+a))\left(3i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b*(-1+cos(b*x+a))*(3*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)-3*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-3*Elliptic
Pi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1
+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-3*EllipticPi(((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*
x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin
(b*x+a))/sin(b*x+a))^(1/2)+2*cos(b*x+a)^2*2^(1/2)-2*cos(b*x+a)*2^(1/2))*(co
s(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(1/2)/sin(b*x+a)^3*2^(1/2)
```

Maxima [A]
time = 0.56, size = 194, normalized size = 0.85

$$\frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{8\,(d\tan(bx+a))^{\frac{3}{2}}d^4}{d^2\tan(bx+a)^2+d^2}}{16\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b
*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) +
sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x +
 a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(b*x + a
))^(3/2)*d^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1903 vs. $2(171) = 342$.
time = 66.04, size = 1903, normalized size = 8.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")

[Out] 1/64*(12*sqrt(2)*b*(d^2/b^4)^(1/4)*arctan((sqrt(4*b^2*d^3*sqrt(d^2/b^4)*cos
(b*x + a)*sin(b*x + a) + d^4 - 2*(sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x +
 a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin
(b*x + a)/cos(b*x + a)))*(2*d^2*cos(b*x + a)*sin(b*x + a) + b^2*d*sqrt(d^2/
b^4) + (sqrt(2)*b^3*(d^2/b^4)^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*d*(d^2/b^4)^
(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + (sqrt
(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/
b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(2*d^4*cos(b*
x + a)^2 - d^4)) + 12*sqrt(2)*b*(d^2/b^4)^(1/4)*arctan(-(sqrt(4*b^2*d^3*sqr
t(d^2/b^4)*cos(b*x + a)*sin(b*x + a) + d^4 + 2*(sqrt(2)*b^3*d^2*(d^2/b^4)^(
3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)
^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(2*d^2*cos(b*x + a)*sin(b*x + a) + b
^2*d*sqrt(d^2/b^4) - (sqrt(2)*b^3*(d^2/b^4)^(3/4)*cos(b*x + a)^2 + sqrt(2)*
b*d*(d^2/b^4)^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x
+ a))) - (sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(
2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))
/(2*d^4*cos(b*x + a)^2 - d^4)) + 12*sqrt(2)*b*(d^2/b^4)^(1/4)*arctan(-1/2*(
2*d^4*sin(b*x + a) - sqrt(4*b^2*d^3*sqrt(d^2/b^4)*cos(b*x + a)*sin(b*x + a)
 + d^4 + 2*(sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) + sqr
t(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)
))*(sqrt(2)*b^3*(d^2/b^4)^(3/4)*cos(b*x + a) + sqrt(2)*b*d*(d^2/b^4)^(1/4)*
sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + (sqrt(2)*b^3*d^2*(d^2/b^4
)^(3/4)*cos(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*sin(b*x + a))*sqrt(d*s
in(b*x + a)/cos(b*x + a)) - 4*(b^2*d^3*cos(b*x + a)^3 - b^2*d^3*cos(b*x + a
))*sqrt(d^2/b^4))/((2*d^4*cos(b*x + a)^2 - d^4)*sin(b*x + a))) + 12*sqrt(2)
*b*(d^2/b^4)^(1/4)*arctan(1/2*(2*d^4*sin(b*x + a) + sqrt(4*b^2*d^3*sqrt(d^2
/b^4)*cos(b*x + a)*sin(b*x + a) + d^4 - 2*(sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*
cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^2)*s
qrt(d*sin(b*x + a)/cos(b*x + a)))*(sqrt(2)*b^3*(d^2/b^4)^(3/4)*cos(b*x + a)
 + sqrt(2)*b*d*(d^2/b^4)^(1/4)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x +
a)) - (sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a) + sqrt(2)*b*d^3*(d^2/b^
4)^(1/4)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^3*cos(b
*x + a)^3 - b^2*d^3*cos(b*x + a))*sqrt(d^2/b^4))/((2*d^4*cos(b*x + a)^2 - d
^4)*sin(b*x + a))) - 3*sqrt(2)*b*(d^2/b^4)^(1/4)*log(2916*b^2*d^3*sqrt(d^2/
b^4)*cos(b*x + a)*sin(b*x + a) + 729*d^4 + 1458*(sqrt(2)*b^3*d^2*(d^2/b^4)^
(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a
)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 3*sqrt(2)*b*(d^2/b^4)^(1/4)*log(2
916*b^2*d^3*sqrt(d^2/b^4)*cos(b*x + a)*sin(b*x + a) + 729*d^4 - 1458*(sqrt(
2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b
^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 3*sqrt(2)*b*
(d^2/b^4)^(1/4)*log(729/4*b^2*d^3*sqrt(d^2/b^4)*cos(b*x + a)*sin(b*x + a) +
 729/16*d^4 + 729/8*(sqrt(2)*b^3*d^2*(d^2/b^4)^(3/4)*cos(b*x + a)*sin(b*x +
 a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos
```

```
(b*x + a))) + 3*sqrt(2)*b*(d^2/b^4)^(1/4)*log(729/4*b^2*d^3*sqrt(d^2/b^4)*c
os(b*x + a)*sin(b*x + a) + 729/16*d^4 - 729/8*(sqrt(2)*b^3*d^2*(d^2/b^4)^(3
/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/b^4)^(1/4)*cos(b*x + a)^
2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 32*sqrt(d*sin(b*x + a)/cos(b*x + a)
)*cos(b*x + a)*sin(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\tan(a+bx)}\,\sin^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*sin(a + b*x)**2, x)
```

Giac [A]
time = 0.58, size = 219, normalized size = 0.96

$$-\frac{\frac{8\sqrt{d\tan(bx+a)}\,d^3\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b}-\frac{6\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}-\frac{6\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{3\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{b}-\frac{3\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{b}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*(8*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)*
b) - 6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sq
rt(d*tan(b*x + a)))/sqrt(abs(d)))/b - 6*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*sq
rt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 3*s
qrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(
abs(d)) + abs(d))/b - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) - sqrt(2)*s
qrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(a+b\,x)^2\,\sqrt{d\tan(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2), x)
```

3.56 $\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=18

$$-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[Out] -2*d/b/(d*tan(b*x+a))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 30}

$$-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sqrt{d\tan(a+bx)}\,dx &= \frac{d\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= -\frac{2d}{b\sqrt{d\tan(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 18, normalized size = 1.00

$$-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.
time = 0.36, size = 38, normalized size = 2.11

method	result	size
default	$-\frac{2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\,\cos(bx+a)}{b\sin(bx+a)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*(d*sin(b*x+a)/cos(b*x+a))^(1/2)*cos(b*x+a)/sin(b*x+a)
```

Maxima [A]
time = 0.34, size = 23, normalized size = 1.28

$$-\frac{2\sqrt{d\tan\left(bx+a\right)}}{b\tan\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(d*tan(b*x + a))/(b*tan(b*x + a))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.
time = 0.37, size = 37, normalized size = 2.06

$$-\frac{2\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}\cos\left(bx+a\right)}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)/(b*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\tan\left(a+bx\right)}\csc^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**2, x)
```

Giac [A]
time = 0.71, size = 16, normalized size = 0.89

$$-\frac{2\,d}{\sqrt{d\tan\left(bx+a\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2*d/(sqrt(d*tan(b*x + a))*b)
```

Mupad [B]
time = 3.05, size = 48, normalized size = 2.67

$$-\frac{\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{d\,\sin\left(2\,a+2\,b\,x\right)}{\cos\left(2\,a+2\,b\,x\right)+1}}}{b\,{\sin\left(a+b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^2,x)
```

```
[Out] -(sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2))/(b*
sin(a + b*x)^2)
```

3.57 $\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=41

$$-\frac{2d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[Out] -2*d/b/(d*tan(b*x+a))^(1/2)-2/5*d^3/b/(d*tan(b*x+a))^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 14}

$$-\frac{2d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d^3)/(5*b*(d*Tan[a + b*x])^(5/2)) - (2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(a+bx)\sqrt{d\tan(a+bx)}\,dx &= \frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{7/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\ &= \frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{7/2}}+\frac{1}{x^{3/2}}\right)\,dx,x,d\tan(a+bx)\right)}{b}\\ &= -\frac{2d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 30, normalized size = 0.73

$$-\frac{2d(4+\csc^2(a+bx))}{5b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sqrt[d*Tan[a + b*x]],x]
[Out] (-2*d*(4 + Csc[a + b*x]^2))/(5*b*Sqrt[d*Tan[a + b*x]])
```

Maple [A]
time = 0.39, size = 50, normalized size = 1.22

method	result	size
default	$\frac{2(4(\cos^2(bx+a))-5)\cos(bx+a)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{5b\sin(bx+a)^3}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5/b*(4*cos(b*x+a)^2-5)*cos(b*x+a)*(d*sin(b*x+a)/cos(b*x+a))^(1/2)/sin(b*x
+a)^3
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.80

$$-\frac{2\left(5\,d^2\tan\left(bx+a\right)^2+d^2\right)d}{5\,\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2/5*(5*d^2*tan(b*x + a)^2 + d^2)*d/((d*tan(b*x + a))^(5/2)*b)
```

Fricas [A]
time = 0.36, size = 63, normalized size = 1.54

$$-\frac{2\left(4\,\cos\left(bx+a\right)^3-5\,\cos\left(bx+a\right)\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{5\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(4*cos(b*x + a)^3 - 5*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))/
((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\tan{(a+bx)}}\,\csc^4{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**4, x)
```

Giac [A]
time = 0.70, size = 43, normalized size = 1.05

$$-\frac{2\left(5\,d^4\tan\left(bx+a\right)^2+d^4\right)}{5\,\sqrt{d\tan\left(bx+a\right)}\,bd^3\tan\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*(5*d^4*tan(b*x + a)^2 + d^4)/(sqrt(d*tan(b*x + a))*b*d^3*tan(b*x + a)^
2)
```

Mupad [B]
time = 6.53, size = 102, normalized size = 2.49

$$\frac{8\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,2\mathrm{i}-\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{5\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^4,x)
```

```
[Out] (8*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*(exp(
a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*2i - exp(a*6i + b*x*6i)*1i - 1i))/(5
*b*(exp(a*2i + b*x*2i) - 1)^3)
```

3.58 $\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=63

$$-\frac{2d^5}{9b(d\tan(a+bx))^{9/2}}-\frac{4d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

```
[Out] -2*d/b/(d*tan(b*x+a))^(1/2)-2/9*d^5/b/(d*tan(b*x+a))^(9/2)-4/5*d^3/b/(d*tan
(b*x+a))^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 276}

$$-\frac{2d^5}{9b(d\tan(a+bx))^{9/2}}-\frac{4d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d^5)/(9*b*(d*Tan[a + b*x])^(9/2)) - (4*d^3)/(5*b*(d*Tan[a + b*x])^(5/2)
) - (2*d)/(b*Sqrt[d*Tan[a + b*x]])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^6(a+bx)\sqrt{d\tan(a+bx)}\,dx &= \frac{d\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{11/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\ &= \frac{d\text{Subst}\left(\int \left(\frac{d^4}{x^{11/2}}+\frac{2d^2}{x^{7/2}}+\frac{1}{x^{3/2}}\right)dx,x,d\tan(a+bx)\right)}{b}\\ &= -\frac{2d^5}{9b(d\tan(a+bx))^{9/2}}-\frac{4d^3}{5b(d\tan(a+bx))^{5/2}}-\frac{2d}{b\sqrt{d\tan(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 50, normalized size = 0.79

$$\frac{2d(-21+20\cos(2(a+bx))-4\cos(4(a+bx)))\csc^4(a+bx)}{45b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6*Sqrt[d*Tan[a + b*x]],x]
[Out] (2*d*(-21 + 20*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4)/(45*b
*Sqrt[d*Tan[a + b*x]])
```

Maple [A]
time = 0.40, size = 60, normalized size = 0.95

method	result	size
default	$-\frac{2(32(\cos^4(bx+a))-72(\cos^2(bx+a))+45)\cos(bx+a)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{45b\sin(bx+a)^5}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/45/b*(32*cos(b*x+a)^4-72*cos(b*x+a)^2+45)*cos(b*x+a)*(d*sin(b*x+a)/cos(b
*x+a))^(1/2)/sin(b*x+a)^5
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.76

$$-\frac{2\left(45\,d^4\tan\left(bx+a\right)^4+18\,d^4\tan\left(bx+a\right)^2+5\,d^4\right)d}{45\,\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2/45*(45*d^4*tan(b*x + a)^4 + 18*d^4*tan(b*x + a)^2 + 5*d^4)*d/((d*tan(b*x
 + a))^(9/2)*b)
```

Fricas [A]
time = 0.38, size = 82, normalized size = 1.30

$$-\frac{2\left(32\,\cos\left(bx+a\right)^5-72\,\cos\left(bx+a\right)^3+45\,\cos\left(bx+a\right)\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{45\,\left(b\cos\left(bx+a\right)^4-2\,b\cos\left(bx+a\right)^2+b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(32*cos(b*x + a)^5 - 72*cos(b*x + a)^3 + 45*cos(b*x + a))*sqrt(d*sin(
b*x + a)/cos(b*x + a))/((b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)*sin(b*x
 + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 58, normalized size = 0.92

$$-\frac{2\left(45\,d^{6}\tan\left(bx+a\right)^{4}+18\,d^{6}\tan\left(bx+a\right)^{2}+5\,d^{6}\right)}{45\,\sqrt{d\tan\left(bx+a\right)}\,bd^{5}\tan\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/45*(45*d^6*tan(b*x + a)^4 + 18*d^6*tan(b*x + a)^2 + 5*d^6)/(sqrt(d*tan(b
*x + a))*b*d^5*tan(b*x + a)^4)
```

Mupad [B]
time = 7.02, size = 356, normalized size = 5.65

$$-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{45\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}+\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{45\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2}-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{15\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3}-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{9\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^4}-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^6,x)
```

```
[Out] ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2)*64i)/(45*b*(exp(a*2i + b*x*2i) - 1)^2) - ((exp(a*2i + b*x
*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2
)*64i)/(45*b*(exp(a*2i + b*x*2i) - 1)) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(ex
p(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*32i)/(15*b*(exp(
a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)
*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*64i)/(9*b*(exp(a*2i + b*x*2i) -
1)^4) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a
*2i + b*x*2i) + 1))^(1/2)*32i)/(9*b*(exp(a*2i + b*x*2i) - 1)^5)
```

3.59 $\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=105

$$-\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{5\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{12b}$$

```
[Out] -5/6*d*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)-1/3*d*sin(b*x+a)^3/b/(d*tan(b*x+a)
)^(1/2)-5/12*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ellip
ticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2678, 2681, 2653, 2720}

$$-\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}}-\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}}+\frac{5\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-5*d*Sin[a + b*x])/(6*b*Sqrt[d*Tan[a + b*x]]) - (d*Sin[a + b*x]^3)/(3*b*Sq
rt[d*Tan[a + b*x]]) + (5*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin
[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(12*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sin^3(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{5}{6}\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}} - \frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{5}{12}\int \csc(a+bx)\sqrt{d\,\text{ta}$$

$$= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}} - \frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{\left(5\sqrt{\cos(a+bx)}\sqrt{d\,\text{t}}\right.}{}$$

$$= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}} - \frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{1}{12}\left(5\csc(a+bx)\sqrt{\sin}\right.$$

$$= -\frac{5d\sin(a+bx)}{6b\sqrt{d\tan(a+bx)}} - \frac{d\sin^3(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{5\csc(a+bx)F\left(a-\frac{\pi}{4}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.20, size = 139, normalized size = 1.32

$$-\frac{\cos(2(a+bx))\sec(a+bx)\left(-5\sqrt[4]{-1}F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\sec^2(a+bx)+(-6+\cos(2(a+bx)))\sqrt{\sec^2(a+bx)}\sqrt{\tan(a+bx)}\right)\sqrt{d\tan(a+bx)}}{6b\sqrt{\sec^2(a+bx)}\sqrt{\tan(a+bx)}\left(-1+\tan^2(a+bx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/6*(Cos[2*(a + b*x)]*Sec[a + b*x]*(-5*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)
^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sec[a + b*x]^2 + (-6 + Cos[2*(a + b*x)])*Sq
rt[Sec[a + b*x]^2]*Sqrt[Tan[a + b*x]])*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Sec[a
+ b*x]^2]*Sqrt[Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

Maple [A]
time = 0.38, size = 216, normalized size = 2.06

method	result

default	$\frac{(-1+\cos(bx+a))\left(2(\cos^4(bx+a))\sqrt{2}-5\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sin(bx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/12/b*(-1+cos(b*x+a))*(2*cos(b*x+a)^4*2^(1/2)-5*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^3*2^(1/2)-7*cos(b*x+a)^2*2^(1/2)+7*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(1/2)/sin(b*x+a)^4*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*tan(b*x + a))*sin(b*x + a)^3, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")

[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a), x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(1/2),x)

[Out] Timed out

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeEv
aluation time: 5.02Done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)^3\,\sqrt{d\tan(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2), x)
```

3.60 $\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=75

$$-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{2b}$$

```
[Out] -d*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)-1/2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(
1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^
(1/2)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2678, 2681, 2653, 2720}

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{2b}-\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -((d*Sin[a + b*x])/(b*Sqrt[d*Tan[a + b*x]])) + (Csc[a + b*x]*EllipticF[a -
Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(2*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx &= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{1}{2}\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx\\ &= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{\left(\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{}{\sqrt{\cos(a+}}}{2\sqrt{\sin(a+bx)}}\\ &= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{1}{2}\Big(\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\\ &= -\frac{d\sin(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{2b}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.88, size = 57, normalized size = 0.76

$$\frac{\cos(a+bx)\left(-1+{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Cos[a + b*x]*(-1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[
Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(92) = 184.
time = 0.33, size = 188, normalized size = 2.51

method	result
default	$-\frac{(-1+\cos(bx+a))\left(\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sin(bx+a)\,\mathrm{EllipticF}\left(\sqrt{\frac{1-}{}}\right.\right.}{2b\sin(bx+a)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(-1+cos(b*x+a))*(((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*s
in(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2
))+cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/
cos(b*x+a))^(1/2)/sin(b*x+a)^4*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\tan{(a + bx)}}\,\sin{(a + bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*sin(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)\,\,\sqrt{d\tan(a+b\,x)}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(d*tan(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)*(d*tan(a + b*x))^(1/2), x)
```

3.61 $\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=47

$$\frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] -csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1
/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2681, 2653, 2720}

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Ta
n[a + b*x]])/b
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx = \frac{\left(\sqrt{\cos(a+bx)}\,\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\,\sqrt{\sin(a+bx)}}\,dx}{\sqrt{\sin(a+bx)}}$$
$$= \left(\csc(a+bx)\sqrt{\sin(2a+2bx)}\,\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,d$$
$$= \frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\,\sqrt{d\tan(a+bx)}}{b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.20, size = 73, normalized size = 1.55

$$-\frac{2\sqrt[4]{-1}\cos(a+bx)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\sqrt{\sec^2(a+bx)}\,\sqrt{d\tan(a+bx)}}{b\sqrt{\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sqrt[d*Tan[a + b*x]],x]
[Out] (-2*(-1)^(1/4)*Cos[a + b*x]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x
]]], -1]*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]])/(b*Sqrt[Tan[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(69) = 138$.
time = 0.38, size = 157, normalized size = 3.34

method	result
default	$-\frac{\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\,(-1+\cos(bx+a))\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticF}\left(\right.}{b\sin(bx+a)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/b*(d*sin(b*x+a)/cos(b*x+a))^(1/2)*(-1+cos(b*x+a))*((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2*2^(1/2))*(cos(b*x+a)+1)^2/sin(b*x+a)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.09, size = 52, normalized size = 1.11

$$-\frac{\sqrt{i\,d}\,\mathrm{ellipticF}(\cos{(bx+a)}+i\,\sin{(bx+a)}\,,-1)+\sqrt{-i\,d}\,\mathrm{ellipticF}(\cos{(bx+a)}-i\,\sin{(bx+a)}\,,-1)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(I*d)*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1) + sqrt(-I*d)*ellip
ticF(cos(b*x + a) - I*sin(b*x + a), -1))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\tan{(a+bx)}}\,\csc{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{d\tan{(a+b\,x)}}}{\sin{(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x),x)
```

```
[Out] int((d*tan(a + b*x))^(1/2)/sin(a + b*x), x)
```

3.62 $\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=77

$$-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}$$

```
[Out] -2/3*d*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-2/3*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2
*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2679, 2681, 2653, 2720}

$$\frac{2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}-\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d*Csc[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (2*Csc[a + b*x]*EllipticF[
a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx &= -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2}{3}\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx\\ &= -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{}{\sqrt{\cos(a}}}{3\sqrt{\sin(a+bx)}}\\ &= -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{1}{3}\Big(2\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+b}\\ &= -\frac{2d\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx}}{3b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.66, size = 115, normalized size = 1.49

$$\frac{2\cos(2(a+bx))\csc^3(a+bx)(d\tan(a+bx))^{3/2}\left(\sqrt{\sec^2(a+bx)}+2\sqrt[4]{-1}F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\tan^{\frac{3}{2}}(a+bx)\right)}{3bd\sqrt{\sec^2(a+bx)}\left(-1+\tan^2(a+bx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (2*Cos[2*(a + b*x)]*Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2)*(Sqrt[Sec[a + b*x
]^2] + 2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]
*Tan[a + b*x]^(3/2)))/(3*b*d*Sqrt[Sec[a + b*x]^2]*(-1 + Tan[a + b*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(92) = 184.
time = 0.40, size = 297, normalized size = 3.86

method	result
default	$\frac{(-1+\cos(bx+a))^2\left(2\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*(-1+cos(b*x+a))^2*(2*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b
*x+a)*cos(b*x+a)+2*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*
x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-co
s(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(1/2)/sin(b*x+
a)^6*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^3, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.10, size = 111, normalized size = 1.44

$$-\frac{2\left(\left(\cos\left(bx+a\right)^{2}-1\right)\sqrt{i\,d}\,\mathrm{ellipticF}\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right),-1\right)+\left(\cos\left(bx+a\right)^{2}-1\right)\sqrt{-i\,d}\,\mathrm{ellipticF}\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right),-1\right)-\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}\cos\left(bx+a\right)\right)}{3\left(b\cos\left(bx+a\right)^{2}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*ellipticF(cos(b*x + a) + I*sin(b*x + a
), -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*ellipticF(cos(b*x + a) - I*sin(b*x
 + a), -1) - sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a))/(b*cos(b*x + a
)^2 - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\tan\left(a+bx\right)}\,\csc^{3}\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\tan\left(a+b\,x\right)}}{{\sin\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^3, x)
```

3.63 $\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx$

Optimal. Leaf size=105

$$-\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{4\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{7b}$$

```
[Out] -4/7*d*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-2/7*d*csc(b*x+a)^3/b/(d*tan(b*x+a)
)^(1/2)-4/7*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ellipt
icF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2679, 2681, 2653, 2720}

$$-\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}}-\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}}+\frac{4\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-4*d*Csc[a + b*x])/(7*b*Sqrt[d*Tan[a + b*x]]) - (2*d*Csc[a + b*x]^3)/(7*b*
Sqrt[d*Tan[a + b*x]]) + (4*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[S
in[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(7*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^5(a+bx)\sqrt{d\tan(a+bx)}\,dx = -\frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}} + \frac{6}{7}\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx$$

$$= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}} - \frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}} + \frac{4}{7}\int \csc(a+bx)\sqrt{d\tan(}$$

$$= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}} - \frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}} + \frac{\left(4\sqrt{\cos(a+bx)}\sqrt{d\,\mathrm{ta}}\right.}{}$$

$$= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}} - \frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}} + \frac{1}{7}\left(4\csc(a+bx)\sqrt{\sin(2a}\right.$$

$$= -\frac{4d\csc(a+bx)}{7b\sqrt{d\tan(a+bx)}} - \frac{2d\csc^3(a+bx)}{7b\sqrt{d\tan(a+bx)}} + \frac{4\csc(a+bx)F\left(a-\frac{\pi}{4}+\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.51, size = 124, normalized size = 1.18

$$-\frac{2d\cos(2(a+bx))\csc^3(a+bx)\left((-2+\cos(2(a+bx)))\sec^2(a+bx)^{3/2}-4\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,\sqrt{\tan(a+bx)}\right)\middle|-1\right)\tan^{\frac{7}{2}}(a+bx)\right)}{7b\sqrt{\sec^2(a+bx)}\,\sqrt{d\tan(a+bx)}\,\left(-1+\tan^2(a+bx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d*Cos[2*(a + b*x)]*Csc[a + b*x]^3*((-2 + Cos[2*(a + b*x)])*(Sec[a + b*x
]^2)^(3/2) - 4*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]
], -1]*Tan[a + b*x]^(7/2)))/(7*b*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]]*
(-1 + Tan[a + b*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(116) = 232.
time = 0.40, size = 550, normalized size = 5.24

method	result
default	$-\frac{(-1+\cos(bx+a))^2\left(4\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,(\cos^3(bx+a))\sin(bx+a)\,\mathrm{I}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/b*(-1+cos(b*x+a))^2*(4*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/
2)*cos(b*x+a)^3*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))+4*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin
(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))
*cos(b*x+a)^2-4*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*
2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*cos(b
*x+a)-4*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b
*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*Ellipt
icF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^
3*2^(1/2)+3*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^
(1/2)/sin(b*x+a)^8*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^5, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.11, size = 155, normalized size = 1.48

$$-\frac{2\left(2\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\sqrt{id}\,\mathrm{ellipticF}(\cos(bx+a)+i\sin(bx+a),-1)+2\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\sqrt{-id}\,\mathrm{ellipticF}(\cos(bx+a)-i\sin(bx+a),-1)-\left(2\cos(bx+a)^3-3\cos(bx+a)\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\right)}{7\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/7*(2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(I*d)*ellipticF(cos(b*x
 + a) + I*sin(b*x + a), -1) + 2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqr
t(-I*d)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1) - (2*cos(b*x + a)^3 -
```

```
3*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a)^4 - 2*b*
cos(b*x + a)^2 + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\tan(a+bx)}\csc^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**5*(d*tan(b*x+a))**(1/2),x)
[Out] Integral(sqrt(d*tan(a + b*x))*csc(a + b*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(b*x + a))*csc(b*x + a)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\tan(a+b\,x)}}{\sin(a+b\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^5,x)
[Out] int((d*tan(a + b*x))^(1/2)/sin(a + b*x)^5, x)
```

3.64 $\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=277

$$\frac{45d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b}-\frac{45d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b}+\frac{45d^{3/2}\log\left(\sqrt{d}\right.}{}$$

```
[Out] 45/64*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-45/6
4*d^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+45/128*d
^(3/2)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2
)-45/128*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a)
)/b*2^(1/2)+45/16*d*(d*tan(b*x+a))^(1/2)/b-9/16*cos(b*x+a)^2*(d*tan(b*x+a))
^(5/2)/b/d-1/4*cos(b*x+a)^4*(d*tan(b*x+a))^(9/2)/b/d^3
```

Rubi [A]
time = 0.14, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2671, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{45d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b}-\frac{45d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}\,b}+\frac{45d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,b}-\frac{45d^{3/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,b}-\frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3}+\frac{45d\sqrt{d\tan(a+bx)}}{16b}-\frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (45*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]
*b) - (45*d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*S
qrt[2]*b) + (45*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d
*Tan[a + b*x]]])/(64*Sqrt[2]*b) - (45*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a +
 b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (45*d*Sqrt[d*Tan[a
+ b*x]])/(16*b) - (9*Cos[a + b*x]^2*(d*Tan[a + b*x])^(5/2))/(16*b*d) - (Cos
[a + b*x]^4*(d*Tan[a + b*x])^(9/2))/(4*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d\text{Subst}\left(\int \frac{x^{11/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3} + \frac{(9d)\text{Subst}\left(\int \frac{x^{7/2}}{(d^2+x^2)^2}\,dx, x, d\tan\right.}{8b}$$

$$= -\frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{9/2}}{4bd^3}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+}{}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+}{}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+}{}$$

$$= \frac{45d\sqrt{d\tan(a+bx)}}{16b} - \frac{9\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{16bd} - \frac{\cos^4(a+}{}$$

$$= \frac{45d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}b} - \frac{45d^3}{}$$

$$= \frac{45d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}b} - \frac{45d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{32\sqrt{2}}$$

Mathematica [A]
time = 0.82, size = 123, normalized size = 0.44

$$-\frac{d\csc(a+bx)\left(-143\sin(a+bx)-45\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}+45\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}-14\sin(3(a+bx))+\sin(5(a+bx))\right)\sqrt{d\tan(a+bx)}}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/64*(d*Csc[a + b*x]*(-143*Sin[a + b*x] - 45*ArcSin[Cos[a + b*x] - Sin[a +
 b*x]]*Sqrt[Sin[2*(a + b*x)]] + 45*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[S
in[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]] - 14*Sin[3*(a + b*x)] + Sin[5*(a +
 b*x)])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 3.80, size = 702, normalized size = 2.53

method	result
default	$\frac{(-1+\cos(bx+a))\left(45i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sin(bx+a)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \sqrt{\frac{1-\cos(bx+a)+\sin(}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/b*(-1+cos(b*x+a))*(45*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)-45*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(
b*x+a))^(1/2)-8*2^(1/2)*cos(b*x+a)^5-90*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin
(b*x+a))^(1/2)*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))+45*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1
-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*
x+a))^(1/2)+45*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(
1/2)+8*cos(b*x+a)^4*2^(1/2)+34*cos(b*x+a)^3*2^(1/2)-34*cos(b*x+a)^2*2^(1/2)
+64*cos(b*x+a)*2^(1/2)-64*2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a
)/cos(b*x+a))^(3/2)/sin(b*x+a)^5*2^(1/2)
```

Maxima [A]
time = 0.49, size = 235, normalized size = 0.85

$$-\frac{90\sqrt{2}\,d^{\frac{13}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{d}}\right)+90\sqrt{2}\,d^{\frac{13}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{d}}\right)+45\sqrt{2}\,d^{\frac{13}{2}}\log\left(d\tan\left(bx+a\right)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{d}+d\right)-45\sqrt{2}\,d^{\frac{13}{2}}\log\left(d\tan\left(bx+a\right)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{d}+d\right)-256\sqrt{d\tan\left(bx+a\right)}\,d^{6}-\frac{8\left(17\left(d\tan(bx+a)\right)^{\frac{5}{2}}d^{8}+13\sqrt{d\tan\left(bx+a\right)}\,d^{10}\right)}{d^{4}\tan(bx+a)^{4}+2\,d^{4}\tan(bx+a)^{2}+d^{4}}}{128\,bd^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/128*(90*sqrt(2)*d^(13/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*
tan(b*x + a)))/sqrt(d)) + 90*sqrt(2)*d^(13/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*
sqrt(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 45*sqrt(2)*d^(13/2)*log(d*tan(
b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 45*sqrt(2)*d^(13/2)*
log(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 256*sqrt(d
*tan(b*x + a))*d^6 - 8*(17*(d*tan(b*x + a))^(5/2)*d^8 + 13*sqrt(d*tan(b*x +
 a))*d^10)/(d^4*tan(b*x + a)^4 + 2*d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1580 vs. $2(213) = 426$.
time = 42.87, size = 1580, normalized size = 5.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/256*(90*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(1/2*(2*d^10*sin(b*x + a) + sqrt(
4*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + d^10 + 2*(sqrt(2)*(d^6/
b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^
5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(sqrt(2)*(d^6/b^4)^(1/
4)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*si
n(b*x + a)/cos(b*x + a)) - 4*(b^2*d^7*cos(b*x + a)^3 - b^2*d^7*cos(b*x + a)
)*sqrt(d^6/b^4) + (sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a) + sqrt(2)*(d^
6/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((2*d
^10*cos(b*x + a)^2 - d^10)*sin(b*x + a))) + 90*sqrt(2)*(d^6/b^4)^(1/4)*b*ar
ctan(-1/2*(2*d^10*sin(b*x + a) - sqrt(4*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*
sin(b*x + a) + d^10 - 2*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x
 + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)
/cos(b*x + a)))*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/
b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^7
*cos(b*x + a)^3 - b^2*d^7*cos(b*x + a))*sqrt(d^6/b^4) - (sqrt(2)*(d^6/b^4)^
(1/4)*b*d^8*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sq
rt(d*sin(b*x + a)/cos(b*x + a)))/((2*d^10*cos(b*x + a)^2 - d^10)*sin(b*x +
a))) - 90*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(1/2*(sqrt(4*sqrt(d^6/b^4)*b^2*d^
7*cos(b*x + a)*sin(b*x + a) + d^10 - 2*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b
*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt
(d*sin(b*x + a)/cos(b*x + a)))*(2*d^5*sin(b*x + a) + (sqrt(2)*(d^6/b^4)^(1/
4)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*si
n(b*x + a)/cos(b*x + a))) - (sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a) - s
qrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x +
a)))/(d^10*sin(b*x + a))) - 90*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(-1/2*(sqrt(
4*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + d^10 + 2*(sqrt(2)*(d^6/
b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^
5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(2*d^5*sin(b*x + a) -
```

```
(sqrt(2)*(d^6/b^4)^(1/4)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*s
in(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + (sqrt(2)*(d^6/b^4)^(1/4)*
b*d^8*cos(b*x + a) - sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sqrt(d*s
in(b*x + a)/cos(b*x + a)))/(d^10*sin(b*x + a))) - 45*sqrt(2)*(d^6/b^4)^(1/4
)*b*log(33215062500*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + 83037
65625*d^10 + 16607531250*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*
x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a
)/cos(b*x + a))) + 45*sqrt(2)*(d^6/b^4)^(1/4)*b*log(33215062500*sqrt(d^6/b^
4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + 8303765625*d^10 - 16607531250*(sqrt(
2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4
)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 16*(4*d*cos(
b*x + a)^4 - 17*d*cos(b*x + a)^2 - 32*d)*sqrt(d*sin(b*x + a)/cos(b*x + a)))
/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.53, size = 252, normalized size = 0.91

$$-\frac{1}{128}\,d\left(\frac{90\,\sqrt{2}\,\sqrt{|d|}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}+2\,\sqrt{d\tan\left(bx+a\right)}\right)}{2\,\sqrt{|d|}}\right)}{b}+\frac{90\,\sqrt{2}\,\sqrt{|d|}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}-2\,\sqrt{d\tan\left(bx+a\right)}\right)}{2\,\sqrt{|d|}}\right)}{b}+\frac{45\,\sqrt{2}\,\sqrt{|d|}\,\log\left(d\tan\left(bx+a\right)+\sqrt{2}\,\sqrt{d\tan\left(bx+a\right)}\,\sqrt{|d|}+|d|\right)}{b}-\frac{45\,\sqrt{2}\,\sqrt{|d|}\,\log\left(d\tan\left(bx+a\right)-\sqrt{2}\,\sqrt{d\tan\left(bx+a\right)}\,\sqrt{|d|}+|d|\right)}{b}-\frac{256\,\sqrt{d\tan\left(bx+a\right)}}{b}-\frac{8\left(17\,\sqrt{d\tan\left(bx+a\right)}\,d^4\tan\left(bx+a\right)^2+13\,\sqrt{d\tan\left(bx+a\right)}\,d^4\right)}{\left(d^2\tan\left(bx+a\right)^2+d^2\right)^2 b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] -1/128*d*(90*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
+ 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 90*sqrt(2)*sqrt(abs(d))*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/
b + 45*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a
))*sqrt(abs(d)) + abs(d))/b - 45*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) -
sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 256*sqrt(d*tan(b*x
+ a))/b - 8*(17*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a)^2 + 13*sqrt(d*tan(b*x
 + a))*d^4)/((d^2*tan(b*x + a)^2 + d^2)^2*b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,x\right)^4\,{\left(d\,\mathrm{tan}\left(a+b\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2), x)
```

3.65 $\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=247

$$\frac{5d^{3/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}-\frac{5d^{3/2}\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}+\frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

```
[Out] 5/8*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-5/8*d^
(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+5/16*d^(3/2)
*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)-5/16
*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1
/2)+5/2*d*(d*tan(b*x+a))^(1/2)/b-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(5/2)/b/d
```

Rubi [A]
time = 0.12, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2671, 294, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{5d^{3/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}-\frac{5d^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}\,b}+\frac{5d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,b}-\frac{5d^{3/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,b}+\frac{5d\sqrt{d\tan(a+bx)}}{2b}-\frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (5*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b
) - (5*d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[
2]*b) + (5*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[
a + b*x]]])/(8*Sqrt[2]*b) - (5*d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] +
 Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) + (5*d*Sqrt[d*Tan[a + b*x]])/
(2*b) - (Cos[a + b*x]^2*(d*Tan[a + b*x])^(5/2))/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{d\text{Subst}\left(\int \frac{x^{7/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} + \frac{(5d)\text{Subst}\left(\int \frac{x^{3/2}}{d^2+x^2}\,dx, x, d\tan(a+\right.}{4b}$$

$$= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} - \frac{(5d^3)\,\text{Subst}\left(\right.}{}$$

$$= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} - \frac{(5d^3)\,\text{Subst}\left(\right.}{}$$

$$= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} - \frac{(5d^2)\,\text{Subst}\left(\right.}{}$$

$$= \frac{5d\sqrt{d\tan(a+bx)}}{2b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{5/2}}{2bd} + \frac{(5d^{3/2})\,\text{Subst}}{}$$

$$= \frac{5d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\,\tan(a+bx)-\sqrt{2}\,\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}\,b} - \frac{5d^{3/2}\,\text{lo}}{}$$

$$= \frac{5d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b} - \frac{5d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d}}{}\right.}{4\sqrt{2}\,b}$$

Mathematica [A]
time = 0.57, size = 113, normalized size = 0.46

$$\frac{d\csc(a+bx)\left(17\sin(a+bx)+5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}+\sin(3(a+bx))\right)\sqrt{d\tan(a+bx)}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d*Csc[a + b*x]*(17*Sin[a + b*x] + 5*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sq
rt[Sin[2*(a + b*x)]] - 5*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a +
b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/(8
*b)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.32, size = 676, normalized size = 2.74

method	result
default	$\frac{(-1+\cos(bx+a))\left(5i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sin(bx+a)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(-1+cos(b*x+a))*(5*I*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b
*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/
2*I,1/2*2^(1/2))-5*I*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/
2*2^(1/2))+5*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/
2*I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/
2)+5*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*
2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-10*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*EllipticF(((1-co
s(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)^3*2^(1/2)-
2*cos(b*x+a)^2*2^(1/2)+8*cos(b*x+a)*2^(1/2)-8*2^(1/2))*cos(b*x+a)*(cos(b*x+
a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)^5*2^(1/2)
```

Maxima [A]
time = 0.50, size = 204, normalized size = 0.83

$$-\frac{10\sqrt{2}\,d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+10\sqrt{2}\,d^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+5\sqrt{2}\,d^{\frac{9}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)-5\sqrt{2}\,d^{\frac{9}{2}}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)-\frac{8\sqrt{d\tan(bx+a)}\,d^6}{d^2\tan(bx+a)^2+d^2}-32\sqrt{d\tan(bx+a)}\,d^4}{16\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(10*sqrt(2)*d^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(b*x + a)))/sqrt(d)) + 10*sqrt(2)*d^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
```

```
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 5*sqrt(2)*d^(9/2)*log(d*tan(b*x +
 a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 5*sqrt(2)*d^(9/2)*log(d*t
an(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x
+ a))*d^6/(d^2*tan(b*x + a)^2 + d^2) - 32*sqrt(d*tan(b*x + a))*d^4)/(b*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1568 vs. $2(187) = 374$.
time = 42.11, size = 1568, normalized size = 6.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(10*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(1/2*(2*d^10*sin(b*x + a) + sqrt(4
*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + d^10 + 2*(sqrt(2)*(d^6/b
^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5
*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(sqrt(2)*(d^6/b^4)^(1/4
)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*sin
(b*x + a)/cos(b*x + a)) - 4*(b^2*d^7*cos(b*x + a)^3 - b^2*d^7*cos(b*x + a))
*sqrt(d^6/b^4) + (sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a) + sqrt(2)*(d^6
/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((2*d^
10*cos(b*x + a)^2 - d^10)*sin(b*x + a))) + 10*sqrt(2)*(d^6/b^4)^(1/4)*b*arc
tan(-1/2*(2*d^10*sin(b*x + a) - sqrt(4*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*s
in(b*x + a) + d^10 - 2*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x
+ a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/
cos(b*x + a)))*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/b
^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^7*
cos(b*x + a)^3 - b^2*d^7*cos(b*x + a))*sqrt(d^6/b^4) - (sqrt(2)*(d^6/b^4)^(
1/4)*b*d^8*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sqr
t(d*sin(b*x + a)/cos(b*x + a)))/((2*d^10*cos(b*x + a)^2 - d^10)*sin(b*x + a
))) - 10*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(1/2*(sqrt(4*sqrt(d^6/b^4)*b^2*d^7
*cos(b*x + a)*sin(b*x + a) + d^10 - 2*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*
x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(
d*sin(b*x + a)/cos(b*x + a)))*(2*d^5*sin(b*x + a) + (sqrt(2)*(d^6/b^4)^(1/4
)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*sin
(b*x + a)/cos(b*x + a))) - (sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a) - sq
rt(2)*(d^6/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a
)))/(d^10*sin(b*x + a))) - 10*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(-1/2*(sqrt(4
*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + d^10 + 2*(sqrt(2)*(d^6/b
^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5
*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(2*d^5*sin(b*x + a) - (
sqrt(2)*(d^6/b^4)^(1/4)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*si
n(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + (sqrt(2)*(d^6/b^4)^(1/4)*b
*d^8*cos(b*x + a) - sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sqrt(d*si
```

```
n(b*x + a)/cos(b*x + a)))/(d^10*sin(b*x + a))) - 5*sqrt(2)*(d^6/b^4)^(1/4)*
b*log(62500*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + 15625*d^10 +
31250*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d
^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) +
5*sqrt(2)*(d^6/b^4)^(1/4)*b*log(62500*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*si
n(b*x + a) + 15625*d^10 - 31250*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)
*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(
b*x + a)/cos(b*x + a))) + 16*(d*cos(b*x + a)^2 + 4*d)*sqrt(d*sin(b*x + a)/c
os(b*x + a)))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 226, normalized size = 0.91

$$-\frac{1}{16}\,d\left(\frac{10\,\sqrt{2}\,\sqrt{|d|}\,\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{|d|}+2\,\sqrt{d\tan\left(bx+a\right)}\right)}{2\,\sqrt{|d|}}\right)}{b}+\frac{10\,\sqrt{2}\,\sqrt{|d|}\,\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{|d|}-2\,\sqrt{d\tan\left(bx+a\right)}\right)}{2\,\sqrt{|d|}}\right)}{b}+\frac{5\,\sqrt{2}\,\sqrt{|d|}\,\log\left(d\tan\left(bx+a\right)+\sqrt{2}\,\sqrt{d\tan\left(bx+a\right)}\,\sqrt{|d|}+|d|\right)}{b}-\frac{5\,\sqrt{2}\,\sqrt{|d|}\,\log\left(d\tan\left(bx+a\right)-\sqrt{2}\,\sqrt{d\tan\left(bx+a\right)}\,\sqrt{|d|}+|d|\right)}{b}-\frac{8\,\sqrt{d\tan\left(bx+a\right)}\,d^{2}}{\left(d^{2}\tan\left(bx+a\right)^{2}+d^{2}\right)b}-\frac{32\,\sqrt{d\tan\left(bx+a\right)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*d*(10*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) +
 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 10*sqrt(2)*sqrt(abs(d))*arctan(-
1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b
 + 5*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))
*sqrt(abs(d)) + abs(d))/b - 5*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) - sqr
t(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*sqrt(d*tan(b*x + a))
*d^2/((d^2*tan(b*x + a)^2 + d^2)*b) - 32*sqrt(d*tan(b*x + a))/b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,x\right)^2\,{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2), x)
```

3.66 $\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=18

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 2*d*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 30}

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= \frac{2d\sqrt{d\tan(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 18, normalized size = 1.00

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(16) = 32$.
time = 0.33, size = 58, normalized size = 3.22

method	result	size
default	$\frac{2\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}\cos(bx+a)(-1+\cos(bx+a))^{2}(\cos(bx+a)+1)^{2}}{b\sin(bx+a)^{5}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*(d*sin(b*x+a)/cos(b*x+a))^(3/2)*cos(b*x+a)*(-1+cos(b*x+a))^2*(cos(b*x+a
)+1)^2/sin(b*x+a)^5
```

Maxima [A]
time = 0.28, size = 23, normalized size = 1.28

$$\frac{2\,(d\tan\left(bx+a\right))^{\frac{3}{2}}}{b\tan\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(d*tan(b*x + a))^(3/2)/(b*tan(b*x + a))
```

Fricas [A]
time = 0.40, size = 24, normalized size = 1.33

$$\frac{2\,d\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*d*sqrt(d*sin(b*x + a)/cos(b*x + a))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.66, size = 16, normalized size = 0.89

$$\frac{2\sqrt{d\tan\left(bx+a\right)}\,d}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*tan(b*x + a))*d/b
```

Mupad [B]
time = 2.77, size = 43, normalized size = 2.39

$$\frac{2\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^2,x)
```

```
[Out] (2*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/b
```

3.67 $\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=41

$$-\frac{2d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 2*d*(d*tan(b*x+a))^(1/2)/b-2/3*d^3/b/(d*tan(b*x+a))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 14}

$$\frac{2d\sqrt{d\tan(a+bx)}}{b}-\frac{2d^3}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^3)/(3*b*(d*Tan[a + b*x])^(3/2)) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(a+bx)(d\tan(a+bx))^{3/2}\,dx&=\frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{5/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\&=\frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{5/2}}+\frac{1}{\sqrt{x}}\right)\,dx,x,d\tan(a+bx)\right)}{b}\\&=-\frac{2d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 30, normalized size = 0.73

$$-\frac{2d(-4+\csc^2(a+bx))\sqrt{d\tan(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
[Out] (-2*d*(-4 + Csc[a + b*x]^2)*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Maple [A]
time = 0.36, size = 50, normalized size = 1.22

method	result	size
default	$-\frac{2\left(4\left(\cos^2(bx+a)\right)-3\right)\cos(bx+a)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}}{3b\sin(bx+a)^3}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/b*(4*cos(b*x+a)^2-3)*cos(b*x+a)*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*
x+a)^3
```

Maxima [A]
time = 0.27, size = 34, normalized size = 0.83

$$-\frac{2\,d^3\left(\frac{1}{(d\tan(bx+a))^{\frac{3}{2}}}-\frac{3\,\sqrt{d\tan\left(bx+a\right)}}{d^2}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2/3*d^3*(1/(d*tan(b*x + a))^(3/2) - 3*sqrt(d*tan(b*x + a))/d^2)/b
```

Fricas [A]
time = 0.37, size = 51, normalized size = 1.24

$$\frac{2\left(4\,d\cos\left(bx+a\right)^2-3\,d\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{3\left(b\cos\left(bx+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(4*d*cos(b*x + a)^2 - 3*d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x
 + a)^2 - b)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.64, size = 43, normalized size = 1.05

$$\frac{2}{3}\,d\left(\frac{3\,\sqrt{d\tan\left(bx+a\right)}}{b}-\frac{d}{\sqrt{d\tan\left(bx+a\right)}\,b\tan\left(bx+a\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*d*(3*sqrt(d*tan(b*x + a))/b - d/(sqrt(d*tan(b*x + a))*b*tan(b*x + a)))
```

Mupad [B]
time = 3.49, size = 100, normalized size = 2.44

$$\frac{8\,d\,\sqrt{\frac{d\,\sin\left(2\,a+2\,b\,x\right)}{\cos\left(2\,a+2\,b\,x\right)+1}}\,\left(11\,\cos\left(2\,a+2\,b\,x\right)-5\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)-7\right)}{3\,b\,\left(15\,\cos\left(2\,a+2\,b\,x\right)-6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)-10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^4,x)
```

```
[Out] (8*d*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(11*cos(2*a + 2*b*
x) - 5*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 7))/(3*b*(15*cos(2*a + 2*b*x)
- 6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

3.68 $\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=63

$$-\frac{2d^5}{7b(d\tan(a+bx))^{7/2}}-\frac{4d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 2*d*(d*tan(b*x+a))^(1/2)/b-2/7*d^5/b/(d*tan(b*x+a))^(7/2)-4/3*d^3/b/(d*tan(
b*x+a))^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 276}

$$-\frac{2d^5}{7b(d\tan(a+bx))^{7/2}}-\frac{4d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^5)/(7*b*(d*Tan[a + b*x])^(7/2)) - (4*d^3)/(3*b*(d*Tan[a + b*x])^(3/2)
) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^6(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{d\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{9/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\ &= \frac{d\text{Subst}\left(\int \left(\frac{d^4}{x^{9/2}}+\frac{2d^2}{x^{5/2}}+\frac{1}{\sqrt{x}}\right)dx,x,d\tan(a+bx)\right)}{b}\\ &= -\frac{2d^5}{7b(d\tan(a+bx))^{7/2}}-\frac{4d^3}{3b(d\tan(a+bx))^{3/2}}+\frac{2d\sqrt{d\tan(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 42, normalized size = 0.67

$$-\frac{2d(-32+8\csc^2(a+bx)+3\csc^4(a+bx))\sqrt{d\tan(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d*(-32 + 8*Csc[a + b*x]^2 + 3*Csc[a + b*x]^4)*Sqrt[d*Tan[a + b*x]])/(21
*b)
```

Maple [A]
time = 0.38, size = 60, normalized size = 0.95

method	result	size
default	$\frac{2\left(32\left(\cos^4(bx+a)\right)-56\left(\cos^2(bx+a)\right)+21\right)\cos(bx+a)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}}{21b\sin(bx+a)^5}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/b*(32*cos(b*x+a)^4-56*cos(b*x+a)^2+21)*cos(b*x+a)*(d*sin(b*x+a)/cos(b*
x+a))^(3/2)/sin(b*x+a)^5
```

Maxima [A]
time = 0.27, size = 58, normalized size = 0.92

$$\frac{2\,d^5\left(\frac{21\,\sqrt{d\tan\left(bx+a\right)}}{d^4}-\frac{14\,d^2\tan(bx+a)^2+3\,d^2}{(d\tan(bx+a))^{\frac{7}{2}}\,d^2}\right)}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21*d^5*(21*sqrt(d*tan(b*x + a))/d^4 - (14*d^2*tan(b*x + a)^2 + 3*d^2)/((d
*tan(b*x + a))^(7/2)*d^2))/b
```

Fricas [A]
time = 0.37, size = 71, normalized size = 1.13

$$\frac{2\left(32\,d\cos\left(bx+a\right)^4-56\,d\cos\left(bx+a\right)^2+21\,d\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{21\left(b\cos\left(bx+a\right)^4-2\,b\cos\left(bx+a\right)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(32*d*cos(b*x + a)^4 - 56*d*cos(b*x + a)^2 + 21*d)*sqrt(d*sin(b*x + a)
/cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 64, normalized size = 1.02

$$\frac{2}{21}\,d\left(\frac{21\,\sqrt{d\tan\left(bx+a\right)}}{b}-\frac{14\,d^4\tan\left(bx+a\right)^2+3\,d^4}{\sqrt{d\tan\left(bx+a\right)}\,bd^3\tan\left(bx+a\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/21*d*(21*sqrt(d*tan(b*x + a))/b - (14*d^4*tan(b*x + a)^2 + 3*d^4)/(sqrt(d
*tan(b*x + a))*b*d^3*tan(b*x + a)^3))
```

Mupad [B]
time = 5.88, size = 292, normalized size = 4.63

$$-\frac{\left(\frac{20\,d}{21\,b}-\frac{64\,d\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}{21\,b}\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1}+\frac{20\,d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{21\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2}-\frac{24\,d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{7\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3}-\frac{16\,d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{7\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^6,x)
```

```
[Out] (20*d*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
 + b*x*2i) + 1))^(1/2))/(21*b*(exp(a*2i + b*x*2i) - 1)^2) - (((20*d)/(21*b)
 - (64*d*exp(a*2i + b*x*2i))/(21*b))*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(ex
p(a*2i + b*x*2i) + 1))^(1/2))/(exp(a*2i + b*x*2i) - 1) - (24*d*(exp(a*2i +
b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(
1/2))/(7*b*(exp(a*2i + b*x*2i) - 1)^3) - (16*d*(exp(a*2i + b*x*2i) + 1)*(-(
d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*b*(exp(
a*2i + b*x*2i) - 1)^4)
```

3.69 $\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}-\frac{7d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 7/2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*P
i+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*
sin(b*x+a)^3*(d*tan(b*x+a))^(1/2)/b+7/3*d^3*sin(b*x+a)^3/b/(d*tan(b*x+a))^(
3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2674, 2678, 2681, 2652, 2719}

$$\frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}-\frac{7d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (7*d^3*Sin[a + b*x]^3)/(3*b*(d*Tan[a + b*x])^(3/2)) - (7*d^2*EllipticE[a -
Pi/4 + b*x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x
]]) + (2*d*Sin[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
```

```
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
&& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sin^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} - (7d^2)\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{1}{2}(7d^2)\int$$

$$= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(7d^2\sqrt{\sin(\right.}{}$$

$$= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} + \frac{2d\sin^3(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{(7d^2\sin(a-}{2\sqrt{\sin(2}}$$

$$= \frac{7d^3\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}} - \frac{7d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.44, size = 90, normalized size = 0.82

$$\frac{\left(-28\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sec(a+bx)+2\cos(a+bx)(13+\cos(2(a+bx)))\sqrt{\sec^2(a+bx)}\right)(d\tan(a+bx))^{3/2}}{12b\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] ((-28*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x] + 2*Co
s[a + b*x]*(13 + Cos[2*(a + b*x)])*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(
3/2))/(12*b*Sqrt[Sec[a + b*x]^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(123) = 246$.
time = 0.30, size = 540, normalized size = 4.91

method	result
default	$-\frac{(-1+\cos(bx+a))^2 \left(2(\cos^4(bx+a))\sqrt{2}+21\cos(bx+a)\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \sqrt{\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/b*(-1+cos(b*x+a))^2*(2*cos(b*x+a)^4*2^(1/2)+21*cos(b*x+a)*EllipticF((
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-
1+sin(b*x+a))/sin(b*x+a))^(1/2)-42*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(
b*x+a))^(1/2)+21*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2
*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin
(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-42*EllipticE(((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1
+sin(b*x+a))/sin(b*x+a))^(1/2)-11*cos(b*x+a)^2*2^(1/2)+21*cos(b*x+a)*2^(1/2
)-12*2^(1/2))*(d*sin(b*x+a)/cos(b*x+a))^(3/2)*cos(b*x+a)*(cos(b*x+a)+1)^2/s
in(b*x+a)^6*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(d*cos(b*x + a)^2 - d)*sqrt(d*tan(b*x + a))*sin(b*x + a)*tan(b*x
+ a), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument Typeex
t_reduce Error: Bad Argument TypeEvaluation time: 9.79Done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^3\,{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2), x)
```

3.70 $\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=76

$$-\frac{3d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\,\sqrt{d\tan(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 3*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*si
n(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2674, 2681, 2652, 2719}

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{3d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\,\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-3*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]
]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sin(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \left(3d^2\right)\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(3d^2\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx}}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a}}$$

$$= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(3d^2\sin(a+bx)\right)\int \sqrt{\sin(2a+2bx)}}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{3d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.30, size = 58, normalized size = 0.76

$$-\frac{2\cos(a+bx)\left(-1+\,_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d\tan(a+bx))^{3/2}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]*(-1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sq
rt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(95) = 190.
time = 0.31, size = 526, normalized size = 6.92

method	result
default	$\frac{(-1+\cos(bx+a))^2\left(6\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(-1+cos(b*x+a))^2*(6*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^
(1/2)-3*cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1
/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/s
in(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+6*EllipticE((
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-
1+sin(b*x+a))/sin(b*x+a))^(1/2)-3*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2
)+cos(b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2)+2*2^(1/2))*cos(b*x+a)*(cos(b*x+
a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)^6*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^(3/2)*sin(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(b*x + a))*d*sin(b*x + a)*tan(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(a+bx))^{\frac{3}{2}}\sin(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))**(3/2),x)
[Out] Integral((d*tan(a + b*x))**(3/2)*sin(a + b*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument Typeex
t_reduce Error: Bad Argument TypeEvaluation time: 11.33Done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)\,\left(d\tan(a+b\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(sin(a + b*x)*(d*tan(a + b*x))^(3/2), x)
```

3.71 $\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=76

$$-\frac{2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] 2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*si
n(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2673, 2681, 2652, 2719}

$$\frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}-\frac{2d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^2*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]
]*Sqrt[d*Tan[a + b*x]]) + (2*d*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \left(2d^2\right)\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx \\ &= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(2d^2\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx}}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a}} \\ &= \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(2d^2\sin(a+bx)\right)\int \sqrt{\sin(2a+2bx)}}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\ &= -\frac{2d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)\sqrt{d\tan(a+bx)}}{b}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.31, size = 61, normalized size = 0.80

$$-\frac{2\cos(a+bx)\left(-3+2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]*(-3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*
Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(95) = 190.
time = 0.35, size = 511, normalized size = 6.72

method	result
default	$\frac{(-1+\cos(bx+a))^2\left(2\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1+cos(b*x+a))^2*(2*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1
/2)-cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2
^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b
*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*EllipticE(((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b
*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)-EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-cos(
b*x+a)*2^(1/2)+2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a
))^(3/2)/sin(b*x+a)^6*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(a+bx))^{\frac{3}{2}}\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))**(3/2),x)
[Out] Integral((d*tan(a + b*x))**(3/2)*csc(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(a+b\,x))^{3/2}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2)/sin(a + b*x),x)
[Out] int((d*tan(a + b*x))^(3/2)/sin(a + b*x), x)
```

3.72 $\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=102

$$-\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{4d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] -4*d^2*cos(b*x+a)/b/(d*tan(b*x+a))^(1/2)+4*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/
sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b
*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2*d*csc(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2673, 2681, 2650, 2652, 2719}

$$-\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{4d^2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-4*d^2*Cos[a + b*x])/(b*Sqrt[d*Tan[a + b*x]]) - (4*d^2*EllipticE[a - Pi/4
+ b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]]) + (
2*d*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
```

```
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} + \left(2d^2\right)\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} + \frac{\left(2d^2\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+b}}{\sin^{\frac{3}{2}}(a+bx)}}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx}}$$

$$= -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{\left(4d^2\sqrt{\sin(a\right.}}{}$$

$$= -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a+bx)\sqrt{d\tan(a+bx)}}{b} - \frac{(4d^2\sin(a+}{\sqrt{\sin(2a}}$$

$$= -\frac{4d^2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{4d^2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} + \frac{2d\csc(a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.62, size = 71, normalized size = 0.70

$$-\frac{2\cos(a+bx)\left(-6+3\csc^2(a+bx)+4\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

[Out] (-2*Cos[a + b*x]*(-6 + 3*Csc[a + b*x]^2 + 4*Hypergeometric2F1[3/4, 3/2, 7/4
, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*(d*Tan[a + b*x])^(3/2))/(3*b)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(119) = 238$.
time = 0.38, size = 494, normalized size = 4.84

method	result
default	$-\frac{\left(-4\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/b*(-4*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),
1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*cos(b*x+a)
*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+c
os(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*(
(cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-4*EllipticE(((1-cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(
b*x+a))^(1/2)+2*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*
2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*cos(b*x+a)*2^(
1/2)-2^(1/2))*cos(b*x+a)*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)^2*2^(1/
2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^3, x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(a+b\,x))^{3/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*tan(a + b*x))^(3/2)/sin(a + b*x)^3, x)
```

3.73 $\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=277

$$\frac{77d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b}-\frac{77d^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b}-\frac{77d^{5/2}\log\left(\sqrt{d}\right.}{}$$

```
[Out] 77/64*d^(5/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-77/6
4*d^(5/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-77/128*d
^(5/2)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2
)+77/128*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a)
)/b*2^(1/2)+77/48*d*(d*tan(b*x+a))^(3/2)/b-11/16*cos(b*x+a)^2*(d*tan(b*x+a)
)^(7/2)/b/d-1/4*cos(b*x+a)^4*(d*tan(b*x+a))^(11/2)/b/d^3
```

Rubi [A]
time = 0.14, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2671, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{77d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b}-\frac{77d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}\,b}-\frac{77d^{5/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,b}+\frac{77d^{5/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,b}-\frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3}+\frac{77d(d\tan(a+bx))^{3/2}}{48b}-\frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (77*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]
*b) - (77*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*S
qrt[2]*b) - (77*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d
*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (77*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a +
 b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b) + (77*d*(d*Tan[a + b*
x])^(3/2))/(48*b) - (11*Cos[a + b*x]^2*(d*Tan[a + b*x])^(7/2))/(16*b*d) - (
Cos[a + b*x]^4*(d*Tan[a + b*x])^(11/2))/(4*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^4(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\text{Subst}\left(\int \frac{x^{13/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{11/2}}{4bd^3} + \frac{(11d)\text{Subst}\left(\int \frac{x^{9/2}}{(d^2+x^2)^2}\,dx, x, d\,\text{ta}\right.}{8b}$$

$$= -\frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{11/}}{4bd^3}$$

$$= \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a}{}$$

$$= \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a}{}$$

$$= \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a}{}$$

$$= \frac{77d(d\tan(a+bx))^{3/2}}{48b} - \frac{11\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{16bd} - \frac{\cos^4(a}{}$$

$$= -\frac{77d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}\,b} + \frac{77}{}$$

$$= \frac{77d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b} - \frac{77d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{32\sqrt{2}}$$

Mathematica [A]

time = 0.61, size = 142, normalized size = 0.51

$$\frac{d\left(128+204\cos^2(a+bx)+231\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\cot(a+bx)\csc(a+bx)\sqrt{\sin(2(a+bx))}+231\cot(a+bx)\csc(a+bx)\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}-6\cot(a+bx)\sin(4(a+bx))\right)(d\tan(a+bx))^{3/2}}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (d*(128 + 204*Cos[a + b*x]^2 + 231*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Cot[
a + b*x]*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]] + 231*Cot[a + b*x]*Csc[a + b*x
]*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a +
 b*x)]] - 6*Cot[a + b*x]*Sin[4*(a + b*x)])*(d*Tan[a + b*x])^(3/2))/(192*b)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 590, normalized size = 2.13

method	result
default	$-\frac{(-1+\cos(bx+a))\left(231i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192/b*(-1+cos(b*x+a))*(231*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b
*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x
+a))^(1/2)*cos(b*x+a)-231*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)*cos(b*x+a)+24*2^(1/2)*cos(b*x+a)^5+231*EllipticPi(((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+
a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b
*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)+231*EllipticPi(((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)*cos(b*x+a)-24*cos(b*x+a)^4*2^(1/2)-114*cos(b*x+a)^3*2^(1
/2)+114*cos(b*x+a)^2*2^(1/2)-64*cos(b*x+a)*2^(1/2)+64*2^(1/2))*cos(b*x+a)*(
cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^5*2^(1/2)
```

Maxima [A]
time = 0.49, size = 240, normalized size = 0.87

$$-\frac{231\,d^8\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-256\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}d^6-\frac{24\left(19\left(d\tan(bx+a)\right)^{\frac{7}{2}}d^8+15\left(d\tan(bx+a)\right)^{\frac{3}{2}}d^{10}\right)}{d^4\tan(bx+a)^4+2\,d^4\tan(bx+a)^2+d^4}}{384\,bd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/384*(231*d^8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*t
an(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
```

```
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b
*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 256*(d*tan(b
*x + a))^(3/2)*d^6 - 24*(19*(d*tan(b*x + a))^(7/2)*d^8 + 15*(d*tan(b*x + a)
)^(3/2)*d^10)/(d^4*tan(b*x + a)^4 + 2*d^4*tan(b*x + a)^2 + d^4))/(b*d^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1997 vs. $2(213) = 426$.
time = 64.93, size = 1997, normalized size = 7.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(924*sqrt(2)*(d^10/b^4)^(1/4)*b*arctan((sqrt(d^16 + 4*sqrt(d^10/b^4)
*b^2*d^11*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*co
s(b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d^8*cos(b*x + a)*sin(b*x + a))*
sqrt(d*sin(b*x + a)/cos(b*x + a)))*(2*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(
d^10/b^4)*b^2*d^3 + (sqrt(2)*(d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x +
a) + sqrt(2)*(d^10/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b
*x + a))) - (sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10
/b^4)^(3/4)*b^3*d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x
+ a)))/(2*d^16*cos(b*x + a)^2 - d^16))*cos(b*x + a) + 924*sqrt(2)*(d^10/b^4
)^(1/4)*b*arctan(-(sqrt(d^16 + 4*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b
*x + a) + 2*(sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10
/b^4)^(3/4)*b^3*d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x
+ a)))*(2*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(d^10/b^4)*b^2*d^3 - (sqrt(2)
*(d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^10/b^4)^(3/4
)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + (sqrt(2)*(d^10/b
^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d^8*cos(b*x
+ a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(2*d^16*cos(b*x + a)^
2 - d^16))*cos(b*x + a) - 924*sqrt(2)*(d^10/b^4)^(1/4)*b*arctan(-1/2*(2*d^1
6*sin(b*x + a) - sqrt(d^16 + 4*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x
 + a) + 2*(sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b
^4)^(3/4)*b^3*d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x +
a)))*(sqrt(2)*(d^10/b^4)^(1/4)*b*d^5*sin(b*x + a) + sqrt(2)*(d^10/b^4)^(3/4
)*b^3*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^11*cos(b*x
 + a)^3 - b^2*d^11*cos(b*x + a))*sqrt(d^10/b^4) + (sqrt(2)*(d^10/b^4)^(1/4)
*b*d^13*sin(b*x + a) + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d^8*cos(b*x + a))*sqrt(
d*sin(b*x + a)/cos(b*x + a)))/((2*d^16*cos(b*x + a)^2 - d^16)*sin(b*x + a))
)*cos(b*x + a) - 924*sqrt(2)*(d^10/b^4)^(1/4)*b*arctan(1/2*(2*d^16*sin(b*x
+ a) + sqrt(d^16 + 4*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x + a) - 2*
(sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/4)*
b^3*d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(sqrt
```

```
(2)*(d^10/b^4)^(1/4)*b*d^5*sin(b*x + a) + sqrt(2)*(d^10/b^4)^(3/4)*b^3*cos(
b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^11*cos(b*x + a)^3 -
b^2*d^11*cos(b*x + a))*sqrt(d^10/b^4) - (sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*si
n(b*x + a) + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d^8*cos(b*x + a))*sqrt(d*sin(b*x
+ a)/cos(b*x + a)))/((2*d^16*cos(b*x + a)^2 - d^16)*sin(b*x + a)))*cos(b*x
+ a) + 231*sqrt(2)*(d^10/b^4)^(1/4)*b*cos(b*x + a)*log(208422380089*d^16 +
833689520356*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x + a) + 4168447601
78*(sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/
4)*b^3*d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) -
231*sqrt(2)*(d^10/b^4)^(1/4)*b*cos(b*x + a)*log(208422380089*d^16 + 8336895
20356*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x + a) - 416844760178*(sqr
t(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*
d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 231*sqr
t(2)*(d^10/b^4)^(1/4)*b*cos(b*x + a)*log(208422380089/16*d^16 + 20842238008
9/4*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x + a) + 208422380089/8*(sqr
t(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*
d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 231*sqr
t(2)*(d^10/b^4)^(1/4)*b*cos(b*x + a)*log(208422380089/16*d^16 + 20842238008
9/4*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x + a) - 208422380089/8*(sqr
t(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*
d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 32*(12*
d^2*cos(b*x + a)^4 - 57*d^2*cos(b*x + a)^2 - 32*d^2)*sqrt(d*sin(b*x + a)/co
s(b*x + a))*sin(b*x + a))/(b*cos(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 278, normalized size = 1.00

$$-\frac{1}{384}d^2\left(\frac{462\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{|d|}}\right)}{bd}+\frac{462\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{|d|}}\right)}{bd}-\frac{231\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan\left(bx+a\right)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{bd}+\frac{231\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan\left(bx+a\right)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{|d|}+|d|\right)}{bd}-\frac{256\sqrt{d\tan\left(bx+a\right)}\tan\left(bx+a\right)}{b}-\frac{24\left(19\sqrt{d\tan\left(bx+a\right)}d^4\tan\left(bx+a\right)^3+15\sqrt{d\tan\left(bx+a\right)}d^4\tan\left(bx+a\right)\right)}{\left(d^2\tan\left(bx+a\right)^2+d^2\right)^2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -1/384*d^2*(462*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d
)) + 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 462*sqrt(2)*abs(d)^(3/2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(a
```

```
bs(d)))/(b*d) - 231*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(
d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) + 231*sqrt(2)*abs(d)^(3/2)*log
(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d)
 - 256*sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 24*(19*sqrt(d*tan(b*x + a))*d^
4*tan(b*x + a)^3 + 15*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a))/((d^2*tan(b*x
+ a)^2 + d^2)^2*b))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,x\right)^4\,{\left(d\,\tan\left(a+b\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2), x)
```

3.74 $\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=247

$$\frac{7d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}-\frac{7d^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}-\frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

```
[Out] 7/8*d^(5/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-7/8*d^
(5/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-7/16*d^(5/2)
*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)+7/16
*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1
/2)+7/6*d*(d*tan(b*x+a))^(3/2)/b-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(7/2)/b/d
```

Rubi [A]
time = 0.12, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2671, 294, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{7d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}-\frac{7d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}\,b}-\frac{7d^{5/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,b}+\frac{7d^{5/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,b}+\frac{7d(d\tan(a+bx))^{3/2}}{6b}-\frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (7*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b
) - (7*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[
2]*b) - (7*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[
a + b*x]]])/(8*Sqrt[2]*b) + (7*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] +
 Sqrt[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) + (7*d*(d*Tan[a + b*x])^(3/2)
)/(6*b) - (Cos[a + b*x]^2*(d*Tan[a + b*x])^(7/2))/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{d\text{Subst}\left(\int \frac{x^{9/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} + \frac{(7d)\text{Subst}\left(\int \frac{x^{5/2}}{d^2+x^2}\,dx, x, d\tan(a+ \ldots\right.}{4b}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} - \frac{(7d^3)\,\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} - \frac{(7d^3)\,\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} + \frac{(7d^3)\,\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= \frac{7d(d\tan(a+bx))^{3/2}}{6b} - \frac{\cos^2(a+bx)(d\tan(a+bx))^{7/2}}{2bd} - \frac{(7d^{5/2})\,\text{Subs}\ldots}{\ldots}$$

$$= -\frac{7d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\,\tan(a+bx)-\sqrt{2}\,\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}\,b} + \frac{7d^{5/2}\ldots}{\ldots}$$

$$= \frac{7d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b} - \frac{7d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d\ldots}}{\ldots}\right)}{4\sqrt{2}\,b}$$

Mathematica [A]
time = 0.42, size = 126, normalized size = 0.51

$$\frac{d\left(16+12\cos^2(a+bx)+21\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\cot(a+bx)\csc(a+bx)\sqrt{\sin(2(a+bx))}+21\cot(a+bx)\csc(a+bx)\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}\right)(d\tan(a+bx))^{3/2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (d*(16 + 12*Cos[a + b*x]^2 + 21*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Cot[a +
 b*x]*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]] + 21*Cot[a + b*x]*Csc[a + b*x]*Lo
g[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x
)]])*(d*Tan[a + b*x])^(3/2))/(24*b)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.28, size = 564, normalized size = 2.28

method	result
default	$-\frac{(-1+\cos(bx+a))\left(21i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/b*(-1+cos(b*x+a))*(21*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(
1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a)
)/sin(b*x+a))^(1/2)-21*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin
(b*x+a))^(1/2)+21*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*cos
(b*x+a)+21*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)
-6*cos(b*x+a)^3*2^(1/2)+6*cos(b*x+a)^2*2^(1/2)-8*cos(b*x+a)*2^(1/2)+8*2^(1/
2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^
5*2^(1/2)
```

Maxima [A]
time = 0.50, size = 209, normalized size = 0.85

$$-\frac{21\,d^{6}\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{24\,(d\tan(bx+a))^{\frac{3}{2}}d^{6}}{d^{2}\tan(bx+a)^{2}+d^{2}}-32\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}d^{4}}{48\,bd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/48*(21*d^6*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a)
```

```
+ sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) - 24*(d*tan(b*x
+ a))^(3/2)*d^6/(d^2*tan(b*x + a)^2 + d^2) - 32*(d*tan(b*x + a))^(3/2)*d^4)
/(b*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1984 vs. $2(187) = 374$.
time = 65.68, size = 1984, normalized size = 8.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(84*sqrt(2)*(d^10/b^4)^(1/4)*b*arctan((sqrt(d^16 + 4*sqrt(d^10/b^4)*b
^2*d^11*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*cos(
b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d^8*cos(b*x + a)*sin(b*x + a))*sq
rt(d*sin(b*x + a)/cos(b*x + a)))*(2*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(d^
10/b^4)*b^2*d^3 + (sqrt(2)*(d^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a)
 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x
 + a))) - (sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b
^4)^(3/4)*b^3*d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x +
a)))/(2*d^16*cos(b*x + a)^2 - d^16))*cos(b*x + a) + 84*sqrt(2)*(d^10/b^4)^(
1/4)*b*arctan(-(sqrt(d^16 + 4*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x
+ a) + 2*(sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^
4)^(3/4)*b^3*d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a
)))*(2*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(d^10/b^4)*b^2*d^3 - (sqrt(2)*(d
^10/b^4)^(1/4)*b*d^5*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^10/b^4)^(3/4)*b
^3*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + (sqrt(2)*(d^10/b^4)
^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d^8*cos(b*x + a
)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(2*d^16*cos(b*x + a)^2 -
 d^16))*cos(b*x + a) - 84*sqrt(2)*(d^10/b^4)^(1/4)*b*arctan(-1/2*(2*d^16*si
n(b*x + a) - sqrt(d^16 + 4*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x + a
) + 2*(sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^4)^
(3/4)*b^3*d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))
*(sqrt(2)*(d^10/b^4)^(1/4)*b*d^5*sin(b*x + a) + sqrt(2)*(d^10/b^4)^(3/4)*b^
3*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^11*cos(b*x + a
)^3 - b^2*d^11*cos(b*x + a))*sqrt(d^10/b^4) + (sqrt(2)*(d^10/b^4)^(1/4)*b*d
^13*sin(b*x + a) + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d^8*cos(b*x + a))*sqrt(d*si
n(b*x + a)/cos(b*x + a)))/((2*d^16*cos(b*x + a)^2 - d^16)*sin(b*x + a)))*co
s(b*x + a) - 84*sqrt(2)*(d^10/b^4)^(1/4)*b*arctan(1/2*(2*d^16*sin(b*x + a)
+ sqrt(d^16 + 4*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x + a) - 2*(sqrt
(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d
^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(sqrt(2)*(
d^10/b^4)^(1/4)*b*d^5*sin(b*x + a) + sqrt(2)*(d^10/b^4)^(3/4)*b^3*cos(b*x +
```

```
a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^11*cos(b*x + a)^3 - b^2*d
^11*cos(b*x + a))*sqrt(d^10/b^4) - (sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*sin(b*x
 + a) + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d^8*cos(b*x + a))*sqrt(d*sin(b*x + a)/
cos(b*x + a)))/((2*d^16*cos(b*x + a)^2 - d^16)*sin(b*x + a)))*cos(b*x + a)
+ 21*sqrt(2)*(d^10/b^4)^(1/4)*b*cos(b*x + a)*log(117649*d^16 + 470596*sqrt(
d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x + a) + 235298*(sqrt(2)*(d^10/b^4)^(
1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d^8*cos(b*x + a)*
sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) - 21*sqrt(2)*(d^10/b^4)^(1
/4)*b*cos(b*x + a)*log(117649*d^16 + 470596*sqrt(d^10/b^4)*b^2*d^11*cos(b*x
 + a)*sin(b*x + a) - 235298*(sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2
 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b
*x + a)/cos(b*x + a))) + 21*sqrt(2)*(d^10/b^4)^(1/4)*b*cos(b*x + a)*log(117
649/16*d^16 + 117649/4*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x + a) +
117649/8*(sqrt(2)*(d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^
4)^(3/4)*b^3*d^8*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a
))) - 21*sqrt(2)*(d^10/b^4)^(1/4)*b*cos(b*x + a)*log(117649/16*d^16 + 11764
9/4*sqrt(d^10/b^4)*b^2*d^11*cos(b*x + a)*sin(b*x + a) - 117649/8*(sqrt(2)*(
d^10/b^4)^(1/4)*b*d^13*cos(b*x + a)^2 + sqrt(2)*(d^10/b^4)^(3/4)*b^3*d^8*co
s(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + 32*(3*d^2*cos
(b*x + a)^2 + 4*d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))*sin(b*x + a))/(b*cos
(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.52, size = 252, normalized size = 1.02

$$\frac{1}{48}\left(\frac{24\sqrt{d\tan(bx+a)}\,d^2\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b}-\frac{42\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd}-\frac{42\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd}+\frac{21\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{bd}-\frac{21\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{bd}+\frac{32\sqrt{d\tan(bx+a)}\tan(bx+a)}{b}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*(24*sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)*
b) - 42*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*s
qrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) - 42*sqrt(2)*abs(d)^(3/2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(
b*d) + 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x
```

```
+ a))*sqrt(abs(d)) + abs(d))/(b*d) - 21*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x
+ a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) + 32*sqrt(
d*tan(b*x + a))*tan(b*x + a)/b)*d^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,x\right)^2\,{\left(d\,\tan(a+b\,x)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2), x)
```

3.75 $\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=20

$$\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] 2/3*d*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 30}

$$\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)(d\tan(a+bx))^{5/2}\,dx &= \frac{d\text{Subst}\left(\int \sqrt{x}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= \frac{2d(d\tan(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 20, normalized size = 1.00

$$\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.
time = 0.32, size = 38, normalized size = 1.90

method	result	size
default	$\frac{2\cos(bx+a)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{5}{2}}}{3b\sin(bx+a)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/b*cos(b*x+a)*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)
```

Maxima [A]
time = 0.27, size = 23, normalized size = 1.15

$$\frac{2\,(d\tan\left(bx+a\right))^{\frac{5}{2}}}{3\,b\tan\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(d*tan(b*x + a))^(5/2)/(b*tan(b*x + a))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(16) = 32$.
time = 0.40, size = 40, normalized size = 2.00

$$\frac{2\,d^2\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}\,\sin\left(bx+a\right)}{3\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*d^2*sqrt(d*sin(b*x + a)/cos(b*x + a))*sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 24, normalized size = 1.20

$$\frac{2\sqrt{d\tan\left(bx+a\right)}\,d^{2}\tan\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)/b
```

Mupad [B]
time = 2.48, size = 56, normalized size = 2.80

$$\frac{2\,d^{2}\,\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{d\,\sin\left(2\,a+2\,b\,x\right)}{\cos\left(2\,a+2\,b\,x\right)+1}}}{3\,b\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^2,x)
```

```
[Out] (2*d^2*sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)
)/(3*b*(cos(2*a + 2*b*x) + 1))
```

3.76 $\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=41

$$-\frac{2d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] -2*d^3/b/(d*tan(b*x+a))^(1/2)+2/3*d*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A]

time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 14}

$$\frac{2d(d\tan(a+bx))^{3/2}}{3b}-\frac{2d^3}{b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^3)/(b*Sqrt[d*Tan[a + b*x]]) + (2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(a+bx)(d\tan(a+bx))^{5/2}\,dx&=\frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{3/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\&=\frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{3/2}}+\sqrt{x}\right)\,dx,x,d\tan(a+bx)\right)}{b}\\&=-\frac{2d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 32, normalized size = 0.78

$$-\frac{2d(-1+3\cot^2(a+bx))\,(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d*(-1 + 3*Cot[a + b*x]^2)*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Maple [A]
time = 0.32, size = 50, normalized size = 1.22

method	result	size
default	$-\frac{2\left(4\left(\cos^2(bx+a)\right)-1\right)\cos(bx+a)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{5}{2}}}{3b\sin(bx+a)^3}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b*(4*cos(b*x+a)^2-1)*cos(b*x+a)*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*
x+a)^3
```

Maxima [A]
time = 0.26, size = 36, normalized size = 0.88

$$-\frac{2\,d^3\left(\frac{3}{\sqrt{d\tan(bx+a)}}-\frac{(d\tan(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*d^3*(3/sqrt(d*tan(b*x + a)) - (d*tan(b*x + a))^(3/2)/d^2)/b
```

Fricas [A]
time = 0.37, size = 58, normalized size = 1.41

$$-\frac{2\left(4\,d^2\cos(bx+a)^2-d^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{3\,b\cos(bx+a)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(4*d^2*cos(b*x + a)^2 - d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(
b*x + a)*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 42, normalized size = 1.02

$$\frac{2}{3}\,d^{2}\left(\frac{\sqrt{d\tan\left(bx+a\right)}\,\tan\left(bx+a\right)}{b}-\frac{3\,d}{\sqrt{d\tan\left(bx+a\right)}\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*d^2*(sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 3*d/(sqrt(d*tan(b*x + a))*b)
)
```

Mupad [**B**]
time = 2.72, size = 64, normalized size = 1.56

$$-\frac{4\,d^2\,\left(\sin\left(2\,a+2\,b\,x\right)+\sin\left(4\,a+4\,b\,x\right)\right)\,\sqrt{\frac{d\,\sin\left(2\,a+2\,b\,x\right)}{\cos\left(2\,a+2\,b\,x\right)+1}}}{3\,b\,{\sin\left(2\,a+2\,b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^4,x)
```

```
[Out] -(4*d^2*(sin(2*a + 2*b*x) + sin(4*a + 4*b*x))*((d*sin(2*a + 2*b*x))/(cos(2*
a + 2*b*x) + 1))^(1/2))/(3*b*sin(2*a + 2*b*x)^2)
```

3.77 $\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=63

$$-\frac{2d^5}{5b(d\tan(a+bx))^{5/2}}-\frac{4d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] -4*d^3/b/(d*tan(b*x+a))^(1/2)-2/5*d^5/b/(d*tan(b*x+a))^(5/2)+2/3*d*(d*tan(b
*x+a))^(3/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2671, 276}

$$-\frac{2d^5}{5b(d\tan(a+bx))^{5/2}}-\frac{4d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^5)/(5*b*(d*Tan[a + b*x])^(5/2)) - (4*d^3)/(b*Sqrt[d*Tan[a + b*x]]) +
(2*d*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^6(a+bx)(d\tan(a+bx))^{5/2}\,dx&=\frac{d\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{7/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\&=\frac{d\text{Subst}\left(\int \left(\frac{d^4}{x^{7/2}}+\frac{2d^2}{x^{3/2}}+\sqrt{x}\right)\,dx,x,d\tan(a+bx)\right)}{b}\\&=-\frac{2d^5}{5b(d\tan(a+bx))^{5/2}}-\frac{4d^3}{b\sqrt{d\tan(a+bx)}}+\frac{2d(d\tan(a+bx))^{3/2}}{3b}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 42, normalized size = 0.67

$$-\frac{2d(-5+3\cot^2(a+bx)\,(9+\csc^2(a+bx)))\,(d\tan(a+bx))^{3/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d*(-5 + 3*Cot[a + b*x]^2*(9 + Csc[a + b*x]^2))*(d*Tan[a + b*x])^(3/2))/
(15*b)
```

Maple [A]
time = 0.36, size = 60, normalized size = 0.95

method	result	size
default	$\frac{2\left(32\left(\cos^4(bx+a)\right)-40\left(\cos^2(bx+a)\right)+5\right)\cos(bx+a)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{5}{2}}}{15b\sin(bx+a)^5}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/b*(32*cos(b*x+a)^4-40*cos(b*x+a)^2+5)*cos(b*x+a)*(d*sin(b*x+a)/cos(b*x
+a))^(5/2)/sin(b*x+a)^5
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.89

$$\frac{2\,d^5\left(\frac{5\,(d\tan(bx+a))^{\frac{3}{2}}}{d^4}-\frac{3\left(10\,d^2\tan(bx+a)^2+d^2\right)}{(d\tan(bx+a))^{\frac{5}{2}}\,d^2}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*d^5*(5*(d*tan(b*x + a))^(3/2)/d^4 - 3*(10*d^2*tan(b*x + a)^2 + d^2)/((
d*tan(b*x + a))^(5/2)*d^2))/b
```

Fricas [A]
time = 0.39, size = 82, normalized size = 1.30

$$-\frac{2\left(32\,d^2\cos\left(bx+a\right)^4-40\,d^2\cos\left(bx+a\right)^2+5\,d^2\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{15\left(b\cos\left(bx+a\right)^3-b\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(32*d^2*cos(b*x + a)^4 - 40*d^2*cos(b*x + a)^2 + 5*d^2)*sqrt(d*sin(b*
x + a)/cos(b*x + a))/((b*cos(b*x + a)^3 - b*cos(b*x + a))*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.59, size = 70, normalized size = 1.11

$$\frac{2}{15}\,d^2\left(\frac{5\,\sqrt{d\tan\left(bx+a\right)}\,\tan\left(bx+a\right)}{b}-\frac{3\,\left(10\,d^3\tan\left(bx+a\right)^2+d^3\right)}{\sqrt{d\tan\left(bx+a\right)}\,bd^2\tan\left(bx+a\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*d^2*(5*sqrt(d*tan(b*x + a))*tan(b*x + a)/b - 3*(10*d^3*tan(b*x + a)^2
+ d^3)/(sqrt(d*tan(b*x + a))*b*d^2*tan(b*x + a)^2))
```

Mupad [**B**]
time = 5.46, size = 134, normalized size = 2.13

$$\frac{32\,d^2\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,2\mathrm{i}-\mathrm{e}^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}\,2\mathrm{i}-2\mathrm{i}\right)}{15\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^6,x)
```

```
[Out] (32*d^2*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*
(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*6i)*2i - ex
p(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(exp(a*2i + b*x
*2i) + 1))
```

3.78 $\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=137

$$\frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}}+\frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{5d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{4b}$$

```
[Out] 5/2*d^3*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)+d^3*sin(b*x+a)^3/b/(d*tan(b*x+a))
^(1/2)+5/4*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ell
ipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)
/b+2/3*d*sin(b*x+a)^3*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A]
time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2674, 2678, 2681, 2653, 2720}

$$\frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}}+\frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}}-\frac{5d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{4b}+\frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (5*d^3*Sin[a + b*x])/(2*b*Sqrt[d*Tan[a + b*x]]) + (d^3*Sin[a + b*x]^3)/(b*S
qrt[d*Tan[a + b*x]]) - (5*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqr
t[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(4*b) + (2*d*Sin[a + b*x]^3*(d*Ta
n[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
```

```
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sin^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \left(3d^2\right)\int \sin^3(a+bx)\sqrt{d\tan(a+$$

$$= \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{2}\left(5d^2\right)\int \mathrm{s}$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+}{3b}$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+}{3b}$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} + \frac{2d\sin^3(a+bx)(d\tan(a+}{3b}$$

$$= \frac{5d^3\sin(a+bx)}{2b\sqrt{d\tan(a+bx)}} + \frac{d^3\sin^3(a+bx)}{b\sqrt{d\tan(a+bx)}} - \frac{5d^2\csc(a+bx)F\left(a-\frac{\pi}{4}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.24, size = 153, normalized size = 1.12

$$-\frac{\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(120\sqrt[4]{-1}\cos(2(a+bx))F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)+(22+77\cos(2(a+bx))+22\cos(4(a+bx))-\cos(6(a+bx)))\sqrt{\sec^2(a+bx)}\sqrt{\tan(a+bx)}\right)(d\tan(a+bx))^{5/2}}{48b\tan^{\frac{3}{2}}(a+bx)\left(-1+\tan^2(a+bx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/48*(Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(120*(-1)^(1/4)*Cos[2*(a + b*x)]*E
llipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1] + (22 + 77*Cos[2*(a
+ b*x)] + 22*Cos[4*(a + b*x)] - Cos[6*(a + b*x)])*Sqrt[Sec[a + b*x]^2]*Sqrt
[Tan[a + b*x]])*(d*Tan[a + b*x])^(5/2))/(b*Tan[a + b*x]^(3/2)*(-1 + Tan[a +
 b*x]^2))
```

Maple [A]
time = 0.29, size = 246, normalized size = 1.80

method	result
default	$-\frac{(-1+\cos(bx+a))\left(2\sqrt{2}\,(\cos^5(bx+a))-15\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a}{\sin(b}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/b*(-1+cos(b*x+a))*(2*2^(1/2)*cos(b*x+a)^5-15*EllipticF(((1-cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)*sin(b*x+a)*cos(b*x+a)-2*cos(b*x+a)^4*2^(1/2)-13*cos(b*x+
a)^3*2^(1/2)+13*cos(b*x+a)^2*2^(1/2)-4*cos(b*x+a)*2^(1/2)+4*2^(1/2))*cos(b*
x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^6*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(b*x + a)^2 - d^2)*sqrt(d*tan(b*x + a))*sin(b*x + a)*tan(
b*x + a)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument Typeex
t_reduce Error: Bad Argument TypeEvaluation time: 9.84Done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^3\,{\left(d\,\tan(a+b\,x)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2), x)
```

3.79 $\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=108

$$\frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}}-\frac{5d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{6b}+\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] 5/3*d^3*sin(b*x+a)/b/(d*tan(b*x+a))^(1/2)+5/6*d^2*csc(b*x+a)*(sin(a+1/4*Pi+
b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*
b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*d*sin(b*x+a)*(d*tan(b*x+a))^(3/2)
/b
```

Rubi [A]
time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2674, 2678, 2681, 2653, 2720}

$$\frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}}-\frac{5d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{6b}+\frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (5*d^3*Sin[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) - (5*d^2*Csc[a + b*x]*Ellip
ticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(6*b)
+ (2*d*Sin[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
```

```
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sin(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{3}\left(5d^2\right)\int \sin(a+bx)\sqrt{d\tan(a+$$

$$= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{6}\left(5d^2\right)\int \mathrm{c}$$

$$= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{\left(5d^2\sqrt{\cos(}\right.}{}$$

$$= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\sin(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{6}\left(5d^2\csc(a\right.$$

$$= \frac{5d^3\sin(a+bx)}{3b\sqrt{d\tan(a+bx)}} - \frac{5d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sqrt{\sin(2a+2b}}{6b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.37, size = 133, normalized size = 1.23

$$-\frac{\cos(2(a+bx))\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(10\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)+(7+3\cos(2(a+bx)))\sqrt{\sec^2(a+bx)}\sqrt{\tan(a+bx)}\right)(d\tan(a+bx))^{5/2}}{6b\tan^{\frac{3}{2}}(a+bx)\left(-1+\tan^2(a+bx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/6*(Cos[2*(a + b*x)]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(10*(-1)^(1/4)*Ell
ipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1] + (7 + 3*Cos[2*(a + b*
```

```
x)])*Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(5/2))/(b*Ta
n[a + b*x]^(3/2)*(-1 + Tan[a + b*x]^2))
```

Maple [A]
time = 0.27, size = 220, normalized size = 2.04

method	result
default	$\frac{(-1+\cos(bx+a))\left(5\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/b*(-1+cos(b*x+a))*(5*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x
+a)*cos(b*x+a)+3*cos(b*x+a)^3*2^(1/2)-3*cos(b*x+a)^2*2^(1/2)+2*cos(b*x+a)*2
^(1/2)-2*2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/
2)/sin(b*x+a)^6*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*sin(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeEv
aluation time: 5.15Done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,{\left(d\,\tan(a+b\,x)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(d*tan(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)*(d*tan(a + b*x))^(5/2), x)
```

3.80 $\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=80

$$-\frac{d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] 1/3*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(
cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*
d*csc(b*x+a)*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2674, 2681, 2653, 2720}

$$\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/3*(d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*
Sqrt[d*Tan[a + b*x]])/b + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{3}d^2\int \csc(a+bx)\sqrt{d\tan(a+bx)}$$

$$= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{\left(d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx}\right.}{3\sqrt{\mathrm{si}}}$$

$$= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} - \frac{1}{3}\left(d^2\csc(a+bx)\sqrt{\sin(2a+2bx)}\right.$$

$$= -\frac{d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.44, size = 71, normalized size = 0.89

$$\frac{2d^2\cos(a+bx)\left(\sec^2(a+bx)-\,_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d^2*Cos[a + b*x]*(Sec[a + b*x]^2 - Hypergeometric2F1[1/4, 1/2, 5/4, -Tan
[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(95) = 190$.
time = 0.32, size = 192, normalized size = 2.40

method	result
default	$\frac{(-1+\cos(bx+a))\left(\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx-}{}}\right.}{3b\sin(bx+a)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*(-1+cos(b*x+a))*(EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a
)*cos(b*x+a)+cos(b*x+a)*2^(1/2)-2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin
(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^6*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.12, size = 101, normalized size = 1.26

$$\frac{\sqrt{i\,d}\,d^2\cos\left(bx+a\right)\mathrm{ellipticF}\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right),-1\right)+\sqrt{-i\,d}\,d^2\cos\left(bx+a\right)\mathrm{ellipticF}\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right),-1\right)+2\,d^2\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{3\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(I*d)*d^2*cos(b*x + a)*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1
) + sqrt(-I*d)*d^2*cos(b*x + a)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1
) + 2*d^2*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\tan(a+b\,x)\right)^{5/2}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x),x)
```

```
[Out] int((d*tan(a + b*x))^(5/2)/sin(a + b*x), x)
```

3.81 $\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=80

$$\frac{2d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

```
[Out] -2/3*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF
(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3
*d*csc(b*x+a)*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A]
time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2673, 2681, 2653, 2720}

$$\frac{2d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}+\frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqr
t[d*Tan[a + b*x]])/(3*b) + (2*d*Csc[a + b*x]*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(2d^2\right)\int \csc(a+bx)\sqrt{d\tan(a-}$$

$$= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{\left(2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+}\right.}{3\sqrt{s}}$$

$$= \frac{2d\csc(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(2d^2\csc(a+bx)\sqrt{\sin(2a+2bx}\right.$$

$$= \frac{2d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.38, size = 71, normalized size = 0.89

$$\frac{2d^2\cos(a+bx)\left(\sec^2(a+bx)+2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*d^2*Cos[a + b*x]*(Sec[a + b*x]^2 + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -T
an[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(95) = 190.
time = 0.35, size = 192, normalized size = 2.40

method	result
default	$-\frac{(-1+\cos(bx+a))\left(2\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos}{}}\right.}{3b\sin(bx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(-1+cos(b*x+a))*(2*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*
x+a)*cos(b*x+a)-cos(b*x+a)*2^(1/2)+2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*
sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^6*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.09, size = 101, normalized size = 1.26

$$-\frac{2\left(\sqrt{i\,d}\,d^{2}\cos\left(bx+a\right)\text{ellipticF}\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right),-1\right)+\sqrt{-i\,d}\,d^{2}\cos\left(bx+a\right)\text{ellipticF}\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right),-1\right)-d^{2}\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}\right)}{3\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(I*d)*d^2*cos(b*x + a)*ellipticF(cos(b*x + a) + I*sin(b*x + a), -
1) + sqrt(-I*d)*d^2*cos(b*x + a)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -
1) - d^2*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(a+b\,x))^{5/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^3,x)
[Out] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^3, x)
```

3.82 $\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=110

$$-\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{4d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3b}+\frac{2d\csc^3(a+bx)(}{3}$$

```
[Out] -4/3*d^3*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-4/3*d^2*csc(b*x+a)*(sin(a+1/4*Pi
+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2
*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b+2/3*d*csc(b*x+a)^3*(d*tan(b*x+a))^(3
/2)/b
```

Rubi [A]

time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2673, 2679, 2681, 2653, 2720}

$$-\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}}+\frac{4d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3b}+\frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-4*d^3*Csc[a + b*x])/(3*b*Sqrt[d*Tan[a + b*x]]) + (4*d^2*Csc[a + b*x]*Elli
pticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b)
 + (2*d*Csc[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
```

```
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^5(a+bx)(d\tan(a+bx))^{5/2}\,dx &= \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \left(2d^2\right)\int \csc^3(a+bx)\sqrt{d\tan(a\ldots}\\ &= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(4d^2\right)\ldots\\ &= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{\left(4d^2\sqrt{\ldots}\right.}{\ldots}\\ &= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^3(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(4d^2\,\mathrm{c}\ldots\right.\\ &= -\frac{4d^3\csc(a+bx)}{3b\sqrt{d\tan(a+bx)}} + \frac{4d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sqrt{\sin(2a+\ldots}}{3b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.54, size = 110, normalized size = 1.00

$$-\frac{2d\csc^3(a+bx)\left(\cos(2(a+bx))\sqrt{\sec^2(a+bx)}+2\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\sin(2(a+bx))\sqrt{\tan(a+bx)}\right)(d\tan(a+bx))^{3/2}}{3b\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*(d*Tan[a + b*x])^(5/2),x]
```

[Out] (-2*d*Csc[a + b*x]^3*(Cos[2*(a + b*x)]*Sqrt[Sec[a + b*x]^2] + 2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sin[2*(a + b*x)]*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(3*b*Sqrt[Sec[a + b*x]^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(121) = 242$.
time = 0.36, size = 316, normalized size = 2.87

method	result
default	$\frac{(-1+\cos(bx+a))^2 \left(4\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sin(bx+a)\,\mathrm{EllipticF}\left(\sqrt{\frac{1-c}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/3/b*(-1+cos(b*x+a))^2*(4*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^2+4*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*cos(b*x+a)-2*cos(b*x+a)^2*2^(1/2)+2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^8*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")

[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^5, x)

Fricas [C] Result contains complex when optimal does not.
time = 0.10, size = 160, normalized size = 1.45

$$-\frac{2\left(2\left(d^2\cos(bx+a)^3 - d^2\cos(bx+a)\right)\sqrt{i\,d}\,\mathrm{ellipticF}(\cos(bx+a) + i\,\sin(bx+a), -1) + 2\left(d^2\cos(bx+a)^3 - d^2\cos(bx+a)\right)\sqrt{-i\,d}\,\mathrm{ellipticF}(\cos(bx+a) - i\,\sin(bx+a), -1) - \left(2\,d^2\cos(bx+a)^2 - d^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\right)}{3\left(b\cos(bx+a)^3 - b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")

[Out] -2/3*(2*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*sqrt(I*d)*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1) + 2*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*sqr

```
t(-I*d)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1) - (2*d^2*cos(b*x + a)^
2 - d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a)^3 - b*cos(b*x +
 a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**5*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^5*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(a+b\,x))^{5/2}}{\sin\left(a+b\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^5,x)
```

```
[Out] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^5, x)
```

3.83 $\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx$

Optimal. Leaf size=140

$$-\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}}-\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}}+\frac{40d^2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a}{21b}$$

```
[Out] -40/21*d^3*csc(b*x+a)/b/(d*tan(b*x+a))^(1/2)-20/21*d^3*csc(b*x+a)^3/b/(d*ta
n(b*x+a))^(1/2)-40/21*d^2*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*
Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*
x+a))^(1/2)/b+2/3*d*csc(b*x+a)^5*(d*tan(b*x+a))^(3/2)/b
```

Rubi [A]
time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2673, 2679, 2681, 2653, 2720}

$$-\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}}-\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}}+\frac{40d^2\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{21b}+\frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^7*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-40*d^3*Csc[a + b*x])/(21*b*Sqrt[d*Tan[a + b*x]]) - (20*d^3*Csc[a + b*x]^3
)/(21*b*Sqrt[d*Tan[a + b*x]]) + (40*d^2*Csc[a + b*x]*EllipticF[a - Pi/4 + b
*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(21*b) + (2*d*Csc[a + b
*x]^5*(d*Tan[a + b*x])^(3/2))/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
```

```
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^7(a+bx)(d\tan(a+bx))^{5/2}\,dx = \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{3}\left(10d^2\right)\int \csc^5(a+bx)\sqrt{d\tan(}$$

$$= -\frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^5(a+bx)(d\tan(a+bx))^{3/2}}{3b} + \frac{1}{7}(20d$$

$$= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}} - \frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^5(a+bx)(d\,\mathrm{ta}}{3b}$$

$$= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}} - \frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^5(a+bx)(d\,\mathrm{ta}}{3b}$$

$$= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}} - \frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{2d\csc^5(a+bx)(d\,\mathrm{ta}}{3b}$$

$$= -\frac{40d^3\csc(a+bx)}{21b\sqrt{d\tan(a+bx)}} - \frac{20d^3\csc^3(a+bx)}{21b\sqrt{d\tan(a+bx)}} + \frac{40d^2\csc(a+bx)F(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.79, size = 130, normalized size = 0.93

$$-\frac{d^2\csc(a+bx)\left(\left(1+10\cos(2(a+bx))-5\cos(4(a+bx))\right)\csc^3(a+bx)\sec(a+bx)\sqrt{\sec^2(a+bx)}+80\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\sqrt{\tan(a+bx)}\right)\sqrt{d\tan(a+bx)}}{21b\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^7*(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/21*(d^2*Csc[a + b*x]*((1 + 10*Cos[2*(a + b*x)] - 5*Cos[4*(a + b*x)])*Csc
[a + b*x]^3*Sec[a + b*x]*Sqrt[Sec[a + b*x]^2] + 80*(-1)^(1/4)*EllipticF[I*A
rcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Tan[a + b*x]])*Sqrt[d*Tan[a
 + b*x]])/(b*Sqrt[Sec[a + b*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(147) = 294$.
time = 0.42, size = 571, normalized size = 4.08

method	result
default	$-\frac{(-1+\cos(bx+a))^2\left(40\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/b*(-1+cos(b*x+a))^2*(40*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*co
s(b*x+a)^4*sin(b*x+a)+40*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*
cos(b*x+a)^3*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2),1/2*2^(1/2))-40*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b
*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*c
os(b*x+a)^2-40*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2
^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b
*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*cos(b*
x+a)-20*cos(b*x+a)^4*2^(1/2)+30*cos(b*x+a)^2*2^(1/2)-7*2^(1/2))*cos(b*x+a)*
(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(5/2)/sin(b*x+a)^10*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^7, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.11, size = 207, normalized size = 1.48

$$-\frac{2\left(20\left(d^2\cos(bx+a)^5-2\,d^2\cos(bx+a)^3+d^2\cos(bx+a)\right)\sqrt{i\,d}\,\mathrm{ellipticF}(\cos(bx+a)+i\,\sin(bx+a),-1)+20\left(d^2\cos(bx+a)^5-2\,d^2\cos(bx+a)^3+d^2\cos(bx+a)\right)\sqrt{-i\,d}\,\mathrm{ellipticF}(\cos(bx+a)-i\,\sin(bx+a),-1)-\left(20\,d^2\cos(bx+a)^4-30\,d^2\cos(bx+a)^2+7\,d^2\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\right)}{21\left(b\cos(bx+a)^5-2\,b\cos(bx+a)^3+b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/21*(20*(d^2*cos(b*x + a)^5 - 2*d^2*cos(b*x + a)^3 + d^2*cos(b*x + a))*sq
rt(I*d)*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1) + 20*(d^2*cos(b*x + a)
^5 - 2*d^2*cos(b*x + a)^3 + d^2*cos(b*x + a))*sqrt(-I*d)*ellipticF(cos(b*x
+ a) - I*sin(b*x + a), -1) - (20*d^2*cos(b*x + a)^4 - 30*d^2*cos(b*x + a)^2
 + 7*d^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a)^5 - 2*b*cos(b*
x + a)^3 + b*cos(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**7*(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^7*(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(5/2)*csc(b*x + a)^7, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,\tan\left(a+b\,x\right)\right)}^{5/2}}{{\sin\left(a+b\,x\right)}^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^7,x)
```

```
[Out] int((d*tan(a + b*x))^(5/2)/sin(a + b*x)^7, x)
```

3.84 $\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=257

$$-\frac{5\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b\sqrt{d}}+\frac{5\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b\sqrt{d}}-\frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+\right.}{64\sqrt{}}$$

```
[Out] -5/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)+5/64
*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)-5/128*ln(
d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)/d^(1/2)+
5/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)
/d^(1/2)-5/16*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d-1/4*cos(b*x+a)^4*(d*tan
(b*x+a))^(5/2)/b/d^3
```

Rubi [A]
time = 0.12, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2671, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{5\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b\sqrt{d}}+\frac{5\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}\,b\sqrt{d}}-\frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3}-\frac{5\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,b\sqrt{d}}+\frac{5\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,b\sqrt{d}}-\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-5*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*Sqrt[
d]) + (5*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
Sqrt[d]) - (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*Sqrt[d]) + (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*Sqrt[d]) - (5*Cos[a + b*x]^2*Sqrt
[d*Tan[a + b*x]])/(16*b*d) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(5/2))/(4*b*d
^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{d\text{Subst}\left(\int \frac{x^{7/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{(5d)\text{Subst}\left(\int \frac{x^{3/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{(5d)\text{Subst}\Big(\int$$

$$= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{(5d)\text{Subst}\Big(\int$$

$$= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{5\text{Subst}\Big(\int \frac{d-}{d^2}$$

$$= -\frac{5\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{5/2}}{4bd^3} + \frac{5\text{Subst}\Big(\int \frac{}{d-}$$

$$= -\frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}\,b\sqrt{d}} + \frac{5\log\Big(\sqrt{d}+\sqrt{d}\tan}{}$$

$$= -\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b\sqrt{d}} + \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,b\sqrt{d}} - \frac{5}{}$$

Mathematica [A]
time = 0.76, size = 122, normalized size = 0.47

$$\frac{\sec(a+bx)\left(-7\sin(a+bx)-5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}+5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}-6\sin(3(a+bx))+\sin(5(a+bx))\right)}{64b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Sec[a + b*x]*(-7*Sin[a + b*x] - 5*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt
[Sin[2*(a + b*x)]] + 5*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*
```

```
x)]]]*Sqrt[Sin[2*(a + b*x)]] - 6*Sin[3*(a + b*x)] + Sin[5*(a + b*x)]))/(64*
b*Sqrt[d*Tan[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 688, normalized size = 2.68

method	result
default	$\frac{(-1+\cos(bx+a))\left(-5i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sin(bx+a)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a}{\sin(bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/b*(-1+cos(b*x+a))*(-5*I*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1
/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*sin(b*x+a)+5*I*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*El
lipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)*sin(b*x+a)+8*2^(1/2)*cos(b*x+a)^5+10*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(
b*x+a))^(1/2)*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2),1/2*2^(1/2))-5*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+
a))^(1/2)-5*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2
)-8*cos(b*x+a)^4*2^(1/2)-18*cos(b*x+a)^3*2^(1/2)+18*cos(b*x+a)^2*2^(1/2))*(
cos(b*x+a)+1)^2/sin(b*x+a)^3/(d*sin(b*x+a)/cos(b*x+a))^(1/2)/cos(b*x+a)*2^(
1/2)
```

Maxima [A]
time = 0.48, size = 220, normalized size = 0.86

$$\frac{10\sqrt{2}\,d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+10\sqrt{2}\,d^{\frac{9}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+5\sqrt{2}\,d^{\frac{9}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)-5\sqrt{2}\,d^{\frac{9}{2}}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)-\frac{8\left(9\,(d\tan(bx+a))^{\frac{5}{2}}d^{6}+5\sqrt{d\tan(bx+a)}\,d^{8}\right)}{d^{4}\tan(bx+a)^{4}+2\,d^{4}\tan(bx+a)^{2}+d^{4}}}{128\,bd^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/128*(10*sqrt(2)*d^(9/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*ta
n(b*x + a)))/sqrt(d)) + 10*sqrt(2)*d^(9/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 5*sqrt(2)*d^(9/2)*log(d*tan(b*x +
```

```
 a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 5*sqrt(2)*d^(9/2)*log(d*t
an(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*(9*(d*tan(b*x +
 a))^(5/2)*d^6 + 5*sqrt(d*tan(b*x + a))*d^8)/(d^4*tan(b*x + a)^4 + 2*d^4*ta
n(b*x + a)^2 + d^4))/(b*d^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1456 vs. $2(197) = 394$.
time = 38.67, size = 1456, normalized size = 5.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/256*(10*sqrt(2)*b*d*(1/(b^4*d^2))^(1/4)*arctan(1/2*(sqrt(4*b^2*d*sqrt(1/
(b^4*d^2))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)
*cos(b*x + a)^2 + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*
sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*((sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*
sin(b*x + a) + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x +
 a)/cos(b*x + a)) + 2*sin(b*x + a)) - (sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*si
n(b*x + a) - sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x + a
)/cos(b*x + a)))/sin(b*x + a)) + 10*sqrt(2)*b*d*(1/(b^4*d^2))^(1/4)*arctan(
1/2*(sqrt(4*b^2*d*sqrt(1/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*
b^3*d*(1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*co
s(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*((sqrt(2)*b
^3*d*(1/(b^4*d^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b
*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 2*sin(b*x + a)) - (sqrt(2)*b^3
*d*(1/(b^4*d^2))^(3/4)*sin(b*x + a) - sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x
 + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/sin(b*x + a)) + 10*sqrt(2)*b*d*(1
/(b^4*d^2))^(1/4)*arctan(1/2*((sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*sin(b*x +
a) + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(4*b^2*d*sqrt(1/(b^4*d
^2))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*cos(b
*x + a)^2 + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d
*sin(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + (sqrt(
2)*b^3*d*(1/(b^4*d^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*c
os(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d*cos(b*x + a)^3 -
b^2*d*cos(b*x + a))*sqrt(1/(b^4*d^2)) + 2*sin(b*x + a))/((2*cos(b*x + a)^2
- 1)*sin(b*x + a))) + 10*sqrt(2)*b*d*(1/(b^4*d^2))^(1/4)*arctan(1/2*((sqrt(
2)*b^3*d*(1/(b^4*d^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*c
os(b*x + a))*sqrt(4*b^2*d*sqrt(1/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) - 2*(
sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*(1/(b^4*d^2))^
(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*sqr
t(d*sin(b*x + a)/cos(b*x + a)) + (sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*sin(b*x
 + a) + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos
(b*x + a)) + 4*(b^2*d*cos(b*x + a)^3 - b^2*d*cos(b*x + a))*sqrt(1/(b^4*d^2)
```

```
) - 2*sin(b*x + a))/((2*cos(b*x + a)^2 - 1)*sin(b*x + a))) - 5*sqrt(2)*b*d*
(1/(b^4*d^2))^(1/4)*log(4*b^2*d*sqrt(1/(b^4*d^2))*cos(b*x + a)*sin(b*x + a)
 + 2*(sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*(1/(b^4*
d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) +
1) + 5*sqrt(2)*b*d*(1/(b^4*d^2))^(1/4)*log(4*b^2*d*sqrt(1/(b^4*d^2))*cos(b*
x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 +
 sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x +
a)/cos(b*x + a)) + 1) - 16*(4*cos(b*x + a)^4 - 9*cos(b*x + a)^2)*sqrt(d*sin
(b*x + a)/cos(b*x + a)))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)**4/sqrt(d*tan(a + b*x)), x)
```

Giac [A]
time = 0.53, size = 246, normalized size = 0.96

$$\frac{5\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd}+\frac{5\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd}+\frac{5\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{128\,bd}-\frac{5\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{128\,bd}-\frac{9\sqrt{d\tan(bx+a)}\,d^3\tan(bx+a)^2+5\sqrt{d\tan(bx+a)}\,d^3}{16\left(d^2\tan(bx+a)^2+d^2\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 5/64*sqrt(2)*sqrt(abs(d))*arctan(-1
/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b
*d) + 5/128*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*
x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 5/128*sqrt(2)*sqrt(abs(d))*log(d*tan
(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 1/1
6*(9*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)^2 + 5*sqrt(d*tan(b*x + a))*d^3)/
((d^2*tan(b*x + a)^2 + d^2)^2*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(a+b\,x\right)}^4}{\sqrt{d\,\tan\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/(d*tan(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^4/(d*tan(a + b*x))^(1/2), x)
```

3.85 $\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=227

$$-\frac{\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b\sqrt{d}}+\frac{\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b\sqrt{d}}-\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)\right.}{8\sqrt{2}\,b}$$

```
[Out] -1/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)+1/8*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)/d^(1/2)-1/16*ln(d^(
1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)/d^(1/2)+1/1
6*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)/d^(
1/2)-1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d
```

Rubi [A]

time = 0.11, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2671, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b\sqrt{d}}+\frac{\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}\,b\sqrt{d}}-\frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,b\sqrt{d}}+\frac{\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,b\sqrt{d}}-\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/4*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*Sqrt[d])
 + ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*Sqrt[d])
 - Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(8*Sq
rt[2]*b*Sqrt[d]) + Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[
a + b*x]]]/(8*Sqrt[2]*b*Sqrt[d]) - (Cos[a + b*x]^2*Sqrt[d*Tan[a + b*x]])/(2
*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{d\text{Subst}\left(\int \frac{x^{3/2}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\tan(a+bx)\right)}{4b}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{d\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2b}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4b} + \text{Subst}$$

$$= -\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}\,x+x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{8b}$$

$$= -\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}\,b\sqrt{d}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a\right.}{8}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b\sqrt{d}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b\sqrt{d}} - \frac{\log}{}$$

Mathematica [A]
time = 0.67, size = 109, normalized size = 0.48

$$-\frac{\sec(a+bx)\left(\sin(a+bx)+\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}+\sin(3(a+bx))\right)}{8b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/8*(Sec[a + b*x]*(Sin[a + b*x] + ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt
[Sin[2*(a + b*x)]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)
]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)]))/(b*Sqrt[d*Tan[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.33, size = 662, normalized size = 2.92

method	result
default	$\frac{(-1+\cos(bx+a))\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sin(bx+a)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(b}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(-1+cos(b*x+a))*(I*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*
EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*sin(b*x+a)-I*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*Elliptic
Pi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(
b*x+a)-EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*(
(-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*EllipticF(((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-EllipticPi(((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)*((-1+co
s(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b*x+a)^3*2^(1/2)+2*cos(b*x
+a)^2*2^(1/2))*(cos(b*x+a)+1)^2/sin(b*x+a)^3/(d*sin(b*x+a)/cos(b*x+a))^(1/2
)/cos(b*x+a)*2^(1/2)
```

Maxima [A]
time = 0.48, size = 188, normalized size = 0.83

$$\frac{2\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}\,d^{\frac{5}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)-\sqrt{2}\,d^{\frac{5}{2}}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)-\frac{8\sqrt{d\tan(bx+a)}\,d^4}{d^2\tan(bx+a)^2+d^2}}{16\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(b*x
+ a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x + a))*d
^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1442 vs. $2(171) = 342$.

time = 37.86, size = 1442, normalized size = 6.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(2*sqrt(2)*b*d*(1/(b^4*d^2))^(1/4)*arctan(1/2*(sqrt(4*b^2*d*sqrt(1/(b
^4*d^2))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*c
os(b*x + a)^2 + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sq
rt(d*sin(b*x + a)/cos(b*x + a)) + 1)*((sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*si
n(b*x + a) + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x + a
)/cos(b*x + a)) + 2*sin(b*x + a)) - (sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*sin(
b*x + a) - sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x + a)/
cos(b*x + a)))/sin(b*x + a)) + 2*sqrt(2)*b*d*(1/(b^4*d^2))^(1/4)*arctan(1/2
*(sqrt(4*b^2*d*sqrt(1/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3
*d*(1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b
*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*((sqrt(2)*b^3*
d*(1/(b^4*d^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x
+ a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 2*sin(b*x + a)) - (sqrt(2)*b^3*d*
(1/(b^4*d^2))^(3/4)*sin(b*x + a) - sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x +
a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/sin(b*x + a)) + 2*sqrt(2)*b*d*(1/(b^
4*d^2))^(1/4)*arctan(1/2*((sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*sin(b*x + a) +
 sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(4*b^2*d*sqrt(1/(b^4*d^2))
*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*cos(b*x +
 a)^2 + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin
(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + (sqrt(2)*b
^3*d*(1/(b^4*d^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b
*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d*cos(b*x + a)^3 - b^2*
d*cos(b*x + a))*sqrt(1/(b^4*d^2)) + 2*sin(b*x + a))/((2*cos(b*x + a)^2 - 1)
*sin(b*x + a))) + 2*sqrt(2)*b*d*(1/(b^4*d^2))^(1/4)*arctan(1/2*((sqrt(2)*b^
3*d*(1/(b^4*d^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*
x + a))*sqrt(4*b^2*d*sqrt(1/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(
2)*b^3*d*(1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*(1/(b^4*d^2))^(1/4)
*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*s
in(b*x + a)/cos(b*x + a)) + (sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*sin(b*x + a)
 + sqrt(2)*b*(1/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x
+ a)) + 4*(b^2*d*cos(b*x + a)^3 - b^2*d*cos(b*x + a))*sqrt(1/(b^4*d^2)) - 2
*sin(b*x + a))/((2*cos(b*x + a)^2 - 1)*sin(b*x + a))) - sqrt(2)*b*d*(1/(b^4
*d^2))^(1/4)*log(4*b^2*d*sqrt(1/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) + 2*(s
qrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*(1/(b^4*d^2))^(
1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + sq
rt(2)*b*d*(1/(b^4*d^2))^(1/4)*log(4*b^2*d*sqrt(1/(b^4*d^2))*cos(b*x + a)*si
n(b*x + a) - 2*(sqrt(2)*b^3*d*(1/(b^4*d^2))^(3/4)*cos(b*x + a)^2 + sqrt(2)*
```

```
b*(1/(b^4*d^2))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*
x + a)) + 1) + 16*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2)/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/sqrt(d*tan(a + b*x)), x)
```

Giac [A]
time = 0.70, size = 218, normalized size = 0.96

$$\frac{\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd}+\frac{\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd}+\frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{16\,bd}-\frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{16\,bd}-\frac{\sqrt{d\tan(bx+a)}\,d}{2\left(d^2\tan(bx+a)^2+d^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(
d*tan(b*x + a)))/sqrt(abs(d)))/(b*d) + 1/8*sqrt(2)*sqrt(abs(d))*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d
) + 1/16*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x +
 a))*sqrt(abs(d)) + abs(d))/(b*d) - 1/16*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x
 + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d) - 1/2*sqr
t(d*tan(b*x + a))*d/((d^2*tan(b*x + a)^2 + d^2)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(a+b\,x\right)}^2}{\sqrt{d\,\tan\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(d*tan(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^2/(d*tan(a + b*x))^(1/2), x)
```

3.86 $\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=20

$$-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[Out] -2/3*d/b/(d*tan(b*x+a))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 30}

$$-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx &= \frac{d\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= -\frac{2d}{3b(d\tan(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 20, normalized size = 1.00

$$-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.
time = 0.53, size = 38, normalized size = 1.90

method	result	size
default	$-\frac{2\cos(bx+a)}{3b\sin(bx+a)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/b*cos(b*x+a)/sin(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(1/2)
```

Maxima [A]
time = 0.27, size = 23, normalized size = 1.15

$$-\frac{2}{3\sqrt{d\tan\left(bx+a\right)}\,b\tan\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3/(sqrt(d*tan(b*x + a))*b*tan(b*x + a))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(16) = 32$.
time = 0.42, size = 46, normalized size = 2.30

$$\frac{2\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}\cos\left(bx+a\right)^{2}}{3\left(bd\cos\left(bx+a\right)^{2}-bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2/(b*d*cos(b*x + a)^2 -
b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**2/sqrt(d*tan(a + b*x)), x)
```

Giac [A]
time = 0.71, size = 23, normalized size = 1.15

$$-\frac{2}{3\sqrt{d\tan(bx+a)}\,b\tan(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/3/(sqrt(d*tan(b*x + a))*b*tan(b*x + a))
```

Mupad [B]
time = 3.33, size = 102, normalized size = 5.10

$$-\frac{2\sqrt{\frac{d\sin(2a+2bx)}{\cos(2a+2bx)+1}}\,(\cos(2a+2bx)+2\cos(4a+4bx)-\cos(6a+6bx)-2)}{3bd\,(15\cos(2a+2bx)-6\cos(4a+4bx)+\cos(6a+6bx)-10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(1/2)),x)
```

```
[Out] -(2*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(cos(2*a + 2*b*x) +
 2*cos(4*a + 4*b*x) - cos(6*a + 6*b*x) - 2))/(3*b*d*(15*cos(2*a + 2*b*x) -
6*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) - 10))
```

3.87 $\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{2d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[Out] -2/7*d^3/b/(d*tan(b*x+a))^(7/2)-2/3*d/b/(d*tan(b*x+a))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2671, 14}

$$-\frac{2d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d^3)/(7*b*(d*Tan[a + b*x])^(7/2)) - (2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx &= \frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{9/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\
&= \frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{9/2}}+\frac{1}{x^{5/2}}\right)\,dx,x,d\tan(a+bx)\right)}{b}\\
&= -\frac{2d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 40, normalized size = 0.93

$$\frac{2d(-5+2\cos(2(a+bx)))\csc^2(a+bx)}{21b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4/Sqrt[d*Tan[a + b*x]],x]
[Out] (2*d*(-5 + 2*Cos[2*(a + b*x)])*Csc[a + b*x]^2)/(21*b*(d*Tan[a + b*x])^(3/2)
)
```

Maple [A]
time = 0.61, size = 50, normalized size = 1.16

method	result	size
default	$\frac{2\left(4\left(\cos^2(bx+a)\right)-7\right)\cos(bx+a)}{21b\sin(bx+a)^3\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/21/b*(4*cos(b*x+a)^2-7)*cos(b*x+a)/sin(b*x+a)^3/(d*sin(b*x+a)/cos(b*x+a))
^(1/2)
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.81

$$-\frac{2\left(7\,d^2\tan\left(bx+a\right)^2+3\,d^2\right)d}{21\,\left(d\tan\left(bx+a\right)\right)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2/21*(7*d^2*tan(b*x + a)^2 + 3*d^2)*d/((d*tan(b*x + a))^(7/2)*b)
```

Fricas [A]
time = 0.41, size = 70, normalized size = 1.63

$$\frac{2\left(4\,\cos\left(bx+a\right)^4-7\,\cos\left(bx+a\right)^2\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{21\left(bd\cos\left(bx+a\right)^4-2\,bd\cos\left(bx+a\right)^2+bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*cos(b*x + a)^4 - 7*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)
)/(b*d*cos(b*x + a)^4 - 2*b*d*cos(b*x + a)^2 + b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**4/sqrt(d*tan(a + b*x)), x)
```

Giac [A]
time = 0.58, size = 45, normalized size = 1.05

$$-\frac{2\left(7\,d^3\tan(bx+a)^2+3\,d^3\right)}{21\,\sqrt{d\tan(bx+a)}\,bd^3\tan(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/21*(7*d^3*tan(b*x + a)^2 + 3*d^3)/(sqrt(d*tan(b*x + a))*b*d^3*tan(b*x +
a)^3)
```

Mupad [B]
time = 7.19, size = 530, normalized size = 12.33

$$\frac{344\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{105\,b\,d\left(e^{a\,2i+b\,x\,2i}-1\right)}+\frac{40\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{21\,b\,d\left(e^{a\,2i+b\,x\,2i}-1\right)^2}+\frac{24\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{35\,b\,d\left(e^{a\,2i+b\,x\,2i}-1\right)^3}-\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,304i}{105\,b\,d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}+\frac{16\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\,d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2}+\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,144i}{35\,b\,d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3}-\frac{16\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\,d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(1/2)),x)
```

```
[Out] (344*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
+ b*x*2i) + 1))^(1/2))/(105*b*d*(exp(a*2i + b*x*2i) - 1)) + (40*(exp(a*2i +
 b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^
(1/2))/(21*b*d*(exp(a*2i + b*x*2i) - 1)^2) + (24*(exp(a*2i + b*x*2i) + 1)*(
-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35*b*d*
(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*
x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*304i)/(105*b*d*(exp(a*2i +
b*x*2i)*1i - 1i)) + (16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1
i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(7*b*d*(exp(a*2i + b*x*2i)*1i - 1
i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a
*2i + b*x*2i) + 1))^(1/2)*144i)/(35*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (
16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
b*x*2i) + 1))^(1/2))/(7*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^4)
```

3.88 $\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=65

$$-\frac{2d^5}{11b(d\tan(a+bx))^{11/2}}-\frac{4d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

```
[Out] -2/11*d^5/b/(d*tan(b*x+a))^(11/2)-4/7*d^3/b/(d*tan(b*x+a))^(7/2)-2/3*d/b/(d
*tan(b*x+a))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 276}

$$-\frac{2d^5}{11b(d\tan(a+bx))^{11/2}}-\frac{4d^3}{7b(d\tan(a+bx))^{7/2}}-\frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d^5)/(11*b*(d*Tan[a + b*x])^(11/2)) - (4*d^3)/(7*b*(d*Tan[a + b*x])^(7/
2)) - (2*d)/(3*b*(d*Tan[a + b*x])^(3/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\csc^6(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{d\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{13/2}}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= \frac{d\text{Subst}\left(\int \left(\frac{d^4}{x^{13/2}}+\frac{2d^2}{x^{9/2}}+\frac{1}{x^{5/2}}\right)\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{2d^5}{11b(d\tan(a+bx))^{11/2}} - \frac{4d^3}{7b(d\tan(a+bx))^{7/2}} - \frac{2d}{3b(d\tan(a+bx))^{3/2}}$$

Mathematica [A]
time = 0.18, size = 50, normalized size = 0.77

$$\frac{2d(-45+28\cos(2(a+bx))-4\cos(4(a+bx)))\csc^4(a+bx)}{231b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6/Sqrt[d*Tan[a + b*x]],x]
[Out] (2*d*(-45 + 28*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4)/(231*
b*(d*Tan[a + b*x])^(3/2))
```

Maple [A]
time = 0.45, size = 60, normalized size = 0.92

method	result	size
default	$-\frac{2(32(\cos^4(bx+a))-88(\cos^2(bx+a))+77)\cos(bx+a)}{231b\sin(bx+a)^5\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/231/b*(32*cos(b*x+a)^4-88*cos(b*x+a)^2+77)*cos(b*x+a)/sin(b*x+a)^5/(d*si
n(b*x+a)/cos(b*x+a))^(1/2)
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.74

$$-\frac{2\left(77\,d^4\tan\left(bx+a\right)^4+66\,d^4\tan\left(bx+a\right)^2+21\,d^4\right)d}{231\,\left(d\tan\left(bx+a\right)\right)^{\frac{11}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/231*(77*d^4*tan(b*x + a)^4 + 66*d^4*tan(b*x + a)^2 + 21*d^4)*d/((d*tan(b
*x + a))^(11/2)*b)
```

Fricas [A]
time = 0.39, size = 93, normalized size = 1.43

$$\frac{2\left(32\,\cos\left(bx+a\right)^{6}-88\,\cos\left(bx+a\right)^{4}+77\,\cos\left(bx+a\right)^{2}\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{231\left(bd\cos\left(bx+a\right)^{6}-3\,bd\cos\left(bx+a\right)^{4}+3\,bd\cos\left(bx+a\right)^{2}-bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(32*cos(b*x + a)^6 - 88*cos(b*x + a)^4 + 77*cos(b*x + a)^2)*sqrt(d*si
n(b*x + a)/cos(b*x + a))/(b*d*cos(b*x + a)^6 - 3*b*d*cos(b*x + a)^4 + 3*b*d
*cos(b*x + a)^2 - b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^{6}\left(a+bx\right)}{\sqrt{d\tan\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**6/sqrt(d*tan(a + b*x)), x)
```

Giac [A]
time = 0.54, size = 58, normalized size = 0.89

$$-\frac{2\left(77\,d^{5}\tan\left(bx+a\right)^{4}+66\,d^{5}\tan\left(bx+a\right)^{2}+21\,d^{5}\right)}{231\,\sqrt{d\tan\left(bx+a\right)}\,bd^{5}\tan\left(bx+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2/231*(77*d^5*tan(b*x + a)^4 + 66*d^5*tan(b*x + a)^2 + 21*d^5)/(sqrt(d*tan
(b*x + a))*b*d^5*tan(b*x + a)^5)
```

Mupad [B]
time = 12.40, size = 831, normalized size = 12.78

$$-\frac{41984\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{10395\,b\,d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}-\frac{128\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{35\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2}-\frac{7136\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{1155\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3}-\frac{1216\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{231\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^4}-\frac{160\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{99\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^5}+\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,44864\mathrm{i}}{10395\,b\,d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}-\frac{3904\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{1155\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,1088\mathrm{i}}{165\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^3}+\frac{320\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{21\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^4}+\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,1600\mathrm{i}}{99\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^5}-\frac{64\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{11\,b\,d\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(1/2)),x)
```

```
[Out] ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2)*44864i)/(10395*b*d*(exp(a*2i + b*x*2i)*1i - 1i)) - (128*(
exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*
2i) + 1))^(1/2))/(35*b*d*(exp(a*2i + b*x*2i) - 1)^2) - (7136*(exp(a*2i + b*
x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/
2))/(1155*b*d*(exp(a*2i + b*x*2i) - 1)^3) - (1216*(exp(a*2i + b*x*2i) + 1)*
(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(231*b*
d*(exp(a*2i + b*x*2i) - 1)^4) - (160*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2
i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(99*b*d*(exp(a*2i +
b*x*2i) - 1)^5) - (41984*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*
1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(10395*b*d*(exp(a*2i + b*x*2i) -
 1)) - (3904*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(e
xp(a*2i + b*x*2i) + 1))^(1/2))/(1155*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^2) -
((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2)*1088i)/(165*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^3) + (320*(e
xp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2
i) + 1))^(1/2))/(21*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^4) + ((exp(a*2i + b*x*
2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)
*1600i)/(99*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (64*(exp(a*2i + b*x*2i) +
 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(11
*b*d*(exp(a*2i + b*x*2i)*1i - 1i)^6)
```

3.89 $\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=107

$$-\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{7E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -7/20*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-7/30*d
*sin(b*x+a)^3/b/(d*tan(b*x+a))^(3/2)-1/5*d*sin(b*x+a)^5/b/(d*tan(b*x+a))^(3
/2)
```

Rubi [A]
time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2678, 2681, 2652, 2719}

$$-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}+\frac{7\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^5/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-7*d*Sin[a + b*x]^3)/(30*b*(d*Tan[a + b*x])^(3/2)) - (d*Sin[a + b*x]^5)/(5
*b*(d*Tan[a + b*x])^(3/2)) + (7*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/
(20*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
```

```
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx &= -\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{7}{10}\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{7}{20}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{\left(7\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+}}{20\sqrt{\cos(a+bx)}\,\sqrt{d}}\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{(7\sin(a+bx))\int \sqrt{\sin(2a+2b}}{20\sqrt{\sin(2a+2bx)}\,\sqrt{d\tan(a+}}\\
&= -\frac{7d\sin^3(a+bx)}{30b(d\tan(a+bx))^{3/2}}-\frac{d\sin^5(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{7E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+b}{20b\sqrt{\sin(2a+2bx)}\,\sqrt{d\tan(a}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.88, size = 86, normalized size = 0.80

$$\frac{\sin(a+bx)\left(-20\sin(2(a+bx))+3\sin(4(a+bx))+28\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\,\tan(a+bx)\right)}{120b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^5/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Sin[a + b*x]*(-20*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + 28*Hypergeometri
c2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]))/(1
20*b*Sqrt[d*Tan[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(118) = 236.
time = 0.36, size = 550, normalized size = 5.14

method	result
default	$-\frac{(-1+\cos(bx+a))^2\left(12(\cos^6(bx+a))\sqrt{2}-38(\cos^4(bx+a))\sqrt{2}+42\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120/b*(-1+cos(b*x+a))^2*(12*cos(b*x+a)^6*2^(1/2)-38*cos(b*x+a)^4*2^(1/2)
+42*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2
^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b
*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-21*cos(b*x+a)*Ell
ipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b
*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+42*EllipticE(((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x
+a))^(1/2)-21*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^
(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+47*cos(b*x+a)^2*2^
(1/2)-21*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)/sin(b*x+a)^4/(d*si
n(b*x+a)/cos(b*x+a))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^5/sqrt(d*tan(b*x + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*tan(b*x + a))*sin(b
*x + a)/(d*tan(b*x + a)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**5/(d*tan(b*x+a))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^5/sqrt(d*tan(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^5}{\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/(d*tan(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^5/(d*tan(a + b*x))^(1/2), x)
```

3.90 $\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-1/3*d*s
in(b*x+a)^3/b/(d*tan(b*x+a))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2678, 2681, 2652, 2719}

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] -1/3*(d*Sin[a + b*x]^3)/(b*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 +
b*x, 2]*Sin[a + b*x])/(2*b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx &= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx \\
&= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}\,dx}{2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}} \\
&= -\frac{d\sin^3(a+bx)}{3b(d\tan(a+bx))^{3/2}}+\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.78, size = 98, normalized size = 1.24

$$\frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}\,(\sin(a+bx)+\sin(3(a+bx)))+4\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sec(a+bx)\tan(a+bx)\right)}{12bd\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(Sin[a + b*x] + Sin[3*(a + b*
x)])) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Ta
n[a + b*x]))/(12*b*d*Sqrt[Sec[a + b*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(94) = 188.
time = 0.38, size = 537, normalized size = 6.80

method	result

default	$\frac{(-1+\cos(bx+a))^2\left(2(\cos^4(bx+a))\sqrt{2}-6\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-c}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/b*(-1+cos(b*x+a))^2*(2*cos(b*x+a)^4*2^(1/2)-6*cos(b*x+a)*EllipticE(((1
-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin
(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+
sin(b*x+a))/sin(b*x+a))^(1/2)+3*cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x
+a))^(1/2)-6*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(
1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+3*EllipticF(((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x
+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(
b*x+a))/sin(b*x+a))^(1/2)-5*cos(b*x+a)^2*2^(1/2)+3*cos(b*x+a)*2^(1/2))*(cos
(b*x+a)+1)^2/sin(b*x+a)^4/(d*sin(b*x+a)/cos(b*x+a))^(1/2)/cos(b*x+a)*2^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d*tan(b*x
 + a)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^3}{\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(d*tan(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^3/(d*tan(a + b*x))^(1/2), x)
```

3.91 $\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=47

$$\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),
2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2681, 2652, 2719}

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[
d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx = \frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$
$$= \frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$
$$= \frac{E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.14, size = 60, normalized size = 1.28

$$\frac{2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\,\sin(a+bx)\sqrt{d\tan(a+bx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*S
in[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(69) = 138.
time = 0.35, size = 523, normalized size = 11.13

method	result
default	$-\frac{(-1+\cos(bx+a))^2\left(2\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(-1+cos(b*x+a))^2*(2*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))
/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)-cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/
2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*EllipticE(((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1
```

```
+sin(b*x+a))/sin(b*x+a))^(1/2)-EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+c
os(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/sin(b*x+a)^4/(d*si
n(b*x+a)/cos(b*x+a))^(1/2)/cos(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d*tan(b*x + a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))**(1/2),x)
[Out] Integral(sin(a + b*x)/sqrt(d*tan(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(d*tan(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)/(d*tan(a + b*x))^(1/2), x)
```

3.92 $\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -2*cos(b*x+a)/b/(d*tan(b*x+a))^(1/2)+2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/
4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2*a)^
(1/2)/(d*tan(b*x+a))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2681, 2650, 2652, 2719}

$$-\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*Cos[a + b*x])/(b*Sqrt[d*Tan[a + b*x]]) - (2*EllipticE[a - Pi/4 + b*x, 2
]*Sin[a + b*x])/(b*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx &= \frac{\sqrt{\sin(a+bx)}\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\ &= -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{\left(2\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\ &= -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{(2\sin(a+bx))\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\ &= -\frac{2\cos(a+bx)}{b\sqrt{d\tan(a+bx)}}-\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.33, size = 69, normalized size = 0.96

$$-\frac{2\cos(a+bx)\left(3+2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{3b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*Cos[a + b*x]*(3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*S
qrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(3*b*Sqrt[d*Tan[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(91) = 182.
time = 0.38, size = 482, normalized size = 6.69

method	result

default	$\frac{\left(2\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1\cdots}{\sin(bx\cdots}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/
2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-cos(b*x+a)*Ell
ipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b
*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*EllipticE(((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+
a))^(1/2)-EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2
))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-cos(b*x+a)*2^(1/2))/(d
*sin(b*x+a)/cos(b*x+a))^(1/2)/cos(b*x+a)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(a+bx\right)}{\sqrt{d\tan\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)/sqrt(d*tan(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sqrt(d*tan(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)\,\sqrt{d\tan\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(1/2)),x)
```

```
[Out] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(1/2)), x)
```

3.93 $\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$

Optimal. Leaf size=102

$$-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}}-\frac{4E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -4/5*cos(b*x+a)/b/(d*tan(b*x+a))^(1/2)+4/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(
a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/sin(2*b*x+2
*a)^(1/2)/(d*tan(b*x+a))^(1/2)-2/5*d*csc(b*x+a)/b/(d*tan(b*x+a))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2679, 2681, 2650, 2652, 2719}

$$-\frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}}-\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{4\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (-2*d*Csc[a + b*x])/(5*b*(d*Tan[a + b*x])^(3/2)) - (4*Cos[a + b*x])/(5*b*Sq
rt[d*Tan[a + b*x]]) - (4*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(5*b*Sq
rt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
```

```
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx &= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{2}{5}\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx\\ &= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}+\frac{\left(2\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{5\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\ &= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}}-\frac{\left(4\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+b}}{5\sqrt{\cos(a+bx)}\sqrt{d\tan}}\\ &= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}}-\frac{(4\sin(a+bx))\int \sqrt{\sin(2a+2bx)}}{5\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+b}}\\ &= -\frac{2d\csc(a+bx)}{5b(d\tan(a+bx))^{3/2}}-\frac{4\cos(a+bx)}{5b\sqrt{d\tan(a+bx)}}-\frac{4E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+b}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.72, size = 104, normalized size = 1.02

$$\frac{6(-2+\cos(2(a+bx)))\cot(a+bx)\csc(a+bx)\sqrt{\sec^2(a+bx)}-8\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sec(a+bx)\tan^2(a+bx)}{15b\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sqrt[d*Tan[a + b*x]],x]
```

```
[Out] (6*(-2 + Cos[2*(a + b*x)])*Cot[a + b*x]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2] -
 8*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a + b
*x]^2)/(15*b*Sqrt[Sec[a + b*x]^2]*Sqrt[d*Tan[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 971 vs. $2(113) = 226$.
time = 0.62, size = 972, normalized size = 9.53

method	result	size
default	Expression too large to display	972

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b*(4*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^3-2*((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^3+4*((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(
b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))*cos(b*x+a)^2-2*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a
))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2)
)*cos(b*x+a)^2-4*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*co
s(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2)
)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-4*EllipticE(((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))
^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+
a))/sin(b*x+a))^(1/2)+2*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b*
x+a)^3*2^(1/2)+cos(b*x+a)^2*2^(1/2)+2*cos(b*x+a)*2^(1/2))/cos(b*x+a)/sin(b*
x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/sqrt(d*tan(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/sqrt(d*tan(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(a+b\,x)^3\,\sqrt{d\tan(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(1/2)), x)
```

3.94 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,bd^{3/2}}+\frac{3\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,bd^{3/2}}+\frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+\right.}{64\sqrt{}}$$

```
[Out] -3/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+3/64
*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+3/128*ln(
d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2)-
3/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)
*2^(1/2)+3/16*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d^3-1/4*cos(b*x+a)^4*(d*t
an(b*x+a))^(3/2)/b/d^3
```

Rubi [A]
time = 0.13, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2671, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,bd^{3/2}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}\,bd^{3/2}}+\frac{3\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,bd^{3/2}}-\frac{3\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,bd^{3/2}}-\frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3}+\frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*d^(3/
2)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
d^(3/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*d^(3/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*d^(3/2)) + (3*Cos[a + b*x]^2*(d*T
an[a + b*x])^(3/2))/(16*b*d^3) - (Cos[a + b*x]^4*(d*Tan[a + b*x])^(3/2))/(4
*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{d\text{Subst}\left(\int \frac{x^{5/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} + \frac{(3d)\text{Subst}\left(\int \frac{\sqrt{x}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} + \frac{3\text{Subst}\left(\int \frac{\sqrt{}}{d^2}\right.}{}$$

$$= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} + \frac{3\text{Subst}\left(\int \frac{}{d^2}\right.}{}$$

$$= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} - \frac{3\text{Subst}\left(\int \frac{d}{d^2}\right.}{}$$

$$= \frac{3\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{16bd^3} - \frac{\cos^4(a+bx)(d\tan(a+bx))^{3/2}}{4bd^3} + \frac{3\text{Subst}\left(\int -\right.}{}$$

$$= \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}\,bd^{3/2}} - \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(\right.}{6}$$

$$= -\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,bd^{3/2}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,bd^{3/2}} +$$

Mathematica [A]
time = 0.35, size = 123, normalized size = 0.48

$$\frac{\csc(a+bx)\left(\cos(a+bx)-2\cos(3(a+bx))+\cos(5(a+bx))-3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}\right)\sqrt{d\tan(a+bx)}}{64bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Csc[a + b*x]*(Cos[a + b*x] - 2*Cos[3*(a + b*x)] + Cos[5*(a + b*x)] - 3*Arc
Sin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[Sin[2*(a + b*x)]] - 3*Log[Cos[a + b*x
] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]])*Sqrt[d*T
an[a + b*x]])/(64*b*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.32, size = 550, normalized size = 2.14

method	result
default	$\frac{(-1+\cos(bx+a))\left(3i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/b*(-1+cos(b*x+a))*(3*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)-3*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-8*cos(b*x+
a)^4*2^(1/2)+3*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+3*Elli
pticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*
((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+8*cos(b*x+a)^3*2^(1/2)+6*
cos(b*x+a)^2*2^(1/2)-6*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)^2/si
n(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(3/2)*2^(1/2)
```

Maxima [A]
time = 0.49, size = 225, normalized size = 0.88

$$\frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+\frac{8\left(3\left(d\tan(bx+a)\right)^{\frac{7}{2}}d^4-\left(d\tan(bx+a)\right)^{\frac{3}{2}}d^6\right)}{d^4\tan(bx+a)^4+2\,d^4\tan(bx+a)^2+d^4}}{128\,bd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/128*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
```

```
) - 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x
+ a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) + 8*(3*(d*tan(b*x
 + a))^(7/2)*d^4 - (d*tan(b*x + a))^(3/2)*d^6)/(d^4*tan(b*x + a)^4 + 2*d^4*
tan(b*x + a)^2 + d^4))/(b*d^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1871 vs. $2(197) = 394$.
time = 62.50, size = 1871, normalized size = 7.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/512*(12*sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*arctan((sqrt(4*b^2*d^3*sqrt(1/(
b^4*d^6))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4
)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^
2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*(b^2*d^3*sqrt(1/(b^4*d^6)) + 2*co
s(b*x + a)*sin(b*x + a) + (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)
^2 + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(
b*x + a)/cos(b*x + a))) + (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)
*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(
b*x + a)/cos(b*x + a)))/(2*cos(b*x + a)^2 - 1)) + 12*sqrt(2)*b*d^2*(1/(b^4*
d^6))^(1/4)*arctan(-(sqrt(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x
+ a) + 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + s
qrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x
+ a)) + 1)*(b^2*d^3*sqrt(1/(b^4*d^6)) + 2*cos(b*x + a)*sin(b*x + a) - (sqrt
(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*d*(1/(b^4*d^6))^
(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) - (sqrt
(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/
(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(2*cos(
b*x + a)^2 - 1)) + 12*sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*arctan(1/2*((sqrt(2
)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4
)*sin(b*x + a))*sqrt(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a)
+ 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2
)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))
 + 1)*sqrt(d*sin(b*x + a)/cos(b*x + a)) - (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3
/4)*cos(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*sin
(b*x + a)/cos(b*x + a)) + 4*(b^2*d^3*cos(b*x + a)^3 - b^2*d^3*cos(b*x + a))
*sqrt(1/(b^4*d^6)) - 2*sin(b*x + a))/((2*cos(b*x + a)^2 - 1)*sin(b*x + a)))
 + 12*sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*arctan(1/2*((sqrt(2)*b^3*d^4*(1/(b^
4*d^6))^(3/4)*cos(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*sin(b*x + a))*
sqrt(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3
*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^
```

```
6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*sin
(b*x + a)/cos(b*x + a)) - (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)
 + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*
x + a)) - 4*(b^2*d^3*cos(b*x + a)^3 - b^2*d^3*cos(b*x + a))*sqrt(1/(b^4*d^6
)) + 2*sin(b*x + a))/((2*cos(b*x + a)^2 - 1)*sin(b*x + a))) - 3*sqrt(2)*b*d
^2*(1/(b^4*d^6))^(1/4)*log(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x
 + a) + 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) +
sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x
 + a)) + 1) + 3*sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*log(4*b^2*d^3*sqrt(1/(b^4
*d^6))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*c
os(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*
sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) - 3*sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4
)*log(1/4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) + 1/8*(sqrt(2
)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b
^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1/16) +
3*sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*log(1/4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b
*x + a)*sin(b*x + a) - 1/8*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a
)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin
(b*x + a)/cos(b*x + a)) + 1/16) - 32*(4*cos(b*x + a)^3 - 3*cos(b*x + a))*sq
rt(d*sin(b*x + a)/cos(b*x + a))*sin(b*x + a))/(b*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
[Out] Integral(sin(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.76, size = 257, normalized size = 1.00

$$\frac{\frac{6\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}+\frac{6\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}-\frac{3\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{bd^2}+\frac{3\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{bd^2}+\frac{8\left(3\sqrt{d\tan(bx+a)}\,d^3\tan(bx+a)^3-\sqrt{d\tan(bx+a)}\,d^3\tan(bx+a)\right)}{\left(d^2\tan(bx+a)^2+d^2\right)^2 b}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] 1/128*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*
sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 6*sqrt(2)*abs(d)^(3/2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))
/(b*d^2) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b
*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(
```

```
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + 8*
(3*sqrt(d*tan(b*x + a))*d^3*tan(b*x + a)^3 - sqrt(d*tan(b*x + a))*d^3*tan(b
*x + a))/((d^2*tan(b*x + a)^2 + d^2)^2*b))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,x\right)^4}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(sin(a + b*x)^4/(d*tan(a + b*x))^(3/2), x)
```

3.95 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,bd^{3/2}}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,bd^{3/2}}+\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx\right.}{8\sqrt{2}\,b}$$

```
[Out] -1/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+1/8*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)+1/16*ln(d^(
1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2)-1/1
6*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(
1/2)+1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(3/2)/b/d^3
```

Rubi [A]
time = 0.12, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2671, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,bd^{3/2}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}\,bd^{3/2}}+\frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,bd^{3/2}}-\frac{\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,bd^{3/2}}+\frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/4*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*d^(3/2))
 + ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(4*Sqrt[2]*b*d^(3/2))
 + Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(8*Sq
rt[2]*b*d^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[
a + b*x]]]/(8*Sqrt[2]*b*d^(3/2)) + (Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2))/
(2*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{4bd}$$

$$= \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3} + \frac{\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2bd}$$

$$= \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4bd} + \frac{\text{Sub}}{}$$

$$= \frac{\cos^2(a+bx)(d\tan(a+bx))^{3/2}}{2bd^3} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+b}\right.}{8\sqrt{2}bd^{3/2}}$$

$$= \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a\right.}{8}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} + \frac{\text{lo}}{}$$

Mathematica [A]
time = 0.25, size = 105, normalized size = 0.46

$$-\frac{\left(\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\csc(a+bx)+\csc(a+bx)\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}\sqrt{d\tan(a+bx)}}{8bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/8*((ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Csc[a + b*x] + Csc[a + b*x]*Log[
Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]] - 2*Sqrt[Sin[2*(a + b
*x)]])*Sqrt[Sin[2*(a + b*x)]]*Sqrt[d*Tan[a + b*x]])/(b*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.30, size = 524, normalized size = 2.31

method	result
default	$-\frac{(-1+\cos(bx+a))\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\ }\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b*(-1+cos(b*x+a))*(I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(
1/2)-I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-EllipticPi(((1
-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(b
*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-EllipticPi(((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)-2*cos(b*x+a)^2*2^(1/2)+2*cos(b*x+a)*2^(1/2))*(cos(b*x+a)
+1)^2/cos(b*x+a)^2/sin(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(3/2)*2^(1/2)
```

Maxima [A]
time = 0.50, size = 193, normalized size = 0.85

$$\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+\frac{8(d\tan(bx+a))^{\frac{3}{2}}d^2}{d^2\tan(bx+a)^2+d^2}}{16bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*x
 + a)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
 2*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) + sq
rt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x + a
) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d)) + 8*(d*tan(b*x + a))
^(3/2)*d^2/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1856 vs. $2(171) = 342$.
time = 60.73, size = 1856, normalized size = 8.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")

[Out] 1/64*(4*sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*arctan((sqrt(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*(b^2*d^3*sqrt(1/(b^4*d^6)) + 2*cos(b*x + a)*sin(b*x + a) + (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(2*cos(b*x + a)^2 - 1)) + 4*sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*arctan(-(sqrt(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*(b^2*d^3*sqrt(1/(b^4*d^6)) + 2*cos(b*x + a)*sin(b*x + a) - (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) - (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(2*cos(b*x + a)^2 - 1)) + 4*sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*arctan(1/2*((sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*sin(b*x + a)/cos(b*x + a)) - (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 4*(b^2*d^3*cos(b*x + a)^3 - b^2*d^3*cos(b*x + a))*sqrt(1/(b^4*d^6)) - 2*sin(b*x + a))/((2*cos(b*x + a)^2 - 1)*sin(b*x + a))) + 4*sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*arctan(1/2*((sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*sin(b*x + a)/cos(b*x + a)) - (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^3*cos(b*x + a)^3 - b^2*d^3*cos(b*x + a))*sqrt(1/(b^4*d^6)) + 2*sin(b*x + a))/((2*cos(b*x + a)^2 - 1)*sin(b*x + a))) - sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*log(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*log(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*si

```
n(b*x + a)/cos(b*x + a)) + 1) - sqrt(2)*b*d^2*(1/(b^4*d^6))^(1/4)*log(1/4*b
^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) + 1/8*(sqrt(2)*b^3*d^4*(
1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1
/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1/16) + sqrt(2)*b*d
^2*(1/(b^4*d^6))^(1/4)*log(1/4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b
*x + a) - 1/8*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a
) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos
(b*x + a)) + 1/16) + 32*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)*sin(
b*x + a))/(b*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.62, size = 228, normalized size = 1.00

$$\frac{\frac{8\sqrt{d\tan(bx+a)}\,d\tan(bx+a)}{\left(d^2\tan(bx+a)^2+d^2\right)b}+\frac{2\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}+\frac{2\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{bd^2}-\frac{\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{bd^2}+\frac{\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{bd^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(8*sqrt(d*tan(b*x + a))*d*tan(b*x + a)/((d^2*tan(b*x + a)^2 + d^2)*b)
+ 2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(
d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 2*sqrt(2)*abs(d)^(3/2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d
^2) - sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a)
)*sqrt(abs(d)) + abs(d))/(b*d^2) + sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a)
- sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a+b\,x)^2}{(d\tan(a+b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(sin(a + b*x)^2/(d*tan(a + b*x))^(3/2), x)
```

3.96 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

[Out] -2/5*d/b/(d*tan(b*x+a))^(5/2)

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 30}

$$-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]

[Out] (-2*d)/(5*b*(d*Tan[a + b*x])^(5/2))

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= \frac{d\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= -\frac{2d}{5b(d\tan(a+bx))^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 20, normalized size = 1.00

$$-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.
time = 0.33, size = 38, normalized size = 1.90

method	result	size
default	$-\frac{2\cos(bx+a)}{5b\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}\sin(bx+a)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/b*cos(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)
```

Maxima [A]
time = 0.28, size = 23, normalized size = 1.15

$$-\frac{2}{5\,\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}b\tan\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5/((d*tan(b*x + a))^(3/2)*b*tan(b*x + a))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(16) = 32$.
time = 0.37, size = 58, normalized size = 2.90

$$\frac{2\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}\cos\left(bx+a\right)^{3}}{5\left(bd^{2}\cos\left(bx+a\right)^{2}-bd^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^3/((b*d^2*cos(b*x + a)^2
 - b*d^2)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.68, size = 26, normalized size = 1.30

$$-\frac{2}{5\sqrt{d\tan(bx+a)}\,bd\tan(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/5/(sqrt(d*tan(b*x + a))*b*d*tan(b*x + a)^2)
```

Mupad [B]
time = 6.45, size = 381, normalized size = 19.05

$$-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,14\mathrm{i}}{5\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,8\mathrm{i}}{15\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2}-\frac{16\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{5\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{15\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}+\frac{8\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{5\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] (8*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
b*x*2i) + 1))^(1/2))/(5*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^3) - ((exp(a*2i
+ b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))
^(1/2)*8i)/(15*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) - (16*(exp(a*2i + b*x*2i)
+ 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(5
*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(
a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*32i)/(15*b*d^2*(ex
p(a*2i + b*x*2i)*1i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b
*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*14i)/(5*b*d^2*(exp(a*2i +
b*x*2i) - 1))
```

3.97 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{2d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[Out] -2/9*d^3/b/(d*tan(b*x+a))^(9/2)-2/5*d/b/(d*tan(b*x+a))^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 14}

$$-\frac{2d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
[Out] (-2*d^3)/(9*b*(d*Tan[a + b*x])^(9/2)) - (2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= \frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{11/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\ &= \frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{11/2}}+\frac{1}{x^{7/2}}\right)\,dx,x,d\tan(a+bx)\right)}{b}\\ &= -\frac{2d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 42, normalized size = 0.98

$$\frac{2(4+\csc^2(a+bx)-5\csc^4(a+bx))}{45bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
[Out] (2*(4 + Csc[a + b*x]^2 - 5*Csc[a + b*x]^4))/(45*b*d*Sqrt[d*Tan[a + b*x]])
```

Maple [A]
time = 0.37, size = 50, normalized size = 1.16

method	result	size
default	$\frac{2\left(4\left(\cos^2(bx+a)\right)-9\right)\cos(bx+a)}{45b\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}\sin(bx+a)^3}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/45/b*(4*cos(b*x+a)^2-9)*cos(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*
x+a)^3
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.81

$$-\frac{2\left(9\,d^2\tan\left(bx+a\right)^2+5\,d^2\right)d}{45\,\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2/45*(9*d^2*tan(b*x + a)^2 + 5*d^2)*d/((d*tan(b*x + a))^(9/2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(35) = 70$.
time = 0.40, size = 84, normalized size = 1.95

$$\frac{2\left(4\,\cos\left(bx+a\right)^5-9\,\cos\left(bx+a\right)^3\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{45\left(bd^2\cos\left(bx+a\right)^4-2\,bd^2\cos\left(bx+a\right)^2+bd^2\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(4*cos(b*x + a)^5 - 9*cos(b*x + a)^3)*sqrt(d*sin(b*x + a)/cos(b*x + a)
)/((b*d^2*cos(b*x + a)^4 - 2*b*d^2*cos(b*x + a)^2 + b*d^2)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.70, size = 45, normalized size = 1.05

$$-\frac{2\left(9\,d^4\tan\left(bx+a\right)^2+5\,d^4\right)}{45\,\sqrt{d\tan\left(bx+a\right)}\,bd^5\tan\left(bx+a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/45*(9*d^4*tan(b*x + a)^2 + 5*d^4)/(sqrt(d*tan(b*x + a))*b*d^5*tan(b*x +
a)^4)
```

Mupad [B]
time = 8.15, size = 684, normalized size = 15.91

$$\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,6088\mathrm{i}}{945\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}+\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,4024\mathrm{i}}{945\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2}+\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,200\mathrm{i}}{63\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3}+\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{63\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^4}+\frac{1184\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{189\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}+\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,4192\mathrm{i}}{945\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}-\frac{2176\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{315\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^3}-\frac{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{63\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^4}+\frac{32\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{9\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2)*6088i)/(945*b*d^2*(exp(a*2i + b*x*2i) - 1)) + ((exp(a*2i
+ b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))
^(1/2)*4024i)/(945*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) + ((exp(a*2i + b*x*2i)
 + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*20
0i)/(63*b*d^2*(exp(a*2i + b*x*2i) - 1)^3) + ((exp(a*2i + b*x*2i) + 1)*(-(d*
(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*64i)/(63*b*d^
2*(exp(a*2i + b*x*2i) - 1)^4) + (1184*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*
2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(189*b*d^2*(exp(a*2
i + b*x*2i)*1i - 1i)) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*
```

```
1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*4192i)/(945*b*d^2*(exp(a*2i + b*x
*2i)*1i - 1i)^2) - (2176*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*
1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(315*b*d^2*(exp(a*2i + b*x*2i)*1
i - 1i)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2)*512i)/(63*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)
^4) + (32*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(
a*2i + b*x*2i) + 1))^(1/2))/(9*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^5)
```

3.98 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2d^5}{13b(d\tan(a+bx))^{13/2}}-\frac{4d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

```
[Out] -2/13*d^5/b/(d*tan(b*x+a))^(13/2)-4/9*d^3/b/(d*tan(b*x+a))^(9/2)-2/5*d/b/(d
*tan(b*x+a))^(5/2)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 276}

$$-\frac{2d^5}{13b(d\tan(a+bx))^{13/2}}-\frac{4d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^5)/(13*b*(d*Tan[a + b*x])^(13/2)) - (4*d^3)/(9*b*(d*Tan[a + b*x])^(9/
2)) - (2*d)/(5*b*(d*Tan[a + b*x])^(5/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= \frac{d\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{15/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\ &= \frac{d\text{Subst}\left(\int \left(\frac{d^4}{x^{15/2}}+\frac{2d^2}{x^{11/2}}+\frac{1}{x^{7/2}}\right)\,dx,x,d\tan(a+bx)\right)}{b}\\ &= -\frac{2d^5}{13b(d\tan(a+bx))^{13/2}}-\frac{4d^3}{9b(d\tan(a+bx))^{9/2}}-\frac{2d}{5b(d\tan(a+bx))^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 54, normalized size = 0.83

$$\frac{64+16\csc^2(a+bx)+10\csc^4(a+bx)-90\csc^6(a+bx)}{585bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (64 + 16*Csc[a + b*x]^2 + 10*Csc[a + b*x]^4 - 90*Csc[a + b*x]^6)/(585*b*d*S
qrt[d*Tan[a + b*x]])
```

Maple [A]
time = 0.42, size = 60, normalized size = 0.92

method	result	size
default	$-\frac{2\left(32\left(\cos^4(bx+a)\right)-104\left(\cos^2(bx+a)\right)+117\right)\cos(bx+a)}{585b\sin(bx+a)^5\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/585/b*(32*cos(b*x+a)^4-104*cos(b*x+a)^2+117)*cos(b*x+a)/sin(b*x+a)^5/(d*
sin(b*x+a)/cos(b*x+a))^(3/2)
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.74

$$-\frac{2\left(117\,d^4\tan\left(bx+a\right)^4+130\,d^4\tan\left(bx+a\right)^2+45\,d^4\right)d}{585\,\left(d\tan\left(bx+a\right)\right)^{\frac{13}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/585*(117*d^4*tan(b*x + a)^4 + 130*d^4*tan(b*x + a)^2 + 45*d^4)*d/((d*tan
(b*x + a))^(13/2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(53) = 106.
time = 0.45, size = 109, normalized size = 1.68

$$\frac{2\left(32\cos\left(bx+a\right)^7-104\cos\left(bx+a\right)^5+117\cos\left(bx+a\right)^3\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{585\left(bd^2\cos\left(bx+a\right)^6-3\,bd^2\cos\left(bx+a\right)^4+3\,bd^2\cos\left(bx+a\right)^2-bd^2\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/585*(32*cos(b*x + a)^7 - 104*cos(b*x + a)^5 + 117*cos(b*x + a)^3)*sqrt(d*
sin(b*x + a)/cos(b*x + a))/((b*d^2*cos(b*x + a)^6 - 3*b*d^2*cos(b*x + a)^4
+ 3*b*d^2*cos(b*x + a)^2 - b*d^2)*sin(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**6/(d*tan(a + b*x))**(3/2), x)
```

Giac [**A**]
time = 0.81, size = 58, normalized size = 0.89

$$-\frac{2\left(117\,d^6\tan(bx+a)^4+130\,d^6\tan(bx+a)^2+45\,d^6\right)}{585\,\sqrt{d\tan(bx+a)}\,bd^7\tan(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/585*(117*d^6*tan(b*x + a)^4 + 130*d^6*tan(b*x + a)^2 + 45*d^6)/(sqrt(d*t
an(b*x + a))*b*d^7*tan(b*x + a)^6)
```

Mupad [**B**]
time = 16.41, size = 987, normalized size = 15.18

$$-\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,18368i}{2145\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}-1\right)}-\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,294464i}{45045\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}-1\right)^2}-\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,24608i}{2145\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}-1\right)^3}-\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,135104i}{9009\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}-1\right)^4}-\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,13088i}{1287\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}-1\right)^5}-\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,384i}{143\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}-1\right)^6}-\frac{55808\,\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{6435\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}-\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,7424i}{1155\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2}+\frac{128\,\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{11\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3}+\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,228736i}{9009\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4}-\frac{17152\,\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{429\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^5}-\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,4608i}{143\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^6}+\frac{128\,\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{13\,b\,d^2\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] (128*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
+ b*x*2i) + 1))^(1/2))/(11*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^3) - ((exp(a*
2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) +
1))^(1/2)*294464i)/(45045*b*d^2*(exp(a*2i + b*x*2i) - 1)^2) - ((exp(a*2i +
b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(
1/2)*24608i)/(2145*b*d^2*(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i)
 + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*13
```

```
5104i)/(9009*b*d^2*(exp(a*2i + b*x*2i) - 1)^4) - ((exp(a*2i + b*x*2i) + 1)*
(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*13088i)/
(1287*b*d^2*(exp(a*2i + b*x*2i) - 1)^5) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(e
xp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*384i)/(143*b*d^
2*(exp(a*2i + b*x*2i) - 1)^6) - (55808*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a
*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(6435*b*d^2*(exp(a
*2i + b*x*2i)*1i - 1i)) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i
)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*7424i)/(1155*b*d^2*(exp(a*2i +
b*x*2i)*1i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i
 - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*18368i)/(2145*b*d^2*(exp(a*2i + b*x
*2i) - 1)) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2)*228736i)/(9009*b*d^2*(exp(a*2i + b*x*2i)*1i
- 1i)^4) - (17152*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i
))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(429*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)
^5) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2
i + b*x*2i) + 1))^(1/2)*4608i)/(143*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^6) +
 (128*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i
 + b*x*2i) + 1))^(1/2))/(13*b*d^2*(exp(a*2i + b*x*2i)*1i - 1i)^7)
```

3.99 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}+\frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{12bd^2}$$

```
[Out] -1/6*sin(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+1/3*sin(b*x+a)^3/b/d/(d*tan(b*x+a)
)^(1/2)-1/12*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ellip
ticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
/d^2
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2676, 2678, 2681, 2653, 2720}

$$\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{12bd^2}+\frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}}-\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/6*Sin[a + b*x]/(b*d*Sqrt[d*Tan[a + b*x]]) + Sin[a + b*x]^3/(3*b*d*Sqrt[d
*Tan[a + b*x]]) + (Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a +
 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(12*b*d^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
```

```
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\int \sin(a+bx)\sqrt{d\tan(a+bx)}\,dx}{6d^2}$$

$$= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,\ldots}{12d^2}$$

$$= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\left(\sqrt{\cos(a+bx)}\sqrt{d\tan(a+b\ldots}\right.}{12d^2\ldots}$$

$$= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\left(\csc(a+bx)\sqrt{\sin(2a+2bx)}\ldots\right.}{\ldots}$$

$$= -\frac{\sin(a+bx)}{6bd\sqrt{d\tan(a+bx)}} + \frac{\sin^3(a+bx)}{3bd\sqrt{d\tan(a+bx)}} + \frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\ldots}{12\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.43, size = 102, normalized size = 0.91

$$-\frac{\csc(a+bx)\left(\sqrt{\sec^2(a+bx)}\sin(4(a+bx))+4\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\sqrt{\tan(a+bx)}\right)\sqrt{d\tan(a+bx)}}{24bd^2\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

[Out] -1/24*(Csc[a + b*x]*(Sqrt[Sec[a + b*x]^2]*Sin[4*(a + b*x)] + 4*(-1)^(1/4)*E
llipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Tan[a + b*x]])*
Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sec[a + b*x]^2])

Maple [A]
time = 0.33, size = 222, normalized size = 1.98

method	result
default	$-\frac{(-1+\cos(bx+a))\left(\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sin(bx+a)\,\mathrm{EllipticF}\left(\sqrt{\frac{1-c}{}}\right.\right.}{12b\sin(bx+a)^2\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/12/b*(-1+cos(b*x+a))*(((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*
sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/
2))+2*cos(b*x+a)^4*2^(1/2)-2*cos(b*x+a)^3*2^(1/2)-cos(b*x+a)^2*2^(1/2)+cos(
b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/sin(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(3/
2)/cos(b*x+a)^2*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")

[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^2*tan(b
*x + a)^2), x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^3}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(sin(a + b*x)^3/(d*tan(a + b*x))^(3/2), x)
```

3.100 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2bd\sqrt{d\tan(a+bx)}}$$

```
[Out] sin(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)-1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1
/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(
1/2)/b/d/(d*tan(b*x+a))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2682, 2649, 2653, 2720}

$$\frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] Sin[a + b*x]/(b*d*Sqrt[d*Tan[a + b*x]]) + (EllipticF[a - Pi/4 + b*x, 2]*Sec
[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*d*Sqrt[d*Tan[a + b*x]])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
```

```
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= \frac{\sqrt{\sin(a+bx)}\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sqrt{\sin(a+bx)}}\,dx}{d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{2d\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}} \\
&= \frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{\left(\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{2d\sqrt{d\tan(a+bx)}} \\
&= \frac{\sin(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2bd\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.82, size = 126, normalized size = 1.59

$$\frac{\cos(2(a+bx))\sec(a+bx)\left(\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\sec^2(a+bx)-\sqrt{\sec^2(a+bx)}\sqrt{\tan(a+bx)}\right)\tan^{\frac{3}{2}}(a+bx)}{b\sqrt{\sec^2(a+bx)}\,(d\tan(a+bx))^{3/2}\left(-1+\tan^2(a+bx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Cos[2*(a + b*x)]*Sec[a + b*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*S
qrt[Tan[a + b*x]]], -1]*Sec[a + b*x]^2 - Sqrt[Sec[a + b*x]^2]*Sqrt[Tan[a +
b*x]])*Tan[a + b*x]^(3/2))/(b*Sqrt[Sec[a + b*x]^2]*(d*Tan[a + b*x])^(3/2)*(
-1 + Tan[a + b*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(96) = 192.
time = 0.32, size = 197, normalized size = 2.49

method	result

default	$\frac{(\cos(bx+a)+1)^2(-1+\cos(bx+a))\left(-\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sin(bx+a)\,\mathrm{Ell}\right.}{2b\cos(bx+a)^2\sin(bx+a)^2\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(cos(b*x+a)+1)^2*(-1+cos(b*x+a))*(-((-1+cos(b*x+a))/sin(b*x+a))^(1/2)
*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/si
n(b*x+a))^(1/2)*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))/cos(b*x+a)^2/s
in(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(d*tan(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)/(d*tan(a + b*x))^(3/2), x)
```

3.101 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}-\frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}{3bd^2}$$

```
[Out] -2/3*csc(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+1/3*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2
*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b/d^2
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2677, 2681, 2653, 2720}

$$-\frac{\sqrt{\sin(2a+2bx)}\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{3bd^2}-\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Csc[a + b*x])/(3*b*d*Sqrt[d*Tan[a + b*x]]) - (Csc[a + b*x]*EllipticF[a
- Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(3*b*d^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
```

```
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}} - \frac{\int \csc(a+bx)\sqrt{d\tan(a+bx)}\,dx}{3d^2}$$

$$= -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}} - \frac{\left(\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\ }}}{3d^2\sqrt{\sin(a+bx)}}$$

$$= -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}} - \frac{\left(\csc(a+bx)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}\right)\int \frac{}{\sqrt{\mathrm{si}}}}{3d^2}$$

$$= -\frac{2\csc(a+bx)}{3bd\sqrt{d\tan(a+bx)}} - \frac{\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}\sqrt{d\tan(}}{3bd^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.80, size = 110, normalized size = 1.34

$$\frac{2\cos(2(a+bx))\sec(a+bx)\sqrt{\sec^2(a+bx)}\left(\sqrt{\sec^2(a+bx)}-\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\tan^{\frac{3}{2}}(a+bx)\right)}{3b(d\tan(a+bx))^{3/2}\left(-1+\tan^2(a+bx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*Cos[2*(a + b*x)]*Sec[a + b*x]*Sqrt[Sec[a + b*x]^2]*(Sqrt[Sec[a + b*x]^2]
 - (-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Tan[a
 + b*x]^(3/2)))/(3*b*(d*Tan[a + b*x])^(3/2)*(-1 + Tan[a + b*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(97) = 194.
time = 0.33, size = 302, normalized size = 3.68

method	result

default	$-\frac{(-1+\cos(bx+a))^2\left(\text{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(b}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(-1+cos(b*x+a))^2*(EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*
x+a)*cos(b*x+a)+((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a
)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b
*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/sin(b*x+a)^4/(d*sin(b*x+a)/cos(b*x+a))^(3/2
)/cos(b*x+a)^2*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.10, size = 117, normalized size = 1.43

$$\frac{(\cos(bx+a)^2-1)\sqrt{i d}\,\text{ellipticF}(\cos(bx+a)+i\sin(bx+a),-1)+(\cos(bx+a)^2-1)\sqrt{-i d}\,\text{ellipticF}(\cos(bx+a)-i\sin(bx+a),-1)+2\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)}{3\,(bd^2\cos(bx+a)^2-bd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*ellipticF(cos(b*x + a) + I*sin(b*x + a)
, -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*ellipticF(cos(b*x + a) - I*sin(b*x
+ a), -1) + 2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a))/(b*d^2*cos(b*
x + a)^2 - b*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)\,{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(3/2)), x)
```

3.102 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=112

$$\frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sqrt{\sin(2a+2bx)}\,\sqrt{d\tan(a+bx)}}{21bd^2}$$

```
[Out] 2/21*csc(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)-2/7*csc(b*x+a)^3/b/d/(d*tan(b*x+a)
)^(1/2)+2/21*csc(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ellip
ticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)*(d*tan(b*x+a))^(1/2)/b
/d^2
```

Rubi [A]
time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2677, 2679, 2681, 2653, 2720}

$$-\frac{2\sqrt{\sin(2a+2bx)}\,\csc(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|\,2\right)\sqrt{d\tan(a+bx)}}{21bd^2}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}+\frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*Csc[a + b*x])/(21*b*d*Sqrt[d*Tan[a + b*x]]) - (2*Csc[a + b*x]^3)/(7*b*d*
Sqrt[d*Tan[a + b*x]]) - (2*Csc[a + b*x]*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[S
in[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])/(21*b*d^2)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
```

```
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= -\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}-\frac{\int \csc^3(a+bx)\sqrt{d\tan(a+bx)}\,dx}{7d^2}\\ &= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}-\frac{2\int \csc(a+bx)\sqrt{d\tan(a+bx)}}{21d^2}\\ &= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}-\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+b}{21d^2}\\ &= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}-\left(2\csc(a+bx)\sqrt{\sin(2a+2bx)}\right.\\ &= \frac{2\csc(a+bx)}{21bd\sqrt{d\tan(a+bx)}}-\frac{2\csc^3(a+bx)}{7bd\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{21}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.82, size = 136, normalized size = 1.21

$$\frac{\csc^3(a+bx)\left(\left(1+10\cos(2(a+bx))+\cos(4(a+bx))\right)\sec^2(a+bx)^{3/2}-8\sqrt[4]{-1}\cos(2(a+bx))F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\tan^{\frac{7}{2}}(a+bx)\right)}{42bd\sqrt{\sec^2(a+bx)}\sqrt{d\tan(a+bx)}\left(-1+\tan^2(a+bx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Csc[a + b*x]^3*((1 + 10*Cos[2*(a + b*x)] + Cos[4*(a + b*x)])*(Sec[a + b*x]
^2)^(3/2) - 8*(-1)^(1/4)*Cos[2*(a + b*x)]*EllipticF[I*ArcSinh[(-1)^(1/4)*Sq
rt[Tan[a + b*x]]], -1]*Tan[a + b*x]^(7/2)))/(42*b*d*Sqrt[Sec[a + b*x]^2]*Sq
rt[d*Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(123) = 246$.
time = 0.36, size = 557, normalized size = 4.97

method	result
default	$-\frac{(-1+\cos(bx+a))^2\left(-2\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,(\cos^3(bx+a))\sin(bx+a)\,\mathrm{E}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/b*(-1+cos(b*x+a))^2*(-2*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(
1/2)*cos(b*x+a)^3*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2),1/2*2^(1/2))-2*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*s
in(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2
))*cos(b*x+a)^2+2*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/
2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*cos
(b*x+a)+2*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin
(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*Elli
pticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^
3*2^(1/2)+2*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)^2/sin(b*x+a)^6/
(d*sin(b*x+a)/cos(b*x+a))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.10, size = 161, normalized size = 1.44

$$\frac{2\left((\cos(bx+a)^4-2\cos(bx+a)^2+1)\sqrt{i\,d}\,\mathrm{ellipticF}(\cos(bx+a)+i\sin(bx+a),-1)+(\cos(bx+a)^4-2\cos(bx+a)^2+1)\sqrt{-i\,d}\,\mathrm{ellipticF}(\cos(bx+a)-i\sin(bx+a),-1)-(\cos(bx+a)^3+2\cos(bx+a))\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\right)}{21\,(bd^2\cos(bx+a)^4-2\,bd^2\cos(bx+a)^2+bd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(I*d)*ellipticF(cos(b*x +
 a) + I*sin(b*x + a), -1) + (cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(-I
*d)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1) - (cos(b*x + a)^3 + 2*cos(
b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*d^2*cos(b*x + a)^4 - 2*b*d^
2*cos(b*x + a)^2 + b*d^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/(d*tan(a + b*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(a+b\,x)^3\,(d\tan(a+b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(3/2)), x)
```

3.103 $\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,bd^{5/2}}+\frac{3\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,bd^{5/2}}-\frac{3\log\left(\sqrt{d}+\sqrt{d}\,\tan(a+\right.}{64\sqrt{}}$$

```
[Out] -3/64*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)+3/64
*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)-3/128*ln(
d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(5/2)*2^(1/2)+
3/128*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(5/2)
*2^(1/2)+1/16*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d^3-1/4*cos(b*x+a)^4*(d*t
an(b*x+a))^(1/2)/b/d^3
```

Rubi [A]
time = 0.13, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2671, 294, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,bd^{5/2}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}\,bd^{5/2}}-\frac{3\log\left(\sqrt{d}\,\tan(a+bx)-\sqrt{2}\,\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,bd^{5/2}}+\frac{3\log\left(\sqrt{d}\,\tan(a+bx)+\sqrt{2}\,\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{64\sqrt{2}\,bd^{5/2}}-\frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3}+\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*d^(5/
2)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(32*Sqrt[2]*b*
d^(5/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b
*x]]])/(64*Sqrt[2]*b*d^(5/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqr
t[2]*Sqrt[d*Tan[a + b*x]]])/(64*Sqrt[2]*b*d^(5/2)) + (Cos[a + b*x]^2*Sqrt[d
*Tan[a + b*x]])/(16*b*d^3) - (Cos[a + b*x]^4*Sqrt[d*Tan[a + b*x]])/(4*b*d^3
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
```

```
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{d\text{Subst}\left(\int \frac{x^{3/2}}{(d^2+x^2)^3}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= -\frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{8b}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(\ldots}\right.}{\ldots}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{3\text{Subst}\left(\int \frac{1}{d^2+x^4}\ldots\right.}{\ldots}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} + \frac{3\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\ldots\right.}{\ldots}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{16bd^3} - \frac{\cos^4(a+bx)\sqrt{d\tan(a+bx)}}{4bd^3} - \frac{3\text{Subst}\left(\int \frac{\sqrt{2}\ldots}{-d-\sqrt{\ldots}}\right.}{\ldots}$$

$$= -\frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{64\sqrt{2}\,bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\tan\ldots\right.}{\ldots}$$

$$= -\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,bd^{5/2}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{32\sqrt{2}\,bd^{5/2}} - \ldots$$

Mathematica [A]
time = 0.81, size = 123, normalized size = 0.48

$$-\frac{\csc(a+bx)\left(\sin(a+bx)+3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}+2\sin(3(a+bx))+\sin(5(a+bx))\right)\sqrt{d\tan(a+bx)}}{64bd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/64*(Csc[a + b*x]*(Sin[a + b*x] + 3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*S
qrt[Sin[2*(a + b*x)]] - 3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a +
 b*x)]]]*Sqrt[Sin[2*(a + b*x)]] + 2*Sin[3*(a + b*x)] + Sin[5*(a + b*x)])*Sq
rt[d*Tan[a + b*x]])/(b*d^3)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.31, size = 688, normalized size = 2.68

method	result
default	$-\frac{(-1+\cos(bx+a))\left(3i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sin(bx+a)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin}{\sin(bx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/b*(-1+cos(b*x+a))*(3*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)*sin(b*x+a)-3*I*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*El
lipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)*sin(b*x+a)+8*2^(1/2)*cos(b*x+a)^5+3*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))
/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b
*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)-6*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(
1/2)*sin(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*
2^(1/2))+3*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)
-8*cos(b*x+a)^4*2^(1/2)-2*cos(b*x+a)^3*2^(1/2)+2*cos(b*x+a)^2*2^(1/2))*(cos
(b*x+a)+1)^2/cos(b*x+a)^3/sin(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(5/2)*2^(1/2
)
```

Maxima [A]
time = 0.50, size = 219, normalized size = 0.85

$$\frac{6\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{d}+2\,\sqrt{d\tan\left(bx+a\right)}\right)}{2\,\sqrt{d}}\right)+6\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{d}-2\,\sqrt{d\tan\left(bx+a\right)}\right)}{2\,\sqrt{d}}\right)+3\sqrt{2}\,d^{\frac{5}{2}}\log\left(d\tan\left(bx+a\right)+\sqrt{2}\,\sqrt{d\tan\left(bx+a\right)}\,\sqrt{d}+d\right)-3\sqrt{2}\,d^{\frac{5}{2}}\log\left(d\tan\left(bx+a\right)-\sqrt{2}\,\sqrt{d\tan\left(bx+a\right)}\,\sqrt{d}+d\right)+\frac{8\left(\left(d\tan(bx+a)\right)^{\frac{5}{2}}d^{4}-3\sqrt{d\tan\left(bx+a\right)}\,d^{6}\right)}{d^{4}\tan(bx+a)^{4}+2\,d^{4}\tan(bx+a)^{2}+d^{4}}}{128\,bd^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/128*(6*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan
(b*x + a)))/sqrt(d)) + 6*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(
d) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 3*sqrt(2)*d^(5/2)*log(d*tan(b*x + a
) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 3*sqrt(2)*d^(5/2)*log(d*tan
(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) + 8*((d*tan(b*x + a))
^(5/2)*d^4 - 3*sqrt(d*tan(b*x + a))*d^6)/(d^4*tan(b*x + a)^4 + 2*d^4*tan(b*
x + a)^2 + d^4))/(b*d^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1558 vs. $2(197) = 394$.
time = 39.44, size = 1558, normalized size = 6.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/256*(6*sqrt(2)*b*d^3*(1/(b^4*d^10))^(1/4)*arctan(1/2*(sqrt(4*b^2*d^5*sqr
t(1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^7*(1/(b^4*d^10
))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a)*s
in(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*((sqrt(2)*b^3*d^7*(1/(b
^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x +
 a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 2*sin(b*x + a)) - (sqrt(2)*b^3*d^7
*(1/(b^4*d^10))^(3/4)*sin(b*x + a) - sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos
(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/sin(b*x + a)) + 6*sqrt(2)*b*d
^3*(1/(b^4*d^10))^(1/4)*arctan(1/2*(sqrt(4*b^2*d^5*sqrt(1/(b^4*d^10))*cos(b
*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*cos(b*x + a)
^2 + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*s
in(b*x + a)/cos(b*x + a)) + 1)*((sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*sin(b
*x + a) + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x +
 a)/cos(b*x + a)) - 2*sin(b*x + a)) - (sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)
*sin(b*x + a) - sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*sin
(b*x + a)/cos(b*x + a)))/sin(b*x + a)) + 6*sqrt(2)*b*d^3*(1/(b^4*d^10))^(1/
4)*arctan(1/2*((sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)
*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(4*b^2*d^5*sqrt(1/(b^4*d^10))
*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*cos(b*
x + a)^2 + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sq
rt(d*sin(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + (s
qrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*d^2*(1/(b^4*d^
10))^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^5*cos
(b*x + a)^3 - b^2*d^5*cos(b*x + a))*sqrt(1/(b^4*d^10)) + 2*sin(b*x + a))/((
2*cos(b*x + a)^2 - 1)*sin(b*x + a))) + 6*sqrt(2)*b*d^3*(1/(b^4*d^10))^(1/4)
*arctan(1/2*((sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b
*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(4*b^2*d^5*sqrt(1/(b^4*d^10))*c
```

```
os(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*cos(b*x
+ a)^2 + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt
(d*sin(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + (sqr
t(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*d^2*(1/(b^4*d^10
))^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 4*(b^2*d^5*cos(b
*x + a)^3 - b^2*d^5*cos(b*x + a))*sqrt(1/(b^4*d^10)) - 2*sin(b*x + a))/((2*
cos(b*x + a)^2 - 1)*sin(b*x + a))) - 3*sqrt(2)*b*d^3*(1/(b^4*d^10))^(1/4)*l
og(4*b^2*d^5*sqrt(1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*
d^7*(1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4
)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + 3*sqr
t(2)*b*d^3*(1/(b^4*d^10))^(1/4)*log(4*b^2*d^5*sqrt(1/(b^4*d^10))*cos(b*x +
a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 +
sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*
x + a)/cos(b*x + a)) + 1) + 16*(4*cos(b*x + a)^4 - cos(b*x + a)^2)*sqrt(d*s
in(b*x + a)/cos(b*x + a)))/(b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4{(a + bx)}}{(d\tan{(a + bx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**4/(d*tan(a + b*x))**(5/2), x)
```

Giac [A]
time = 0.62, size = 248, normalized size = 0.96

$$\frac{3\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd^3}+\frac{3\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{64\,bd^3}+\frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{128\,bd^3}-\frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{128\,bd^3}+\frac{\sqrt{d\tan(bx+a)}\,d^2\tan(bx+a)^2-3\sqrt{d\tan(bx+a)}\,d^2}{16\left(d^2\tan(bx+a)^2+d^2\right)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt
(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^3) + 3/64*sqrt(2)*sqrt(abs(d))*arctan(
-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/
(b*d^3) + 3/128*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*ta
n(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^3) - 3/128*sqrt(2)*sqrt(abs(d))*log
(d*tan(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^
3) + 1/16*(sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)^2 - 3*sqrt(d*tan(b*x + a))
*d^2)/((d^2*tan(b*x + a)^2 + d^2)^2*b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(a+b\,x\right)}^4}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/(d*tan(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^4/(d*tan(a + b*x))^(5/2), x)
```

3.104 $\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,bd^{5/2}}+\frac{3\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,bd^{5/2}}-\frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a\cdots\right.}{8\sqrt{\cdots}}$$

```
[Out] -3/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)+3/8*a
rctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)*2^(1/2)-3/16*ln(d^(
1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(5/2)*2^(1/2)+3/1
6*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(5/2)*2^(
1/2)+1/2*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b/d^3
```

Rubi [A]
time = 0.12, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2671, 296, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,bd^{5/2}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}\,bd^{5/2}}-\frac{3\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,bd^{5/2}}+\frac{3\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,bd^{5/2}}+\frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(5/2
)) + (3*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^
(5/2)) - (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x
]]])/(8*Sqrt[2]*b*d^(5/2)) + (3*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2
]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b*d^(5/2)) + (Cos[a + b*x]^2*Sqrt[d*Tan
[a + b*x]])/(2*b*d^3)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)^2}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\tan(a+bx)\right)}{4bd}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} + \frac{3\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2bd}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} + \frac{3\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{4bd^2} + \frac{3\text{Su}}{}$$

$$= \frac{\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2bd^3} - \frac{3\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\tan(a+b}\right.}{8\sqrt{2}\,bd^{5/2}}$$

$$= -\frac{3\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}\,bd^{5/2}} + \frac{3\log\left(\sqrt{d}+\sqrt{d}\,\text{t}\right.}{}$$

$$= -\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,bd^{5/2}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,bd^{5/2}} -$$

Mathematica [A]
time = 0.59, size = 113, normalized size = 0.50

$$\frac{\csc(a+bx)\left(\sin(a+bx)-3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}+3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}+\sin(3(a+bx))\right)\sqrt{d\tan(a+bx)}}{8bd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (Csc[a + b*x]*(Sin[a + b*x] - 3*ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sqrt[Si
n[2*(a + b*x)]] + 3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]
]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/(8*b*d^
3)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.30, size = 662, normalized size = 2.92

method	result
default	$-\frac{(-1+\cos(bx+a))\left(3i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sin(bx+a)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b*(-1+cos(b*x+a))*(3*I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)*sin(b*x+a)-3*I*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*Ell
ipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))
*sin(b*x+a)+3*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1
/2*I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1
/2)-6*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)*Elliptic
F(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+3*EllipticPi(((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a
)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b*x+a)^3*2^(1/2)+
2*cos(b*x+a)^2*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)^3/sin(b*x+a)/(d*sin(b*x
+a)/cos(b*x+a))^(5/2)*2^(1/2)
```

Maxima [A]
time = 0.50, size = 189, normalized size = 0.83

$$\frac{6\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+6\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+3\sqrt{2}\sqrt{d}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)-3\sqrt{2}\sqrt{d}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)+\frac{8\sqrt{d\tan(bx+a)}\,d^2}{d^2\tan(bx+a)^2+d^2}}{16\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(6*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d)) + 6*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + 3*sqrt(2)*sqrt(d)*log(d*tan(b*x + a)
 + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 3*sqrt(2)*sqrt(d)*log(d*tan(
b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) + 8*sqrt(d*tan(b*x + a
))*d^2/(d^2*tan(b*x + a)^2 + d^2))/(b*d^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. $2(171) = 342$.

time = 38.59, size = 1545, normalized size = 6.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(6*sqrt(2)*b*d^3*(1/(b^4*d^10))^(1/4)*arctan(1/2*(sqrt(4*b^2*d^5*sqrt
(1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^7*(1/(b^4*d^10)
)^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a)*si
n(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*((sqrt(2)*b^3*d^7*(1/(b^
4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x +
a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 2*sin(b*x + a)) - (sqrt(2)*b^3*d^7*
(1/(b^4*d^10))^(3/4)*sin(b*x + a) - sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(
b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/sin(b*x + a)) + 6*sqrt(2)*b*d^
3*(1/(b^4*d^10))^(1/4)*arctan(1/2*(sqrt(4*b^2*d^5*sqrt(1/(b^4*d^10))*cos(b*
x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*cos(b*x + a)^
2 + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*si
n(b*x + a)/cos(b*x + a)) + 1)*((sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*sin(b*
x + a) + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x +
a)/cos(b*x + a)) - 2*sin(b*x + a)) - (sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*
sin(b*x + a) - sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*sin(
b*x + a)/cos(b*x + a)))/sin(b*x + a)) + 6*sqrt(2)*b*d^3*(1/(b^4*d^10))^(1/4
)*arctan(1/2*((sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*
b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(4*b^2*d^5*sqrt(1/(b^4*d^10))*
cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*cos(b*x
 + a)^2 + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqr
t(d*sin(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + (sq
rt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*d^2*(1/(b^4*d^1
0))^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^5*cos(
b*x + a)^3 - b^2*d^5*cos(b*x + a))*sqrt(1/(b^4*d^10)) + 2*sin(b*x + a))/((2
*cos(b*x + a)^2 - 1)*sin(b*x + a))) + 6*sqrt(2)*b*d^3*(1/(b^4*d^10))^(1/4)*
arctan(1/2*((sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*
d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(4*b^2*d^5*sqrt(1/(b^4*d^10))*co
s(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*cos(b*x +
 a)^2 + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(
d*sin(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + (sqrt
(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*d^2*(1/(b^4*d^10)
)^(1/4)*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 4*(b^2*d^5*cos(b*
x + a)^3 - b^2*d^5*cos(b*x + a))*sqrt(1/(b^4*d^10)) - 2*sin(b*x + a))/((2*c
os(b*x + a)^2 - 1)*sin(b*x + a))) - 3*sqrt(2)*b*d^3*(1/(b^4*d^10))^(1/4)*lo
g(4*b^2*d^5*sqrt(1/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d
^7*(1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)
*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + 3*sqrt
(2)*b*d^3*(1/(b^4*d^10))^(1/4)*log(4*b^2*d^5*sqrt(1/(b^4*d^10))*cos(b*x + a
```

```
)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^7*(1/(b^4*d^10))^(3/4)*cos(b*x + a)^2 + s
qrt(2)*b*d^2*(1/(b^4*d^10))^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x
 + a)/cos(b*x + a)) + 1) - 16*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a
)^2)/(b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**2/(d*tan(a + b*x))**(5/2), x)
```

Giac [A]
time = 0.60, size = 220, normalized size = 0.97

$$\frac{3\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd^3}+\frac{3\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{8\,bd^3}+\frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{16\,bd^3}-\frac{3\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{16\,bd^3}+\frac{\sqrt{d\tan(bx+a)}}{2\left(d^2\tan(bx+a)^2+d^2\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqrt(
d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^3) + 3/8*sqrt(2)*sqrt(abs(d))*arctan(-1
/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b
*d^3) + 3/16*sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b
*x + a))*sqrt(abs(d)) + abs(d))/(b*d^3) - 3/16*sqrt(2)*sqrt(abs(d))*log(d*t
an(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^3) +
 1/2*sqrt(d*tan(b*x + a))/((d^2*tan(b*x + a)^2 + d^2)*b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(a+b\,x\right)}^2}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(d*tan(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)^2/(d*tan(a + b*x))^(5/2), x)
```

3.105 $\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[Out] -2/7*d/b/(d*tan(b*x+a))^(7/2)
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2671, 30}

$$-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= \frac{d\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= -\frac{2d}{7b(d\tan(a+bx))^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 20, normalized size = 1.00

$$-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.
time = 0.34, size = 38, normalized size = 1.90

method	result	size
default	$-\frac{2\cos(bx+a)}{7b\sin(bx+a)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{5}{2}}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/b*cos(b*x+a)/sin(b*x+a)/(d*sin(b*x+a)/cos(b*x+a))^(5/2)
```

Maxima [A]
time = 0.27, size = 23, normalized size = 1.15

$$-\frac{2}{7\,\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}b\tan\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/7/((d*tan(b*x + a))^(5/2)*b*tan(b*x + a))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(16) = 32$.
time = 0.39, size = 63, normalized size = 3.15

$$-\frac{2\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}\cos\left(bx+a\right)^{4}}{7\left(bd^{3}\cos\left(bx+a\right)^{4}-2\,bd^{3}\cos\left(bx+a\right)^{2}+bd^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^4/(b*d^3*cos(b*x + a)^4
 - 2*b*d^3*cos(b*x + a)^2 + b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(csc(a + b*x)**2/(d*tan(a + b*x))**(5/2), x)
```

Giac [A]
time = 0.62, size = 26, normalized size = 1.30

$$-\frac{2}{7\sqrt{d\tan(bx+a)}\,bd^2\tan(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/7/(sqrt(d*tan(b*x + a))*b*d^2*tan(b*x + a)^3)
```

Mupad [B]
time = 7.40, size = 530, normalized size = 26.50

$$\frac{46\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\,d^3\left(e^{a\,2i+b\,x\,2i}-1\right)}+\frac{12\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{5\,b\,d^3\left(e^{a\,2i+b\,x\,2i}-1\right)^2}+\frac{24\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{35\,b\,d^3\left(e^{a\,2i+b\,x\,2i}-1\right)^3}-\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,48i}{7\,b\,d^3\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}+\frac{144\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{35\,b\,d^3\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^2}+\frac{\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}\,144i}{35\,b\,d^3\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^3}-\frac{16\left(e^{a\,2i+b\,x\,2i}+1\right)\sqrt{-\frac{d\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)}{e^{a\,2i+b\,x\,2i}+1}}}{7\,b\,d^3\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*(d*tan(a + b*x))^(5/2)),x)
```

```
[Out] (46*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
 b*x*2i) + 1))^(1/2))/(7*b*d^3*(exp(a*2i + b*x*2i) - 1)) + (12*(exp(a*2i +
b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(
1/2))/(5*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) + (24*(exp(a*2i + b*x*2i) + 1)*(
-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35*b*d^
3*(exp(a*2i + b*x*2i) - 1)^3) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*48i)/(7*b*d^3*(exp(a*2i +
 b*x*2i)*1i - 1i)) + (144*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)
*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(35*b*d^3*(exp(a*2i + b*x*2i)*1
i - 1i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2)*144i)/(35*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)
^3) - (16*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(
a*2i + b*x*2i) + 1))^(1/2))/(7*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^4)
```

3.106 $\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{2d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[Out] -2/11*d^3/b/(d*tan(b*x+a))^(11/2)-2/7*d/b/(d*tan(b*x+a))^(7/2)
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 14}

$$-\frac{2d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^3)/(11*b*(d*Tan[a + b*x])^(11/2)) - (2*d)/(7*b*(d*Tan[a + b*x])^(7/2)
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx&=\frac{d\text{Subst}\left(\int \frac{d^2+x^2}{x^{13/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\&=\frac{d\text{Subst}\left(\int \left(\frac{d^2}{x^{13/2}}+\frac{1}{x^{9/2}}\right)\,dx,x,d\tan(a+bx)\right)}{b}\\&=-\frac{2d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 50, normalized size = 1.16

$$\frac{2(-9+2\cos(2(a+bx)))\cot^4(a+bx)\csc^2(a+bx)\sqrt{d\tan(a+bx)}}{77bd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*(-9 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^4*Csc[a + b*x]^2*Sqrt[d*Tan[a + b
*x]])/(77*b*d^3)
```

Maple [A]
time = 0.36, size = 50, normalized size = 1.16

method	result	size
default	$\frac{2(4(\cos^2(bx+a))-11)\cos(bx+a)}{77b\sin(bx+a)^3\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{5}{2}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77/b*(4*cos(b*x+a)^2-11)*cos(b*x+a)/sin(b*x+a)^3/(d*sin(b*x+a)/cos(b*x+a)
)^(5/2)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.81

$$-\frac{2\left(11\,d^2\tan\left(bx+a\right)^2+7\,d^2\right)d}{77\,\left(d\tan\left(bx+a\right)\right)^{\frac{11}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/77*(11*d^2*tan(b*x + a)^2 + 7*d^2)*d/((d*tan(b*x + a))^(11/2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.
time = 0.44, size = 91, normalized size = 2.12

$$-\frac{2\left(4\,\cos\left(bx+a\right)^6-11\,\cos\left(bx+a\right)^4\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{77\left(bd^3\cos\left(bx+a\right)^6-3\,bd^3\cos\left(bx+a\right)^4+3\,bd^3\cos\left(bx+a\right)^2-bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*cos(b*x + a)^6 - 11*cos(b*x + a)^4)*sqrt(d*sin(b*x + a)/cos(b*x +
a))/(b*d^3*cos(b*x + a)^6 - 3*b*d^3*cos(b*x + a)^4 + 3*b*d^3*cos(b*x + a)^2
 - b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**4/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(csc(a + b*x)**4/(d*tan(a + b*x))**(5/2), x)
```

Giac [A]
time = 0.82, size = 45, normalized size = 1.05

$$-\frac{2\left(11\,d^3\tan(bx+a)^2+7\,d^3\right)}{77\,\sqrt{d\tan(bx+a)}\,bd^5\tan(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^4/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/77*(11*d^3*tan(b*x + a)^2 + 7*d^3)/(sqrt(d*tan(b*x + a))*b*d^5*tan(b*x +
 a)^5)
```

Mupad [B]
time = 12.14, size = 831, normalized size = 19.33

$$-\frac{14456\,(e^{a\,2i+b\,x\,2i}+1)\sqrt{-\frac{d\,(e^{a\,2i+b\,x\,2i}\,1i-i)}{e^{a\,2i+b\,x\,2i}+1}}}{1155\,b\,d^3\,(e^{a\,2i+b\,x\,2i}-1)}-\frac{7768\,(e^{a\,2i+b\,x\,2i}+1)\sqrt{-\frac{d\,(e^{a\,2i+b\,x\,2i}\,1i-i)}{e^{a\,2i+b\,x\,2i}+1}}}{945\,b\,d^3\,(e^{a\,2i+b\,x\,2i}-1)^2}-\frac{4232\,(e^{a\,2i+b\,x\,2i}+1)\sqrt{-\frac{d\,(e^{a\,2i+b\,x\,2i}\,1i-i)}{e^{a\,2i+b\,x\,2i}+1}}}{495\,b\,d^3\,(e^{a\,2i+b\,x\,2i}-1)^3}-\frac{1328\,(e^{a\,2i+b\,x\,2i}+1)\sqrt{-\frac{d\,(e^{a\,2i+b\,x\,2i}\,1i-i)}{e^{a\,2i+b\,x\,2i}+1}}}{231\,b\,d^3\,(e^{a\,2i+b\,x\,2i}-1)^4}-\frac{160\,(e^{a\,2i+b\,x\,2i}+1)\sqrt{-\frac{d\,(e^{a\,2i+b\,x\,2i}\,1i-i)}{e^{a\,2i+b\,x\,2i}+1}}}{99\,b\,d^3\,(e^{a\,2i+b\,x\,2i}-1)^5}+\frac{(e^{a\,2i+b\,x\,2i}+1)\sqrt{-\frac{d\,(e^{a\,2i+b\,x\,2i}\,1i-i)}{e^{a\,2i+b\,x\,2i}+1}}\,2048i}{165\,b\,d^3\,(e^{a\,2i+b\,x\,2i}\,1i-i)}-\frac{86528\,(e^{a\,2i+b\,x\,2i}+1)\sqrt{-\frac{d\,(e^{a\,2i+b\,x\,2i}\,1i-i)}{e^{a\,2i+b\,x\,2i}+1}}}{10395\,b\,d^3\,(e^{a\,2i+b\,x\,2i}\,1i-i)^2}-\frac{(e^{a\,2i+b\,x\,2i}+1)\sqrt{-\frac{d\,(e^{a\,2i+b\,x\,2i}\,1i-i)}{e^{a\,2i+b\,x\,2i}+1}}\,3904i}{315\,b\,d^3\,(e^{a\,2i+b\,x\,2i}\,1i-i)^3}+\frac{4160\,(e^{a\,2i+b\,x\,2i}+1)\sqrt{-\frac{d\,(e^{a\,2i+b\,x\,2i}\,1i-i)}{e^{a\,2i+b\,x\,2i}+1}}}{231\,b\,d^3\,(e^{a\,2i+b\,x\,2i}\,1i-i)^4}+\frac{(e^{a\,2i+b\,x\,2i}+1)\sqrt{-\frac{d\,(e^{a\,2i+b\,x\,2i}\,1i-i)}{e^{a\,2i+b\,x\,2i}+1}}\,1600i}{99\,b\,d^3\,(e^{a\,2i+b\,x\,2i}\,1i-i)^5}-\frac{64\,(e^{a\,2i+b\,x\,2i}+1)\sqrt{-\frac{d\,(e^{a\,2i+b\,x\,2i}\,1i-i)}{e^{a\,2i+b\,x\,2i}+1}}}{11\,b\,d^3\,(e^{a\,2i+b\,x\,2i}\,1i-i)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^4*(d*tan(a + b*x))^(5/2)),x)
```

```
[Out] ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2)*2048i)/(165*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)) - (7768*(
exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*
2i) + 1))^(1/2))/(945*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) - (4232*(exp(a*2i +
 b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^
(1/2))/(495*b*d^3*(exp(a*2i + b*x*2i) - 1)^3) - (1328*(exp(a*2i + b*x*2i) +
 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(23
1*b*d^3*(exp(a*2i + b*x*2i) - 1)^4) - (160*(exp(a*2i + b*x*2i) + 1)*(-(d*(e
xp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(99*b*d^3*(exp
```

```
(a*2i + b*x*2i) - 1)^5) - (14456*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(1155*b*d^3*(exp(a*2i +
b*x*2i) - 1)) - (86528*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i
 - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(10395*b*d^3*(exp(a*2i + b*x*2i)*1
i - 1i)^2) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2)*3904i)/(315*b*d^3*(exp(a*2i + b*x*2i)*1i - 1
i)^3) + (4160*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2))/(231*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^4)
+ ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i +
b*x*2i) + 1))^(1/2)*1600i)/(99*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (64*
(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x
*2i) + 1))^(1/2))/(11*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^6)
```

3.107 $\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2d^5}{15b(d\tan(a+bx))^{15/2}}-\frac{4d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

```
[Out] -2/15*d^5/b/(d*tan(b*x+a))^(15/2)-4/11*d^3/b/(d*tan(b*x+a))^(11/2)-2/7*d/b/
(d*tan(b*x+a))^(7/2)
```

Rubi [A]

time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2671, 276}

$$-\frac{2d^5}{15b(d\tan(a+bx))^{15/2}}-\frac{4d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^6/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*d^5)/(15*b*(d*Tan[a + b*x])^(15/2)) - (4*d^3)/(11*b*(d*Tan[a + b*x])^(1
1/2)) - (2*d)/(7*b*(d*Tan[a + b*x])^(7/2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^6(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= \frac{d\text{Subst}\left(\int \frac{(d^2+x^2)^2}{x^{17/2}}\,dx,x,d\tan(a+bx)\right)}{b}\\ &= \frac{d\text{Subst}\left(\int \left(\frac{d^4}{x^{17/2}}+\frac{2d^2}{x^{13/2}}+\frac{1}{x^{9/2}}\right)\,dx,x,d\tan(a+bx)\right)}{b}\\ &= -\frac{2d^5}{15b(d\tan(a+bx))^{15/2}}-\frac{4d^3}{11b(d\tan(a+bx))^{11/2}}-\frac{2d}{7b(d\tan(a+bx))^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 60, normalized size = 0.92

$$\frac{2(-117+44\cos(2(a+bx))-4\cos(4(a+bx)))\cot^4(a+bx)\csc^4(a+bx)\sqrt{d\tan(a+bx)}}{1155bd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^6/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*(-117 + 44*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)])*Cot[a + b*x]^4*Csc[a +
 b*x]^4*Sqrt[d*Tan[a + b*x]])/(1155*b*d^3)
```

Maple [A]
time = 0.56, size = 60, normalized size = 0.92

method	result	size
default	$-\frac{2(32(\cos^4(bx+a))-120(\cos^2(bx+a))+165)\cos(bx+a)}{1155b\sin(bx+a)^5\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{5}{2}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1155/b*(32*cos(b*x+a)^4-120*cos(b*x+a)^2+165)*cos(b*x+a)/sin(b*x+a)^5/(d
*sin(b*x+a)/cos(b*x+a))^(5/2)
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.74

$$-\frac{2\left(165\,d^4\tan\left(bx+a\right)^4+210\,d^4\tan\left(bx+a\right)^2+77\,d^4\right)d}{1155\,\left(d\tan\left(bx+a\right)\right)^{\frac{15}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(165*d^4*tan(b*x + a)^4 + 210*d^4*tan(b*x + a)^2 + 77*d^4)*d/((d*ta
n(b*x + a))^(15/2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(53) = 106$.
time = 0.43, size = 114, normalized size = 1.75

$$-\frac{2\left(32\cos\left(bx+a\right)^8-120\cos\left(bx+a\right)^6+165\cos\left(bx+a\right)^4\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{1155\left(bd^3\cos\left(bx+a\right)^8-4\,bd^3\cos\left(bx+a\right)^6+6\,bd^3\cos\left(bx+a\right)^4-4\,bd^3\cos\left(bx+a\right)^2+bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(32*cos(b*x + a)^8 - 120*cos(b*x + a)^6 + 165*cos(b*x + a)^4)*sqrt(
d*sin(b*x + a)/cos(b*x + a))/(b*d^3*cos(b*x + a)^8 - 4*b*d^3*cos(b*x + a)^6
 + 6*b*d^3*cos(b*x + a)^4 - 4*b*d^3*cos(b*x + a)^2 + b*d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**6/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.81, size = 58, normalized size = 0.89

$$-\frac{2\left(165\,d^5\tan\left(bx+a\right)^4+210\,d^5\tan\left(bx+a\right)^2+77\,d^5\right)}{1155\,\sqrt{d\tan\left(bx+a\right)}\,bd^7\tan\left(bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^6/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/1155*(165*d^5*tan(b*x + a)^4 + 210*d^5*tan(b*x + a)^2 + 77*d^5)/(sqrt(d*
tan(b*x + a))*b*d^7*tan(b*x + a)^7)
```

Mupad [B]
time = 14.01, size = 1132, normalized size = 17.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^6*(d*tan(a + b*x))^(5/2)),x)
```

```
[Out] (199232*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*
2i + b*x*2i) + 1))^(1/2))/(12285*b*d^3*(exp(a*2i + b*x*2i) - 1)) + (1581376
*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*
x*2i) + 1))^(1/2))/(135135*b*d^3*(exp(a*2i + b*x*2i) - 1)^2) + (4539104*(ex
p(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i
) + 1))^(1/2))/(225225*b*d^3*(exp(a*2i + b*x*2i) - 1)^3) + (1152*(exp(a*2i
+ b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))
^(1/2))/(35*b*d^3*(exp(a*2i + b*x*2i) - 1)^4) + (74528*(exp(a*2i + b*x*2i)
```

```
+ 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(2
145*b*d^3*(exp(a*2i + b*x*2i) - 1)^5) + (1088*(exp(a*2i + b*x*2i) + 1)*(-(d
*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(55*b*d^3*(
exp(a*2i + b*x*2i) - 1)^6) + (896*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i +
 b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(195*b*d^3*(exp(a*2i +
b*x*2i) - 1)^7) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1
i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*439808i)/(27027*b*d^3*(exp(a*2i + b*x*2
i)*1i - 1i)) + (1573888*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1
i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(135135*b*d^3*(exp(a*2i + b*x*2i)
*1i - 1i)^2) + ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))
/(exp(a*2i + b*x*2i) + 1))^(1/2)*4557824i)/(225225*b*d^3*(exp(a*2i + b*x*2i
)*1i - 1i)^3) - (7168*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i
- 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(165*b*d^3*(exp(a*2i + b*x*2i)*1i -
 1i)^4) - ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp
(a*2i + b*x*2i) + 1))^(1/2)*172288i)/(2145*b*d^3*(exp(a*2i + b*x*2i)*1i - 1
i)^5) + (5376*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2))/(55*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^6) +
 ((exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2)*12544i)/(195*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^7) - (25
6*(exp(a*2i + b*x*2i) + 1)*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b
*x*2i) + 1))^(1/2))/(15*b*d^3*(exp(a*2i + b*x*2i)*1i - 1i)^8)
```

3.108 $\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}}-\frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}+\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{40bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a-}}$$

```
[Out] -3/40*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-1/
20*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)-3/70*sin(b*x+a)^5/b/d/(d*tan(b*x+a
))^(3/2)+1/7*sin(b*x+a)^7/b/d/(d*tan(b*x+a))^(3/2)
```

Rubi [A]

time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2676, 2678, 2681, 2652, 2719}

$$\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{40bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}}-\frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}}-\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^7/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/20*Sin[a + b*x]^3/(b*d*(d*Tan[a + b*x])^(3/2)) - (3*Sin[a + b*x]^5)/(70*
b*d*(d*Tan[a + b*x])^(3/2)) + Sin[a + b*x]^7/(7*b*d*(d*Tan[a + b*x])^(3/2))
 + (3*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(40*b*d^2*Sqrt[Sin[2*a + 2
*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
```

```
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^7(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{3\int \frac{\sin^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{14d^2}$$

$$= -\frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{3\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{20d^2}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{3}{}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{(}{}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{(}{40}$$

$$= -\frac{\sin^3(a+bx)}{20bd(d\tan(a+bx))^{3/2}} - \frac{3\sin^5(a+bx)}{70bd(d\tan(a+bx))^{3/2}} + \frac{\sin^7(a+bx)}{7bd(d\tan(a+bx))^{3/2}} + \frac{}{40}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.64, size = 122, normalized size = 0.85

$$\frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}\,(15\sin(a+bx)+29\sin(3(a+bx))+9\sin(5(a+bx))-5\sin(7(a+bx)))+112\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sec(a+bx)\tan(a+bx)\right)}{2240bd^3\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^7/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(15*Sin[a + b*x] + 29*Sin[3*(
a + b*x)] + 9*Sin[5*(a + b*x)] - 5*Sin[7*(a + b*x)])) + 112*Hypergeometric2
F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a + b*x]))/(2240*b*d^3*
Sqrt[Sec[a + b*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(151) = 302$.
time = 0.38, size = 563, normalized size = 3.91

method	result
default	$-\frac{(-1+\cos(bx+a))^2\left(40\sqrt{2}\,(\cos^8(bx+a))-108(\cos^6(bx+a))\sqrt{2}+82(\cos^4(bx+a))\sqrt{2}-21\cos(bx+a)\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+}{\sin(}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/560/b*(-1+cos(b*x+a))^2*(40*2^(1/2)*cos(b*x+a)^8-108*cos(b*x+a)^6*2^(1/2
)+82*cos(b*x+a)^4*2^(1/2)-21*cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)+42*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2
),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-21*Ellipti
cF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a
))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x
+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+42*EllipticE(((1-cos(b*x+a)+sin(b*x+a))
/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)+7*cos(b*x+a)^2*2^(1/2)-21*cos(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/cos(b
*x+a)^3/sin(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^7/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^6 - 3*cos(b*x + a)^4 + 3*cos(b*x + a)^2 - 1)*sqrt(d
*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b*x + a)^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**7/(d*tan(b*x+a))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^7/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^7}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^7/(d*tan(a + b*x))^(5/2),x)
[Out] int(sin(a + b*x)^7/(d*tan(a + b*x))^(5/2), x)
```

3.109 $\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}+\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{20bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -3/20*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-1/
10*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)+1/5*sin(b*x+a)^5/b/d/(d*tan(b*x+a)
)^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2676, 2678, 2681, 2652, 2719}

$$\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{20bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}}-\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^5/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] -1/10*Sin[a + b*x]^3/(b*d*(d*Tan[a + b*x])^(3/2)) + Sin[a + b*x]^5/(5*b*d*(
d*Tan[a + b*x])^(3/2)) + (3*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(20*
b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
```

```
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sin^5(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{3\int \frac{\sin^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{10d^2}$$

$$= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}} + \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{3\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{20d^2}$$

$$= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}} + \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{\left(3\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(}}{20d^2\sqrt{\cos(a+bx)}}$$

$$= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}} + \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{(3\sin(a+bx))\int \sqrt{\sin(2a}}{20d^2\sqrt{\sin(2a+2bx)}\sqrt{d\,\mathrm{ta}}}$$

$$= -\frac{\sin^3(a+bx)}{10bd(d\tan(a+bx))^{3/2}} + \frac{\sin^5(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(}{20bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\,\mathrm{t}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.13, size = 100, normalized size = 0.88

$$\frac{\sqrt{d\tan(a+bx)}\left(-\sqrt{\sec^2(a+bx)}\,(\sin(3(a+bx))+\sin(5(a+bx)))+8\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sec(a+bx)\tan(a+bx)\right)}{80bd^3\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^5/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (Sqrt[d*Tan[a + b*x]]*(-(Sqrt[Sec[a + b*x]^2]*(Sin[3*(a + b*x)] + Sin[5*(a
+ b*x)])) + 8*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x
]*Tan[a + b*x]))/(80*b*d^3*Sqrt[Sec[a + b*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(125) = 250$.
time = 0.33, size = 550, normalized size = 4.82

method	result
default	$\frac{(-1+\cos(bx+a))^2 \left(4(\cos^6(bx+a))\sqrt{2}-6(\cos^4(bx+a))\sqrt{2}-6\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40/b*(-1+cos(b*x+a))^2*(4*cos(b*x+a)^6*2^(1/2)-6*cos(b*x+a)^4*2^(1/2)-6*c
os(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2
))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+3*cos(b*x+a)*EllipticF
(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))
/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a
)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-6*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1
/2)+3*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(
(-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-cos(b*x+a)^2*2^(1/2)+3*cos
(b*x+a)*2^(1/2))*(cos(b*x+a)+1)^2/cos(b*x+a)^3/sin(b*x+a)^2/(d*sin(b*x+a)/c
os(b*x+a))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^5/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*tan(b*x + a))*sin(b
*x + a)/(d^3*tan(b*x + a)^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**5/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^5/(d*tan(b*x + a))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^5}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/(d*tan(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)^5/(d*tan(a + b*x))^(5/2), x)
```

3.110 $\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}+\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+1/3
*sin(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2676, 2681, 2652, 2719}

$$\frac{\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}+\frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] Sin[a + b*x]^3/(3*b*d*(d*Tan[a + b*x])^(3/2)) + (EllipticE[a - Pi/4 + b*x,
2]*Sin[a + b*x])/(2*b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}+\frac{\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{2d^2}\\
&= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}+\frac{\sqrt{\sin(a+bx)}\int \sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}\,dx}{2d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\
&= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}+\frac{\sin(a+bx)\int \sqrt{\sin(2a+2bx)}\,dx}{2d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}\\
&= \frac{\sin^3(a+bx)}{3bd(d\tan(a+bx))^{3/2}}+\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{2bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}
\end{aligned}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.64, size = 97, normalized size = 1.15

$$\frac{\sqrt{d\tan(a+bx)}\left(\sqrt{\sec^2(a+bx)}\,(\sin(a+bx)+\sin(3(a+bx)))+4\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sec(a+bx)\tan(a+bx)\right)}{12bd^3\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (Sqrt[d*Tan[a + b*x]]*(Sqrt[Sec[a + b*x]^2]*(Sin[a + b*x] + Sin[3*(a + b*x)
]) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]*Tan[a
 + b*x]))/(12*b*d^3*Sqrt[Sec[a + b*x]^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(99) = 198.
time = 0.36, size = 536, normalized size = 6.38

method	result

default	$-\frac{(-1+\cos(bx+a))^2\left(6\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/b*(-1+cos(b*x+a))^2*(6*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)-3*cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))
/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+2*cos(b*x+a
)^4*2^(1/2)+6*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^
(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-3*EllipticF(((1-co
s(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*
x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin
(b*x+a))/sin(b*x+a))^(1/2)+cos(b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2))*(cos(
b*x+a)+1)^2/cos(b*x+a)^3/sin(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(5/2)*2^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b
*x + a)^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^3}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(d*tan(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)^3/(d*tan(a + b*x))^(5/2), x)
```

3.111 $\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}-\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] 3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x)
,2^(1/2))*sin(b*x+a)/b/d^2/sin(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-2*sin(
b*x+a)/b/d/(d*tan(b*x+a))^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2677, 2681, 2652, 2719}

$$-\frac{3\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*Sin[a + b*x])/(b*d*(d*Tan[a + b*x])^(3/2)) - (3*EllipticE[a - Pi/4 + b*
x, 2]*Sin[a + b*x])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
```

```
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}} - \frac{3\int \frac{\sin(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{d^2}\\
&= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}} - \frac{\left(3\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,\sqrt{\sin(a+bx)}\,dx}{d^2\sqrt{\cos(a+bx)}\,\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}} - \frac{(3\sin(a+bx))\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}\,\sqrt{d\tan(a+bx)}}\\
&= -\frac{2\sin(a+bx)}{bd(d\tan(a+bx))^{3/2}} - \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{bd^2\sqrt{\sin(2a+2bx)}\,\sqrt{d\tan(a+bx)}}
\end{aligned}
$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.42, size = 69, normalized size = 0.88

$$
-\frac{2\cos(a+bx)\left(1+\,_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\,\tan^2(a+bx)\right)}{bd^2\sqrt{d\tan(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x]*(1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqr
t[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(b*d^2*Sqrt[d*Tan[a + b*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(97) = 194.
time = 0.32, size = 503, normalized size = 6.45

method	result

default	$\frac{\left(6\cos(bx+a)\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1\cdots}{\sin(bx\cdots}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(6*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),
1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-3*cos(b*x+a)
*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+c
os(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*(
(cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+6*EllipticE(((1-cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(
b*x+a))^(1/2)-3*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*
2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+cos(b*x+a)^2*2^(
1/2)-3*cos(b*x+a)*2^(1/2))*sin(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(5/2)/cos
(b*x+a)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*sin(b*x + a)/(d^3*tan(b*x + a)^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)}{(d\tan(a+b\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(d*tan(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)/(d*tan(a + b*x))^(5/2), x)
```

3.112 $\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}+\frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}}+\frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{5bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] 6/5*cos(b*x+a)/b/d^2/(d*tan(b*x+a))^(1/2)-6/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/s
in(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/d^2/sin(
2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)-2/5*csc(b*x+a)/b/d/(d*tan(b*x+a))^(3/
2)
```

Rubi [A]

time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2677, 2681, 2650, 2652, 2719}

$$\frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}}+\frac{6\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*Csc[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(3/2)) + (6*Cos[a + b*x])/(5*b*d^
2*Sqrt[d*Tan[a + b*x]]) + (6*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(5*
b*d^2*Sqrt[Sin[2*a + 2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
```

```
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} - \frac{3\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{5d^2}$$

$$= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} - \frac{\left(3\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+bx)}}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{5d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}$$

$$= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{\left(6\sqrt{\sin(a+bx)}\right)\int \sqrt{\cos(a}}{5d^2\sqrt{\cos(a+bx)}\,\sqrt{}}$$

$$= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{(6\sin(a+bx))\int \sqrt{\sin(2a+}}{5d^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(}}$$

$$= -\frac{2\csc(a+bx)}{5bd(d\tan(a+bx))^{3/2}} + \frac{6\cos(a+bx)}{5bd^2\sqrt{d\tan(a+bx)}} + \frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a}{5bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.85, size = 105, normalized size = 0.95

$$\frac{2\left(2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sec^2(a+bx)-\left(3-4\csc^2(a+bx)+\csc^4(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sin(a+bx)\sqrt{d\tan(a+bx)}}{5bd^3\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b*x]/(d*Tan[a + b*x])^(5/2),x]

[Out] (2*(2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 - (3 - 4*Csc[a + b*x]^2 + Csc[a + b*x]^4)*Sqrt[Sec[a + b*x]^2])*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/(5*b*d^3*Sqrt[Sec[a + b*x]^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(121) = 242$.

time = 0.38, size = 964, normalized size = 8.76

method	result	size
default	Expression too large to display	964

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/5/b*(3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^3-6*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^3+3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^2-6*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^2-3*cos(b*x+a)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+6*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+3*cos(b*x+a)^3*2^(1/2)-3*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+6*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)+cos(b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2))/(d*sin(b*x+a)/cos(b*x+a))^(5/2)/cos(b*x+a)^3*2^(1/2)

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc{(a+bx)}}{(d\tan{(a+bx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(csc(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)\,{\left(d\,\tan\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(5/2)),x)
```

```
[Out] int(1/(sin(a + b*x)*(d*tan(a + b*x))^(5/2)), x)
```

3.113 $\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=140

$$\frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}+\frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}}+\frac{4E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sin(a+bx)}{15bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}$$

```
[Out] 4/15*cos(b*x+a)/b/d^2/(d*tan(b*x+a))^(1/2)-4/15*(sin(a+1/4*Pi+b*x)^2)^(1/2)
/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*sin(b*x+a)/b/d^2/si
n(2*b*x+2*a)^(1/2)/(d*tan(b*x+a))^(1/2)+2/15*csc(b*x+a)/b/d/(d*tan(b*x+a))^
(3/2)-2/9*csc(b*x+a)^3/b/d/(d*tan(b*x+a))^(3/2)
```

Rubi [A]

time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2677, 2679, 2681, 2650, 2652, 2719}

$$\frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}}+\frac{4\sin(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{15bd^2\sqrt{\sin(2a+2bx)}\sqrt{d\tan(a+bx)}}-\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}}+\frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*Csc[a + b*x])/(15*b*d*(d*Tan[a + b*x])^(3/2)) - (2*Csc[a + b*x]^3)/(9*b*
d*(d*Tan[a + b*x])^(3/2)) + (4*Cos[a + b*x])/(15*b*d^2*Sqrt[d*Tan[a + b*x]]
) + (4*EllipticE[a - Pi/4 + b*x, 2]*Sin[a + b*x])/(15*b*d^2*Sqrt[Sin[2*a +
2*b*x]]*Sqrt[d*Tan[a + b*x]])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
```

```
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx = -\frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} - \frac{\int \frac{\csc^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{3d^2}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} - \frac{2\int \frac{\csc(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{15d^2}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} - \frac{\left(2\sqrt{\sin(a+bx)}\right)\int \frac{\sqrt{\cos(a+}}{\sin^{\frac{3}{2}}(a+}}{15d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a}}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{\left(4\sqrt{\right.}}{}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{(4\sin}{15d^2}$$

$$= \frac{2\csc(a+bx)}{15bd(d\tan(a+bx))^{3/2}} - \frac{2\csc^3(a+bx)}{9bd(d\tan(a+bx))^{3/2}} + \frac{4\cos(a+bx)}{15bd^2\sqrt{d\tan(a+bx)}} + \frac{4}{15bd^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.90, size = 116, normalized size = 0.83

$$\frac{2\left(4\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sec^2(a+bx)+\left(-6+3\csc^2(a+bx)+8\csc^4(a+bx)-5\csc^6(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sin(a+bx)\sqrt{d\tan(a+bx)}}{45bd^3\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (2*(4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 + (-
6 + 3*Csc[a + b*x]^2 + 8*Csc[a + b*x]^4 - 5*Csc[a + b*x]^6)*Sqrt[Sec[a + b*
x]^2])*Sin[a + b*x]*Sqrt[d*Tan[a + b*x]])/(45*b*d^3*Sqrt[Sec[a + b*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1454 vs. $2(147) = 294$.
time = 0.40, size = 1455, normalized size = 10.39

method	result	size
default	Expression too large to display	1455

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45/b*(6*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^5-12*((1-co
s(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a
))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^5+6*((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos
(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))*cos(b*x+a)^4-12*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x
+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/
2))*cos(b*x+a)^4-12*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+
a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^3
+24*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))
/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^3-12*((1-cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^2+24*((1-cos(b*x+a)+sin(b*x+a))/s
in(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2
),1/2*2^(1/2))*cos(b*x+a)^2+6*2^(1/2)*cos(b*x+a)^5+6*cos(b*x+a)*EllipticF((
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-
1+sin(b*x+a))/sin(b*x+a))^(1/2)-12*cos(b*x+a)*EllipticE(((1-cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(
b*x+a))^(1/2)-3*cos(b*x+a)^4*2^(1/2)+6*EllipticF(((1-cos(b*x+a)+sin(b*x+a))
/sin(b*x+a))^(1/2),1/2*2^(1/2))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)-12*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2
))*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)-12*cos(b*x+a)^3*2^(1/2
)-2*cos(b*x+a)^2*2^(1/2)+6*cos(b*x+a)*2^(1/2))/cos(b*x+a)^3/sin(b*x+a)^2/(d
*sin(b*x+a)/cos(b*x+a))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)}{(d\tan(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/(d*tan(a + b*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(a+b\,x)^3\,(d\tan(a+b\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^3*(d*tan(a + b*x))^(5/2)), x)
```

3.114 $\int (a\sin (e+fx))^{5/2}\sqrt {b\tan (e+fx)}\, dx$

Optimal. Leaf size=68

$$-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

```
[Out] -2/5*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(1/2)-8/5*a^2*b*(a*sin(f*x+e))
^(1/2)/f/(b*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2678, 2669}

$$-\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-8*a^2*b*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[b*Tan[e + f*x]]) - (2*b*(a*Sin[e
+ f*x])^(5/2))/(5*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx &= -\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}+\frac{1}{5}\left(4a^2\right)\int \sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+\\ &= -\frac{8a^2b\sqrt{a\sin(e+fx)}}{5f\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 51, normalized size = 0.75

$$-\frac{a^2\sqrt{a\sin(e+fx)}\,(8\cot(e+fx)+\sin(2(e+fx)))\sqrt{b\tan(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -1/5*(a^2*Sqrt[a*Sin[e + f*x]]*(8*Cot[e + f*x] + Sin[2*(e + f*x)])*Sqrt[b*T
an[e + f*x]])/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(56) = 112$.
time = 6.01, size = 493, normalized size = 7.25

method	result
default	$(a\sin(fx+e))^{\frac{5}{2}}\left(4(\cos^3(fx+e))-5\cos(fx+e)\ln\left(-\frac{2(\cos^2(fx+e))\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-(\cos^2(fx+e))+2\cos(fx+e)-2\sqrt{-\frac{\cos(}{(\cos(f}}}{\sin(fx+e)^2}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/f*(a*sin(f*x+e))^(5/2)*(4*cos(f*x+e)^3-5*cos(f*x+e)*ln(-(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)+5*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-5*ln(-(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)+5*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-20*cos(f*x+e))*(b*sin(f*x+e)/cos(f*x+e))^(1
/2)/sin(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

Fricas [**A**]
time = 0.38, size = 71, normalized size = 1.04

$$\frac{2\left(a^{2}\cos\left(fx+e\right)^{3}-5\,a^{2}\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\,\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}{5\,f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a^2*cos(f*x + e)^3 - 5*a^2*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*s
in(f*x + e)/cos(f*x + e))/(f*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARa near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [**B**]
time = 4.33, size = 80, normalized size = 1.18

$$\frac{a^2\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\left(18\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{10\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2),x)
```

```
[Out] (a^2*(a*sin(e + f*x))^(1/2)*(18*sin(2*e + 2*f*x) - sin(4*e + 4*f*x))*((b*si
n(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*f*(cos(2*e + 2*f*x) - 1)
)
```

3.115 $\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal. Leaf size=88

$$-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}+\frac{4a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}$$

```
[Out] -2/3*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(1/2)+4/3*a^2*(cos(1/2*f*x+1/2
*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f
*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2678, 2681, 2720}

$$\frac{4a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*Sqrt[b*Tan[e + f*x]]) + (4*a^2*Sqrt[Cos[
e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e
 + f*x]])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}+\frac{1}{3}(2a^2)\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$$
$$= -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}+\frac{\left(2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int}{3\sqrt{a\sin(e+fx)}}$$
$$= -\frac{2b(a\sin(e+fx))^{3/2}}{3f\sqrt{b\tan(e+fx)}}+\frac{4a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\mathrm{t}}}{3f\sqrt{a\sin(e+fx)}}$$

Mathematica [**A**]
time = 6.70, size = 80, normalized size = 0.91

$$-\frac{2ab\sqrt{a\sin(e+fx)}\left(-2F\left(\frac{1}{2}\mathrm{ArcSin}(\sin(e+fx))\middle|2\right)+\sqrt[4]{\cos^2(e+fx)}\sin(e+fx)\right)}{3f\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*a*b*Sqrt[a*Sin[e + f*x]]*(-2*EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Co
s[e + f*x]^2)^(1/4)*Sin[e + f*x]))/(3*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e
 + f*x]])
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.83, size = 131, normalized size = 1.49

method	result
default	$-\frac{2\left(2i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)+\cos^2(fx+e)-\cos(fx+e)\right)(a\sin(fx+e))^{\frac{3}{2}}\sqrt{\frac{b}{c}}}{3f(\cos(fx+e)-1)\sin(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/f*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)+cos(f*x+e)^2-cos(f*x+e))*(a
*sin(f*x+e))^(3/2)*(b*sin(f*x+e)/cos(f*x+e))^(1/2)/(cos(f*x+e)-1)/sin(f*x+e
)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 110, normalized size = 1.25

$$-\frac{2\left(\sqrt{a\sin\left(fx+e\right)}\,a\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)-\sqrt{2}\,\sqrt{-ab}\,a\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)-\sqrt{2}\,\sqrt{-ab}\,a\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)
 - sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x
 + e)) - sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) - I*s
in(f*x + e)))/f
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARa near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{b\,\mathrm{tan}\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2), x)
```

3.116 $\int \sqrt{a \sin(e+fx)} \sqrt{b \tan(e+fx)} \, dx$

Optimal. Leaf size=30

$$-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[Out] -2*b*(a*sin(f*x+e))^(1/2)/f/(b*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2669}

$$-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]])
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}\,dx = -\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Mathematica [A]
time = 0.15, size = 30, normalized size = 1.00

$$-\frac{2b\sqrt{a\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(26) = 52$.
time = 0.40, size = 295, normalized size = 9.83

method	result
risch	$-\frac{2i\sqrt{a\sin(fx+e)}\,\sqrt{-\frac{ib(\mathrm{e}^{2i(fx+e)}-1)}{\mathrm{e}^{2i(fx+e)}+1}}\,(\mathrm{e}^{2i(fx+e)}+1)}{(\mathrm{e}^{2i(fx+e)}-1)f}$
default	$-\frac{(\cos(fx+e)-1)\left(-4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cos(fx+e)+\ln\left(-\frac{2\left(\cos^2(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\left(\cos^2(fx+e)\right)+2\cos(fx+e)}{\sin(fx+e)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)*(-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+l
n(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(
f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-4*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2))*cos(f*x+e)*(a*sin(f*x+e))^(1/2)*(b*sin(f*x+e)/cos(f*x+e)
)^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/sin(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

Fricas [A]
time = 0.35, size = 52, normalized size = 1.73

$$-\frac{2\sqrt{a\sin(fx+e)}\,\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\,\cos(fx+e)}{f\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/(f*s
in(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(e + f*x))*sqrt(b*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARa near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [B]
time = 2.90, size = 60, normalized size = 2.00

$$\frac{\sin(2\,e+2\,f\,x)\,\sqrt{a\,\sin(e+f\,x)}\,\sqrt{\frac{b\,\sin(2\,e+2\,f\,x)}{2\cos(e+f\,x)^2}}}{f\,\left(\cos(e+f\,x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2),x)
```

```
[Out] (sin(2*e + 2*f*x)*(a*sin(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(2*cos(e + f
*x)^2))^(1/2))/(f*(cos(e + f*x)^2 - 1))
```

3.117 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[Out] 2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2
*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2681, 2720}

$$\frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sq
rt[a*Sin[e + f*x]])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\left(\sqrt{\cos(e+fx)}\,\sqrt{b\tan(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{\sqrt{a\sin(e+fx)}}$$
$$= \frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Mathematica [A]
time = 0.14, size = 60, normalized size = 1.20

$$\frac{2\cos(e+fx)F\left(\frac{1}{2}\text{ArcSin}(\sin(e+fx))\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt[4]{\cos^2(e+fx)}\,\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*Cos[e + f*x]*EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sqrt[b*Tan[e + f*x]])/
(f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 88, normalized size = 1.76

method	result	size
default	$\frac{2i\,\text{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{f\sqrt{a\sin\left(fx+e\right)}\,\sqrt{\frac{1}{\cos(fx+e)+1}}}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/f/(a*sin(f*x+e))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)/(1/(cos(f*x+e)+1))^(1/2)*(b*sin(f*x+e)/cos(f*x+e
))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(a*sin(f*x + e)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 68, normalized size = 1.36

$$\frac{\sqrt{2}\,\sqrt{-ab}\ \mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+\sqrt{2}\,\sqrt{-ab}\ \mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right))}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] (sqrt(2)*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e
)) + sqrt(2)*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x
 + e)))/(a*f)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\tan\left(e+fx\right)}}{\sqrt{a\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(1/2),x)

[Out] Integral(sqrt(b*tan(e + f*x))/sqrt(a*sin(e + f*x)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(a*sin(f*x + e)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{\sqrt{a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2),x)

[Out] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2), x)

3.118 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{\text{ArcTan}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}}-\frac{\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}}$$

```
[Out] -arctan(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/f/(a*sin(
f*x+e))^(1/2)-arctanh(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/
2)/a/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2681, 12, 2645, 335, 218, 212, 209}

$$-\frac{\sqrt{\cos(e+fx)}\,\text{ArcTan}\left(\sqrt{\cos(e+fx)}\right)\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}}-\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((ArcTan[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(a*f
*Sqrt[a*Sin[e + f*x]])) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*S
qrt[b*Tan[e + f*x]])/(a*f*Sqrt[a*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}\,dx}{\sqrt{a\sin(e+fx)}}$$

$$= \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}\,dx}{a\sqrt{a\sin(e+fx)}}$$

$$= -\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx, x, \cos(e+fx)\right)}{af\sqrt{a\sin(e+fx)}}$$

$$= -\frac{\left(2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}}$$

$$= -\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(e+fx)}\right)}{af\sqrt{a\sin(e+fx)}} - \frac{\left(\sqrt{c\right.}}{}$$

$$= -\frac{\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{af\sqrt{a\sin(e+fx)}} - \frac{\tanh^{-1}\left(\sqrt{\cos(e+\right.}}{a}$$

Mathematica [A]
time = 0.32, size = 72, normalized size = 0.67

$$-\frac{b\left(\text{ArcTan}\left(\sqrt[4]{\cos^2(e+fx)}\right)+\tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
[Out] -((b*(ArcTan[(Cos[e + f*x]^2)^(1/4)] + ArcTanh[(Cos[e + f*x]^2)^(1/4)])*Sqr
t[a*Sin[e + f*x]])/(a^2*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(91) = 182.
time = 0.34, size = 185, normalized size = 1.73

method	result

default	$\frac{(\cos(fx+e)-1)\left(\ln\left(-\frac{2\left(2\left(\cos^2(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\left(\cos^2(fx+e)\right)+2\cos(fx+e)-2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-1\right)}{\sin(fx+e)^2}\right)-\text{arctan}\right.}{2f\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,(a\sin(fx+e))^{\frac{3}{2}}\sin(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(cos(f*x+e)-1)*(ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/s
in(f*x+e)^2)-arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))*cos(f*x+e)*(
b*sin(f*x+e)/cos(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(a*sin(
f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(99) = 198.
time = 0.66, size = 447, normalized size = 4.18

$$\left[\frac{2\sqrt{-\frac{b}{a}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{b}{a}}\cos(fx+e)}{(b\cos(fx+e)+b)\sin(fx+e)}\right)+\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(fx+e)^3+4\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{b}{a}}\cos(fx+e)\sin(fx+e)-5b\cos(fx+e)^2-5b\cos(fx+e)+b}{\cos(fx+e)^3+3\cos(fx+e)^2+3\cos(fx+e)+1}\right)}{4af}, \frac{2\sqrt{\frac{b}{a}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{b}{a}}\cos(fx+e)}{(b\cos(fx+e)-b)\sin(fx+e)}\right)+\sqrt{\frac{b}{a}}\log\left(\frac{4\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{b}{a}}-\left(b\cos(fx+e)^2+6b\cos(fx+e)+b\right)\sin(fx+e)}{\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sin(fx+e)}\right)}{4af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*
x + e))*sqrt(-b/a)*cos(f*x + e)/((b*cos(f*x + e) + b)*sin(f*x + e))) + sqrt
(-b/a)*log(-(b*cos(f*x + e)^3 + 4*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(-b/a)*cos(f*x + e)*sin(f*x + e) - 5*b*cos(f*x + e)^2 - 5
*b*cos(f*x + e) + b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) +
1)))/(a*f), 1/4*(2*sqrt(b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(b/a)*cos(f*x + e)/((b*cos(f*x + e) - b)*sin(f*x + e)
)) + sqrt(b/a)*log((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*
```

```
sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/a) - (b*cos(f*x + e)^2 + 6*b*cos(f
*x + e) + b)*sin(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x +
 e))))/(a*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \tan{(e+fx)}}}{(a \sin{(e+fx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x))/(a*sin(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\tan\left(e+f\,x\right)}}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(3/2), x)
```

3.119 $\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{b}{a^2 f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{a^2 f\sqrt{a\sin(e+fx)}}$$

```
[Out] -b/a^2/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+(cos(1/2*f*x+1/2*e)^2)^(
1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1
/2)*(b*tan(f*x+e))^(1/2)/a^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2679, 2681, 2720}

$$\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{a^2 f\sqrt{a\sin(e+fx)}}-\frac{b}{a^2 f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(b/(a^2*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) + (Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x
]])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = -\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{2a^2}$$

$$= -\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int -}{2a^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{b}{a^2f\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan}}{a^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [A]
time = 0.27, size = 79, normalized size = 0.92

$$\frac{b\left(-\sqrt[4]{\cos^2(e+fx)} + F\left(\frac{1}{2}\text{ArcSin}(\sin(e+fx))\middle|2\right)\sin(e+fx)\right)}{a^2f\sqrt[4]{\cos^2(e+fx)}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
[Out] (b*(-(Cos[e + f*x]^2)^(1/4) + EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e +
f*x]))/(a^2*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*
x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.84, size = 178, normalized size = 2.07

method	result
default	$\frac{\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \text{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)\cos(fx+e)+i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \text{E}}{f(a\sin(fx+e))^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^
```

```
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+
e),I)*sin(f*x+e)-cos(f*x+e))*sin(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(1/2)/(a*
sin(f*x+e))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 158, normalized size = 1.84

$$\frac{\left(\sqrt{2}\cos\left(fx+e\right)^{2}-\sqrt{2}\right)\sqrt{-ab}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right))+\left(\sqrt{2}\cos\left(fx+e\right)^{2}-\sqrt{2}\right)\sqrt{-ab}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right))+2\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{2\left(a^{3}f\cos\left(fx+e\right)^{2}-a^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2*((sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4,
0, cos(f*x + e) + I*sin(f*x + e)) + (sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt
(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*sqrt(a
*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/(a^3*f*cos(f
*x + e)^2 - a^3*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(a*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2), x)
```

3.120 $\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=126

$$-\frac{24a^2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f}-\frac{2b(a\sin(e+fx))^{5/2}}{5f}$$

```
[Out] -24/5*a^2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin
(1/2*f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x
+e))^(1/2)-2/5*b*(a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2)/f+12/5*a^2*b*(a*
sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2678, 2674, 2681, 2719}

$$-\frac{24a^2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{12a^2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{5f}-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-24*a^2*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[
e + f*x]]*Sqrt[b*Tan[e + f*x]]) + (12*a^2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan
[e + f*x]])/(5*f) - (2*b*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(5*f)
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
```

```
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx &= -\frac{2b(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{5f}+\frac{1}{5}\left(6a^2\right)\int \sqrt{a\sin(e+\ldots}\\ &= \frac{12a^2b\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}{5f}-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{\ldots}}{5f}\\ &= \frac{12a^2b\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}{5f}-\frac{2b(a\sin(e+fx))^{5/2}\sqrt{\ldots}}{5f}\\ &= -\frac{24a^2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\,\sqrt{b\tan(e+fx)}}+\frac{12a^2b\sqrt{a\sin(e+f\ldots}}{5\ldots}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.69, size = 99, normalized size = 0.79

$$\frac{a^2b\left(\cos^2(e+fx)^{3/4}(11+\cos(2(e+fx)))-12\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}{5f\cos^2(e+fx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^2*b*((Cos[e + f*x]^2)^(3/4)*(11 + Cos[2*(e + f*x)]) - 12*Cos[e + f*x]^2*
Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sqrt
[b*Tan[e + f*x]])/(5*f*(Cos[e + f*x]^2)^(3/4))
```

Maple [C] Result contains complex when optimal does not.
time = 4.17, size = 338, normalized size = 2.68

method	result

default	$\frac{2\left(12i\,\mathrm{EllipticE}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)-12i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+}{\cos(fx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/f*(12*I*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-12*I*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/
sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e
)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-cos(f*x+e)^4+8*cos(f*x+e)^2-12*c
os(f*x+e)+5)*(a*sin(f*x+e))^(5/2)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)*cos(f*x+e
)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 135, normalized size = 1.07

$$\frac{2\left(6\sqrt{2}\sqrt{-ab}\,a^2b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+6\sqrt{2}\sqrt{-ab}\,a^2b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+(a^2b\cos(fx+e)^2+5a^2b)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\right)}{5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(6*sqrt(2)*sqrt(-a*b)*a^2*b*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 6*sqrt(2)*sqrt(-a*b)*a^2*b*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)))
+ (a^2*b*cos(f*x + e)^2 + 5*a^2*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)
/cos(f*x + e)))/f
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARa near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\sin\left(e+f\,x\right))^{5/2}\,\left(b\tan(e+f\,x)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2), x)
```

3.121 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=68

$$\frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

```
[Out] -2/3*b*(a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2)/f+8/3*a^2*b*(b*tan(f*x+e))
^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2678, 2669}

$$\frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (8*a^2*b*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x]]) - (2*b*(a*Sin[e +
 f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(3*f)
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx &= -\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}+\frac{1}{3}\left(4a^2\right)\int \frac{(b\tan(e+}{\sqrt{a\sin(e}}\\ &= \frac{8a^2b\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}-\frac{2b(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{3f}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 45, normalized size = 0.66

$$\frac{a^2 b(7+\cos(2(e+fx)))\sqrt{b\tan(e+fx)}}{3f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^2*b*(7 + Cos[2*(e + f*x)])*Sqrt[b*Tan[e + f*x]])/(3*f*Sqrt[a*Sin[e + f*x
]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(56) = 112$.
time = 0.32, size = 492, normalized size = 7.24

method	result
default	$\left(3\cos(fx+e)\ln\left(-\frac{2\left(\cos^2(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\left(\cos^2(fx+e)\right)+2\cos(fx+e)-2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-1}{\sin(fx+e)^2}\right)\sqrt{-\frac{\cos(fx\text{-}}{(\cos(fx+e}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(3*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3*cos(f*x+e)*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)+3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(
f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^
2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*cos
(f*x+e)^2+12)*(a*sin(f*x+e))^(3/2)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)*cos(f*x+
e)/sin(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

Fricas [**A**]
time = 0.39, size = 62, normalized size = 0.91

$$\frac{2 \left(ab \cos \left(fx+e\right)^{2}+3 \, ab\right) \sqrt{a \sin \left(fx+e\right)} \sqrt{\frac{b \sin \left(fx+e\right)}{\cos \left(fx+e\right)}}}{3 \, f \sin \left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*b*cos(f*x + e)^2 + 3*a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/c
os(f*x + e))/(f*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARa near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [**B**]
time = 3.73, size = 69, normalized size = 1.01

$$\frac{a\,b\,\left(13\,\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{6\,f\,{\sin\left(e+f\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2),x)
```

```
[Out] (a*b*(13*sin(e + f*x) + sin(3*e + 3*f*x))*(a*sin(e + f*x))^(1/2)*((b*sin(2*
e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(6*f*sin(e + f*x)^2)
```

3.122 $\int \sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=84

$$-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}$$

```
[Out] -4*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1
/2)+2*b*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2674, 2681, 2719}

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-4*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x
]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])/
f
```

Rule 2674

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Dist[b^2*((m + n - 1)/(n - 1)), Int[(a*Sin[e + f*x])^m*(b*Tan
[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && Int
egersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}\,dx = \frac{2b\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}{f} - (2b^2)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}$$

$$= \frac{2b\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}{f} - \frac{\left(2b^2\sqrt{a\sin(e+fx)}\right)\int \sqrt{}}{\sqrt{\cos(e+fx)}\,\sqrt{b\,\mathrm{ta}}}$$

$$= -\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\,\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{a\sin(e+fx)}\,\sqrt{b\,\mathrm{t}}}{f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.22, size = 83, normalized size = 0.99

$$\frac{2b\left(\cos^2(e+fx)^{3/4}-\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}{f\cos^2(e+fx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*b*((Cos[e + f*x]^2)^(3/4) - Cos[e + f*x]^2*Hypergeometric2F1[1/4, 1/2, 3
/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(f*(Cos[e +
 f*x]^2)^(3/4))
```

Maple [C] Result contains complex when optimal does not.
time = 0.35, size = 328, normalized size = 3.90

method	result
default	$-\frac{2\left(2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\;\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)\cos(fx+e)-2i\,\mathrm{EllipticE}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(f\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-2*I*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*Elliptic
E(I*(cos(f*x+e)-1)/sin(f*x+e),I)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-2*
I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*Ell
```

ipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)-cos(f*x+e)^2+2*cos(f*x+e)-1)*cos(f*x+e)*(a*sin(f*x+e))^(1/2)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)/sin(f*x+e)^3

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 107, normalized size = 1.27

$$\frac{2\left(\sqrt{2}\sqrt{-ab}\,b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+\sqrt{2}\sqrt{-ab}\,b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+\sqrt{a\sin(fx+e)}\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 2*(sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))/f

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARa near

```
 OSimplification assuming sageVARf near OSimplification assuming sageVARx n
ear OS
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a \sin{(e+fx)}}\,(b\tan(e+fx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2), x)
```

3.123 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal. Leaf size=30

$$\frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[Out] 2*b*(b*tan(f*x+e))^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2669}

$$\frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*b*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[a*Sin[e + f*x]])
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Mathematica [A]
time = 0.07, size = 30, normalized size = 1.00

$$\frac{2b\sqrt{b\tan(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*b*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[a*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(26) = 52$.
time = 0.35, size = 308, normalized size = 10.27

method	result
default	$\frac{(\cos(fx+e)-1)\left(\cos(fx+e)\ln\left(-\frac{2\left(\cos^2(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\left(\cos^2(fx+e)\right)+2\cos(fx+e)-2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-1}{\sin(fx+e)^2}\right)-c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(cos(f*x+e)-1)*(cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)-cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-1)/sin(f*x+e)^2)-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-4
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))
^(3/2)/sin(f*x+e)^3/(a*sin(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(a*sin(f*x + e)), x)
```

Fricas [A]
time = 0.36, size = 49, normalized size = 1.63

$$\frac{2\sqrt{a\sin(fx+e)}\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{af\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e))/(a*f*sin(f*x + e
))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(a*sin(f*x + e)), x)
```

Mupad [**B**]
time = 3.07, size = 39, normalized size = 1.30

$$\frac{2\,b\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{2\,{\cos\left(e+f\,x\right)}^2}}}{f\,\sqrt{a\,\sin\left(e+f\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(1/2),x)
[Out] (2*b*((b*sin(2*e + 2*f*x))/(2*cos(e + f*x)^2))^(1/2))/(f*(a*sin(e + f*x))^(
1/2))
```

3.124 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}$$

```
[Out] -2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/a^2/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e)
)^(1/2)+2*b*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/a^2/f
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2673, 2681, 2719}

$$\frac{2b\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}{a^2f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*b^2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e +
 f*x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x
]])/(a^2*f)
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{2b\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}{a^2f} - \frac{b^2\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{a^2}$$
$$= \frac{2b\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}{a^2f} - \frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\int \sqrt{\cos(e+fx)}\,dx}{a^2\sqrt{\cos(e+fx)}\,\sqrt{b\tan(e+fx)}}$$
$$= -\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\,\sqrt{b\tan(e+fx)}} + \frac{2b\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}{a^2f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.30, size = 92, normalized size = 1.02

$$\frac{\left(2\cos(e+fx)\cos^2(e+fx)^{3/4} - \cos^3(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)(b\tan(e+fx))^{3/2}}{af\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((2*Cos[e + f*x]*(Cos[e + f*x]^2)^(3/4) - Cos[e + f*x]^3*Hypergeometric2F1[
1/4, 1/2, 3/2, Sin[e + f*x]^2])*(b*Tan[e + f*x])^(3/2))/(a*f*(Cos[e + f*x]^
2)^(3/4)*Sqrt[a*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 316, normalized size = 3.51

method	result
default	$-\frac{2\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \text{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)\cos(fx+e)-i\,\text{EllipticE}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(f}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-I*EllipticE(I*(cos(f
```

```
*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-I*Ellipt
icE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e)-1)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)
*cos(f*x+e)/(a*sin(f*x+e))^(3/2)/sin(f*x+e)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 110, normalized size = 1.22

$$\frac{\sqrt{2}\sqrt{-ab}\,b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+\sqrt{2}\sqrt{-ab}\,b\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+2\sqrt{a\sin(fx+e)}\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*b*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*sqrt(a*sin(f
*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^2*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^{3/2}}{(a\,\sin\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(3/2), x)
```

3.125 $\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$\frac{b^2\mathrm{ArcTan}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b^2\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+}}{a^2f\sqrt{a\sin(e+}}$$

```
[Out] b^2*arctan(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f/cos(f*x+e)^(1/2)/(b
*tan(f*x+e))^(1/2)-b^2*arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f
/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)+2*b*(b*tan(f*x+e))^(1/2)/a^2/f/(a*si
n(f*x+e))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2673, 2681, 12, 2645, 335, 304, 209, 212}

$$\frac{b^2\sqrt{a\sin(e+fx)}\,\mathrm{ArcTan}\left(\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b^2\sqrt{a\sin(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (b^2*ArcTan[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(a^3*f*Sqrt[Cos[e + f
*x]]*Sqrt[b*Tan[e + f*x]]) - (b^2*ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e
+ f*x]])/(a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]) + (2*b*Sqrt[b*Tan[
e + f*x]])/(a^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2673

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)/
(a^2*f*(n - 1))), x] - Dist[b^2*((m + 2)/(a^2*(n - 1))), Int[(a*Sin[e + f*x
])^(m + 2)*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && Gt
Q[n, 1] && (LtQ[m, -1] || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*
n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{3/2}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} + \frac{b^2\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{a^2}$$

$$= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} + \frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\int \frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}\,dx}{a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} + \frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\int \sqrt{\cos(e+fx)}\csc(e+fx)\,dx}{a^3\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} - \frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \cos(e+fx)\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} - \frac{\left(2b^2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= \frac{2b\sqrt{b\tan(e+fx)}}{a^2f\sqrt{a\sin(e+fx)}} - \frac{\left(b^2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(e+fx)}\right)}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

$$= \frac{b^2\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{b^2\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Mathematica [A]
time = 0.40, size = 104, normalized size = 0.72

$$\frac{b\left(\text{ArcTan}\left(\sqrt[4]{\cos^2(e+fx)}\right)\cos^2(e+fx)-\tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)\cos^2(e+fx)+2\cos^2(e+fx)^{3/4}\right)\sqrt{b\tan(e+fx)}}{a^2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(a*Sin[e + f*x])^(5/2),x]
[Out] (b*(ArcTan[(Cos[e + f*x]^2)^(1/4)]*Cos[e + f*x]^2 - ArcTanh[(Cos[e + f*x]^2
)^(1/4)]*Cos[e + f*x]^2 + 2*(Cos[e + f*x]^2)^(3/4))*Sqrt[b*Tan[e + f*x]])/(
a^2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]])
```

Maple [A]
time = 0.32, size = 247, normalized size = 1.70

method	result

default	$-\frac{(\cos(fx+e)-1)\left(\cos(fx+e)\ln\left(-\frac{2\left(2\left(\cos^2(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\left(\cos^2(fx+e)\right)+2\cos(fx+e)-2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\right.}{\sin(fx+e)^2}\right.\right.}{2f(a\sin(fx+e))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)*(cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)+arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*c
os(f*x+e)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+4*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)/(a*sin(
f*x+e))^(5/2)/sin(f*x+e)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(135) = 270.
time = 0.64, size = 570, normalized size = 3.93

$$\left[\frac{2ab\sqrt{-\frac{b}{a}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{b}{a}}\cos(fx+e)}{(b\cos(fx+e)+b)\sin(fx+e)}\right)\sin(fx+e)+ab\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(fx+e)^3-4\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{b}{a}}\cos(fx+e)\sin(fx+e)-5b\cos(fx+e)^2-5b\cos(fx+e)+b}{\cos(fx+e)^3+3\cos(fx+e)^2+3\cos(fx+e)+1}\right)\sin(fx+e)+8\sqrt{a\sin(fx+e)}\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{4a^3f\sin(fx+e)}, -\frac{2ab\sqrt{\frac{b}{a}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{b}{a}}\cos(fx+e)}{(b\cos(fx+e)-b)\sin(fx+e)}\right)\sin(fx+e)-ab\sqrt{\frac{b}{a}}\log\left(\frac{4\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{b}{a}}-\left(b\cos(fx+e)^2+6b\cos(fx+e)+b\right)\sin(fx+e)}{\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sin(fx+e)}\right)\sin(fx+e)-8\sqrt{a\sin(fx+e)}\,b\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{4a^3f\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*sqrt(-b/a)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/co
s(f*x + e))*sqrt(-b/a)*cos(f*x + e)/((b*cos(f*x + e) + b)*sin(f*x + e)))*si
n(f*x + e) + a*b*sqrt(-b/a)*log(-(b*cos(f*x + e)^3 - 4*sqrt(a*sin(f*x + e))
*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/a)*cos(f*x + e)*sin(f*x + e) - 5
*b*cos(f*x + e)^2 - 5*b*cos(f*x + e) + b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^
2 + 3*cos(f*x + e) + 1))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*b*sqrt(b*sin
(f*x + e)/cos(f*x + e)))/(a^3*f*sin(f*x + e)), -1/4*(2*a*b*sqrt(b/a)*arctan
(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/a)*cos(f*x
```

```
 + e)/((b*cos(f*x + e) - b)*sin(f*x + e)))*sin(f*x + e) - a*b*sqrt(b/a)*log
((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e
)/cos(f*x + e))*sqrt(b/a) - (b*cos(f*x + e)^2 + 6*b*cos(f*x + e) + b)*sin(f
*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x + e)
 - 8*sqrt(a*sin(f*x + e))*b*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^3*f*sin(f
*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(a*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)/(a*sin(f*x + e))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^{3/2}}{(a\,\sin\left(e+f\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(3/2)/(a*sin(e + f*x))^(5/2), x)
```

3.126 $\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}+\frac{8a^4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{15f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] 8/15*a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*
f*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^
(1/2)-4/15*a^2*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(3/2)-2/9*b*(a*sin(f
*x+e))^(9/2)/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2678, 2681, 2719}

$$\frac{8a^4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{15f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(9/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-4*a^2*b*(a*Sin[e + f*x])^(5/2))/(15*f*(b*Tan[e + f*x])^(3/2)) - (2*b*(a*S
in[e + f*x])^(9/2))/(9*f*(b*Tan[e + f*x])^(3/2)) + (8*a^4*EllipticE[(e + f*
x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(15*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]
])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{9/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}} + \frac{1}{3}\left(2a^2\right)\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$$
$$= -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}} + \frac{1}{15}\left(4a^4\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,d$$
$$= -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}} + \frac{\left(4a^4\sqrt{a\sin(e+fx)}\right)\int\sqrt{\cos}}{15\sqrt{\cos(e+fx)}\sqrt{b\tan(}}$$
$$= -\frac{4a^2b(a\sin(e+fx))^{5/2}}{15f(b\tan(e+fx))^{3/2}} - \frac{2b(a\sin(e+fx))^{9/2}}{9f(b\tan(e+fx))^{3/2}} + \frac{8a^4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e}}{15f\sqrt{\cos(e+fx)}\sqrt{b\tan(e}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.54, size = 100, normalized size = 0.81

$$\frac{a^4\left(\cos^2(e+fx)^{3/4}(-17+5\cos(2(e+fx)))+12\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)\sqrt{a\sin(e+fx)}\,\sin(2(e+fx))}{90f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(9/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (a^4*((Cos[e + f*x]^2)^(3/4)*(-17 + 5*Cos[2*(e + f*x)]) + 12*Hypergeometric
2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)])/
(90*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.73, size = 349, normalized size = 2.84

method	result
default	$-\frac{2\left(5\left(\cos^6(fx+e)\right)+12i\,\mathrm{EllipticE}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)-12i\sqrt{\frac{1}{\cos(fx+e)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45/f*(5*cos(f*x+e)^6+12*I*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f
*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)
-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(
I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+12*I*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*
x+e),I)*sin(f*x+e)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-16*cos(f*x+e)^4
+23*cos(f*x+e)^2-12*cos(f*x+e))*(a*sin(f*x+e))^(9/2)/cos(f*x+e)/sin(f*x+e)^
5/(b*sin(f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(9/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 145, normalized size = 1.18

$$-\frac{2\left(6\sqrt{2}\sqrt{-ab}\,a^4\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))+6\sqrt{2}\sqrt{-ab}\,a^4\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e)))-\left(5\,a^4\cos(fx+e)^4-11\,a^4\cos(fx+e)^2\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\right)}{45\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(6*sqrt(2)*sqrt(-a*b)*a^4*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 6*sqrt(2)*sqrt(-a*b)*a^4*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) -
(5*a^4*cos(f*x + e)^4 - 11*a^4*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqrt(b*
sin(f*x + e)/cos(f*x + e)))/(b*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(9/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,\sin\left(e+f\,x\right))^{9/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(1/2), x)
```

3.127 $\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=68

$$-\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

```
[Out] -8/21*a^2*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(3/2)-2/7*b*(a*sin(f*x+e)
)^(7/2)/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.080$, Rules used = {2678, 2669}

$$-\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-8*a^2*b*(a*Sin[e + f*x])^(3/2))/(21*f*(b*Tan[e + f*x])^(3/2)) - (2*b*(a*S
in[e + f*x])^(7/2))/(7*f*(b*Tan[e + f*x])^(3/2))
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a\sin(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx &= -\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}+\frac{1}{7}\left(4a^2\right)\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx\\ &= -\frac{8a^2b(a\sin(e+fx))^{3/2}}{21f(b\tan(e+fx))^{3/2}}-\frac{2b(a\sin(e+fx))^{7/2}}{7f(b\tan(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 52, normalized size = 0.76

$$\frac{a^3\cos(e+fx)(-11+3\cos(2(e+fx)))\sqrt{a\sin(e+fx)}}{21f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
[Out] (a^3*Cos[e + f*x]*(-11 + 3*Cos[2*(e + f*x)])*Sqrt[a*Sin[e + f*x]])/(21*f*Sq
rt[b*Tan[e + f*x]])
```

Maple [A]
time = 0.60, size = 60, normalized size = 0.88

method	result	size
default	$\frac{2\left(3\left(\cos^2(fx+e)\right)-7\right)(a\sin(fx+e))^{\frac{7}{2}}\cos(fx+e)}{21f\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\,\sin(fx+e)^3}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/21/f*(3*cos(f*x+e)^2-7)*(a*sin(f*x+e))^(7/2)*cos(f*x+e)/(b*sin(f*x+e)/cos
(f*x+e))^(1/2)/sin(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [A]
time = 0.36, size = 77, normalized size = 1.13

$$\frac{2\left(3\,a^3\cos\left(fx+e\right)^4-7\,a^3\cos\left(fx+e\right)^2\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}{21\,bf\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^3*cos(f*x + e)^4 - 7*a^3*cos(f*x + e)^2)*sqrt(a*sin(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e))/(b*f*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(7/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [B]
time = 4.82, size = 88, normalized size = 1.29

$$-\frac{a^3\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{-\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{2\,{\sin\left(e+f\,x\right)}^2-2}}\,\left(22\,\sin\left(e+f\,x\right)+19\,\sin\left(3\,e+3\,f\,x\right)-3\,\sin\left(5\,e+5\,f\,x\right)\right)}{168\,b\,f\,{\sin\left(e+f\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2),x)
[Out] -(a^3*(a*sin(e + f*x))^(1/2)*(-(b*sin(2*e + 2*f*x))/(2*sin(e + f*x)^2 - 2))
^(1/2)*(22*sin(e + f*x) + 19*sin(3*e + 3*f*x) - 3*sin(5*e + 5*f*x)))/(168*b
*f*sin(e + f*x)^2)
```

3.128 $\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=88

$$-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}+\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] 4/5*a^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f
*x+1/2*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(
1/2)-2/5*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2678, 2681, 2719}

$$\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(3/2)) + (4*a^2*Ellipti
cE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan
[e + f*x]])
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a\sin(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx &= -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}+\frac{1}{5}\left(2a^2\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx\\ &= -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}+\frac{\left(2a^2\sqrt{a\sin(e+fx)}\right)\int \sqrt{\cos(e+fx)}\,dx}{5\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\ &= -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{3/2}}+\frac{4a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{5f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.25, size = 87, normalized size = 0.99

$$-\frac{a^2\left(\cos^2(e+fx)^{3/4}-{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)\sqrt{a\sin(e+fx)}\,\sin(2(e+fx))}{5f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -1/5*(a^2*((Cos[e + f*x]^2)^(3/4) - Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e
+ f*x]^2])*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)])/(f*(Cos[e + f*x]^2)^(3/4)
*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 339, normalized size = 3.85

method	result
default	$-\frac{2\left(2i\,\mathrm{EllipticE}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)-2i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+}{\cos(fx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/f*(2*I*sin(f*x+e)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)-2*I*(1/(cos(f*x+
```

```
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/s
in(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-
2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*
(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-cos(f*x+e)^4+3*cos(f*x+e)^2-2*cos(f
*x+e))*(a*sin(f*x+e))^(5/2)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)^3/co
s(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 125, normalized size = 1.42

$$-\frac{2\left(\sqrt{a\sin\left(fx+e\right)}\,a^{2}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+\sqrt{2}\,\sqrt{-ab}\,a^{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)))+\sqrt{2}\,\sqrt{-ab}\,a^{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)))\right)}{5bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(sqrt(a*sin(f*x + e))*a^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x +
e)^2 + sqrt(2)*sqrt(-a*b)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*a^2*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2), x)
```

3.129 $\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=32

$$-\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

```
[Out] -2/3*b*(a*sin(f*x+e))^(3/2)/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2669}

$$-\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*(b*Tan[e + f*x])^(3/2))
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx = -\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

Mathematica [A]
time = 0.15, size = 32, normalized size = 1.00

$$-\frac{2b(a\sin(e+fx))^{3/2}}{3f(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(3/2))/(3*f*(b*Tan[e + f*x])^(3/2))
```

Maple [A]
time = 0.31, size = 48, normalized size = 1.50

method	result	size
default	$-\frac{2(a\sin(fx+e))^{\frac{3}{2}}\cos(fx+e)}{3f\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\,\sin(fx+e)}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3/f*(a*sin(f*x+e))^(3/2)*cos(f*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(
f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(28) = 56$.
time = 0.36, size = 58, normalized size = 1.81

$$-\frac{2\sqrt{a\sin(fx+e)}\,a\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\,\cos(fx+e)^2}{3\,bf\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)^
2/(b*f*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [B]
time = 3.63, size = 69, normalized size = 2.16

$$-\frac{a\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\left(\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{6\,b\,f\,{\sin\left(e+f\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2),x)
[Out] -(a*(a*sin(e + f*x))^(1/2)*(sin(e + f*x) + sin(3*e + 3*f*x))*((b*sin(2*e +
2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(6*b*f*sin(e + f*x)^2)
```

3.130 $\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] 2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2
*e),2^(1/2))*(a*sin(f*x+e))^(1/2)/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2681, 2719}

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e + f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[Cos[e + f*x]]*Sq
rt[b*Tan[e + f*x]])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx &= \frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\\ &= \frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.16, size = 69, normalized size = 1.38

$$\frac{{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}\,\sin(2(e+fx))}{2f\cos^2(e+fx)^{3/4}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (Hypergeometric2F1[1/4, 1/2, 3/2, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*Sin[
2*(e + f*x)])/(2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.69, size = 327, normalized size = 6.54

method	result
default	$\frac{2\sqrt{a\sin(fx+e)}\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)\cos(fx+e)-i\,\mathrm{EllipticE}\left(\frac{i(\cos}{\sin}\right.\right.}{}$
risch	$-\frac{i\sqrt{2}\sqrt{-ia\left(e^{2i(fx+e)}-1\right)e^{-i(fx+e)}}}{f\sqrt{-\frac{ib\left(e^{2i(fx+e)}-1\right)}{e^{2i(fx+e)}+1}}}-\frac{i\left(-\frac{2\left(ab\,e^{2i(fx+e)}+ab\right)}{ab\sqrt{e^{i(fx+e)}\left(ab\,e^{2i(fx+e)}+ab\right)}}+\frac{i\sqrt{-i\left(e^{i(fx+e)}+i\right)}\,\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(a*sin(f*x+e))^(1/2)*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-
I*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)
*sin(f*x+e)-I*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-cos(f*x+e)^2+cos(f*x+e))/
(b*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 75, normalized size = 1.50

$$-\frac{\sqrt{2}\,\sqrt{-ab}\ \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+\sqrt{2}\,\sqrt{-ab}\ \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))}{bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*sqrt(-a*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
f*x + e) + I*sin(f*x + e))) + sqrt(2)*sqrt(-a*b)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(e+fx\right)}}{\sqrt{b\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(e + f*x))/sqrt(b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,\sin\left(e+f\,x\right)}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2), x)
```

3.131 $\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=106

$$\frac{\operatorname{ArcTan}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] arctan(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a/f/cos(f*x+e)^(1/2)/(b*tan(f
*x+e))^(1/2)-arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a/f/cos(f*x+e)^
(1/2)/(b*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2681, 12, 2645, 335, 304, 209, 212}

$$\frac{\sqrt{a\sin(e+fx)}\operatorname{ArcTan}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\sqrt{a\sin(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (ArcTan[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(a*f*Sqrt[Cos[e + f*x]]*S
qrt[b*Tan[e + f*x]]) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(
a*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{\sqrt{a\sin(e+fx)}\int \frac{\sqrt{\cos(e+fx)}\csc(e+fx)}{a}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$
$$= \frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\csc(e+fx)\,dx}{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$
$$= -\frac{\sqrt{a\sin(e+fx)}\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \cos(e+fx)\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$
$$= -\frac{\left(2\sqrt{a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$
$$= -\frac{\sqrt{a\sin(e+fx)}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\cos(e+fx)}\right)}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} + \frac{\sqrt{a\sin}}{}$$
$$= \frac{\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{af\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\tanh^{-1}\left(\sqrt{\cos(e+f}\right.}{af\sqrt{\cos(e+fx)}}$$

Mathematica [A]
time = 0.15, size = 80, normalized size = 0.75

$$\frac{\left(\text{ArcTan}\left(\sqrt[4]{\cos^2(e+fx)}\right)-\tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)\right)\sin(2(e+fx))}{2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
[Out] ((ArcTan[(Cos[e + f*x]^2)^(1/4)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)])*Sin[2*(
e + f*x)])/(2*f*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e +
f*x]])
```

Maple [A]
time = 0.33, size = 177, normalized size = 1.67

method	result
default	$-\frac{(\cos(fx+e)-1)\left(\ln\left(-\frac{2\left(\cos^2(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\left(\cos^2(fx+e)\right)+2\cos(fx+e)-2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-1}{\sin(fx+e)^2}\right)+\arctan\left(\right.\right.}{2f\sqrt{a\sin(fx+e)}\,\sin(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)*(ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/si
n(f*x+e)^2)+arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))/(a*sin(f*x+e)
)^(1/2)/sin(f*x+e)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*sin(f*x+e)/cos(f
*x+e))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(98) = 196$.
time = 0.60, size = 453, normalized size = 4.27

$$\left[\frac{2\sqrt{-ab}\arctan\left(\frac{2\sqrt{-ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)+ab)\sin(fx+e)}\right)-\sqrt{-ab}\log\left(-\frac{ab\cos(fx+e)^3-5ab\cos(fx+e)^2+4\sqrt{-ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-5ab\cos(fx+e)+ab}{\cos(fx+e)^3+3\cos(fx+e)^2+3\cos(fx+e)+1}\right)}{4abf},\ -\frac{2\sqrt{ab}\arctan\left(\frac{2\sqrt{ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)-ab)\sin(fx+e)}\right)-\sqrt{ab}\log\left(\frac{4\sqrt{ab}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}-\left(ab\cos(fx+e)^2+6ab\cos(fx+e)+ab\right)\sin(fx+e)}{\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sin(fx+e)}\right)}{4abf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*sqrt(-a*b)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x
+ e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) + a*b)*sin(f*x + e))) -
sqrt(-a*b)*log(-(a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*s
qrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x
+ e) - 5*a*b*cos(f*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos
(f*x + e) + 1)))/(a*b*f), -1/4*(2*sqrt(a*b)*arctan(2*sqrt(a*b)*sqrt(a*sin(f
*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e)
- a*b)*sin(f*x + e))) - sqrt(a*b)*log((4*sqrt(a*b)*(cos(f*x + e)^2 + cos(f*
x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)) - (a*b*cos(f
*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x + e))/((cos(f*x + e)^2 - 2*co
s(f*x + e) + 1)*sin(f*x + e))))/(a*b*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(a*sin(e + f*x))*sqrt(b*tan(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*sin(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2)),x)
[Out] int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(1/2)), x)
```

3.132 $\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=87

$$-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] -(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*
e),2^(1/2))*(a*sin(f*x+e))^(1/2)/a^2/f/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2
)-b*(a*sin(f*x+e))^(1/2)/a^2/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2679, 2681, 2719}

$$-\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] -((b*Sqrt[a*Sin[e + f*x]])/(a^2*f*(b*Tan[e + f*x])^(3/2))) - (EllipticE[(e
+ f*x)/2, 2]*Sqrt[a*Sin[e + f*x]])/(a^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e +
 f*x]])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{2a^2}$$
$$= -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}} - \frac{\sqrt{a\sin(e+fx)}\int \sqrt{\cos(e+fx)}\,dx}{2a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$
$$= -\frac{b\sqrt{a\sin(e+fx)}}{a^2f(b\tan(e+fx))^{3/2}} - \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{a\sin(e+fx)}}{a^2f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.37, size = 89, normalized size = 1.02

$$-\frac{b\sqrt{a\sin(e+fx)}\left(2\cos^2(e+fx)^{3/4}+{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sin^2(e+fx)\right)}{2a^2f\cos^2(e+fx)^{3/4}(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] -1/2*(b*Sqrt[a*Sin[e + f*x]]*(2*(Cos[e + f*x]^2)^(3/4) + Hypergeometric2F1[
1/4, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x]^2))/(a^2*f*(Cos[e + f*x]^2)^(3/
4)*(b*Tan[e + f*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 315, normalized size = 3.62

method	result
default	$-\frac{\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)\cos(fx+e)-i\,\mathrm{EllipticE}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-I*EllipticE(I*(cos(f
```

```
*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*sin(f*x+e)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-I*Ellipt
icE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*sin(f*x+e)+cos(f*x+e))*sin(f*x+e)/(a*sin(f*x+e))^(3/2)/(
b*sin(f*x+e)/cos(f*x+e))^(1/2)/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 168, normalized size = 1.93

$$\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)^2+\left(\sqrt{2}\cos(fx+e)^2-\sqrt{2}\right)\sqrt{-ab}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+\left(\sqrt{2}\cos(fx+e)^2-\sqrt{2}\right)\sqrt{-ab}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))}{2\left(a^2bf\cos(fx+e)^2-a^2bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)^
2 + (sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + (sqrt(2)*cos(f*x
 + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(f*x + e) - I*sin(f*x + e))))/(a^2*b*f*cos(f*x + e)^2 - a^2*b*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sin(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2)),x)
[Out] int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(1/2)), x)
```

3.133 $\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=146

$$-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}}+\frac{\text{ArcTan}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{a\sin(e+fx)}}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}$$

```
[Out] 1/4*arctan(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f/cos(f*x+e)^(1/2)/(b
*tan(f*x+e))^(1/2)-1/4*arctanh(cos(f*x+e)^(1/2))*(a*sin(f*x+e))^(1/2)/a^3/f
/cos(f*x+e)^(1/2)/(b*tan(f*x+e))^(1/2)-1/2*b/a^2/f/(a*sin(f*x+e))^(1/2)/(b*
tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2679, 2681, 12, 2645, 335, 304, 209, 212}

$$\frac{\sqrt{a\sin(e+fx)}\,\text{ArcTan}\left(\sqrt{\cos(e+fx)}\right)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\sqrt{a\sin(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{4a^3f\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{b}{2a^2f\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] -1/2*b/(a^2*f*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)) + (ArcTan[Sqrt[C
os[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(4*a^3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[
e + f*x]]) - (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[a*Sin[e + f*x]])/(4*a^3*f*Sq
rt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\int \frac{1}{(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx = -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}} + \frac{\int \frac{\quad}{\sqrt{a\sin(e+fx)}\ \cdots}}{4\cdots}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}} + \frac{\sqrt{a\sin(e+fx)}\int \cdots}{4a^2\sqrt{\cos(e+\cdots}}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}} + \frac{\sqrt{a\sin(e+fx)}\int \cdots}{4a^3\sqrt{\cos(e\cdots}}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}} - \frac{\sqrt{a\sin(e+fx)}\,\mathrm{Su}\cdots}{4a^3f\sqrt{\cos(\cdots}}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}} - \frac{\sqrt{a\sin(e+fx)}\,\mathrm{Su}\cdots}{2a^3f\sqrt{\cos\cdots}}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}} - \frac{\sqrt{a\sin(e+fx)}\,\mathrm{Su}\cdots}{4a^3f\sqrt{\cos\cdots}}$$

$$= -\frac{b}{2a^2f\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}} + \frac{\tan^{-1}\left(\sqrt{\cos(e+\cdots}\right.}{4a^3f\sqrt{\cos(e+f\cdots}}$$

Mathematica [A]
time = 0.63, size = 112, normalized size = 0.77

$$\frac{-4\cos^2(e+fx)^{3/4}\cot(e+fx)+\mathrm{ArcTan}\left(\sqrt[4]{\cos^2(e+fx)}\right)\sin(2(e+fx))-\tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)\sin(2(e+fx))}{8a^2f\cos^2(e+fx)^{3/4}\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (-4*(Cos[e + f*x]^2)^(3/4)*Cot[e + f*x] + ArcTan[(Cos[e + f*x]^2)^(1/4)]*Si
n[2*(e + f*x)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)]*Sin[2*(e + f*x)])/(8*a^2*f
*(Cos[e + f*x]^2)^(3/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(120) = 240$.
time = 0.36, size = 319, normalized size = 2.18

method	result

default	$-\frac{\left(4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\ \cos(fx+e)+\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)\cos(fx+e)+\cos(fx+e)\ln\left(-\frac{2\left(\cos^2(fx+e)\right)\sqrt{-\frac{c}{(\cos}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/8/f*(4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)+cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2))*sin(f*x+e)/(a*sin(f*x+e))^(5/2)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(130) = 260.
time = 0.63, size = 659, normalized size = 4.51

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/16*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) + a*b)*sin(f*x + e)))*sin(f*x + e) - sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 5*a*b*cos(f*

```
x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1))*sin
(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*
x + e)^2)/((a^3*b*f*cos(f*x + e)^2 - a^3*b*f)*sin(f*x + e)), -1/16*(2*sqrt(
a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(a*b)*sqrt(a*sin(f*x + e))*sqrt(b*si
n(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) - a*b)*sin(f*x + e
)))*sin(f*x + e) - sqrt(a*b)*(cos(f*x + e)^2 - 1)*log((4*sqrt(a*b)*(cos(f*x
 + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x +
e)) - (a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x + e))/((cos(f
*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x + e) - 8*sqrt(a*sin(
f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)^2)/((a^3*b*f*cos(f
*x + e)^2 - a^3*b*f)*sin(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sin(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{b\,\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2)),x)
[Out] int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(1/2)), x)
```

3.134 $\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{64a^6\sqrt{a\sin(e+fx)}}{585bf\sqrt{b\tan(e+fx)}}-\frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

```
[Out] -16/585*a^4*(a*sin(f*x+e))^(5/2)/b/f/(b*tan(f*x+e))^(1/2)-2/117*a^2*(a*sin(
f*x+e))^(9/2)/b/f/(b*tan(f*x+e))^(1/2)+2/13*(a*sin(f*x+e))^(13/2)/b/f/(b*ta
n(f*x+e))^(1/2)-64/585*a^6*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2676, 2678, 2669}

$$-\frac{64a^6\sqrt{a\sin(e+fx)}}{585bf\sqrt{b\tan(e+fx)}}-\frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(13/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-64*a^6*Sqrt[a*Sin[e + f*x]])/(585*b*f*Sqrt[b*Tan[e + f*x]]) - (16*a^4*(a*
Sin[e + f*x])^(5/2))/(585*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*Sin[e + f*x
])^(9/2))/(117*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(13/2))/(13*
b*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
```

```
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a\sin(e+fx))^{13/2}}{(b\tan(e+fx))^{3/2}}\,dx &= \frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}+\frac{a^2\int(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}\,dx}{13b^2}\\ &= -\frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}+\frac{(8a^4)\int(a\sin(e+fx))^{5/2}\sqrt{}}{117b^2}\\ &= -\frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{13/2}}{13bf\sqrt{b\tan(e+fx)}}+\\ &= -\frac{64a^6\sqrt{a\sin(e+fx)}}{585bf\sqrt{b\tan(e+fx)}}-\frac{16a^4(a\sin(e+fx))^{5/2}}{585bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{9/2}}{117bf\sqrt{b\tan(e+fx)}}+\end{aligned}$$

Mathematica [A]
time = 0.43, size = 67, normalized size = 0.46

$$\frac{a^6\cos^2(e+fx)(-551+340\cos(2(e+fx))-45\cos(4(e+fx)))\sqrt{a\sin(e+fx)}}{2340bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(13/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^6*Cos[e + f*x]^2*(-551 + 340*Cos[2*(e + f*x)] - 45*Cos[4*(e + f*x)])*Sqr
t[a*Sin[e + f*x]])/(2340*b*f*Sqrt[b*Tan[e + f*x]])
```

Maple [A]
time = 0.32, size = 70, normalized size = 0.48

method	result	size
default	$-\frac{2\left(45\left(\cos^4(fx+e)\right)-130\left(\cos^2(fx+e)\right)+117\right)\left(a\sin(fx+e)\right)^{\frac{13}{2}}\cos(fx+e)}{585f\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}\sin(fx+e)^5}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/585/f*(45*cos(f*x+e)^4-130*cos(f*x+e)^2+117)*(a*sin(f*x+e))^(13/2)*cos(f
*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(13/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [A]
time = 0.38, size = 91, normalized size = 0.62

$$-\frac{2\left(45\,a^6\cos\left(fx+e\right)^7-130\,a^6\cos\left(fx+e\right)^5+117\,a^6\cos\left(fx+e\right)^3\right)\sqrt{a\sin\left(fx+e\right)}\,\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}{585\,b^2f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/585*(45*a^6*cos(f*x + e)^7 - 130*a^6*cos(f*x + e)^5 + 117*a^6*cos(f*x +
e)^3)*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))/(b^2*f*sin(f*x
 + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(13/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(13/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(13/2)/(b*tan(f*x + e))^(3/2), x)
```

Mupad [B]
time = 8.60, size = 296, normalized size = 2.03

$$\frac{\left(\cos\left(7\,e+7\,f\,x\right)-\sin\left(7\,e+7\,f\,x\right)\,1{}\mathrm{i}\right)\,\sqrt{\frac{b\,\left(\sin\left(2\,e+2\,f\,x\right)-\cos\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}+1{}\mathrm{i}\right)}{\cos\left(2\,e+2\,f\,x\right)+1+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}}}\,\left(\frac{a^6\,\cos(3\,e+3\,f\,x)\,\sqrt{a\,\sin\left(e+f\,x\right)}\,(\cos(7\,e+7\,f\,x)+\sin(7\,e+7\,f\,x)\,1{}\mathrm{i})\,217{}\mathrm{i}}{9360\,b^2\,f}-\frac{a^6\,\cos(5\,e+5\,f\,x)\,\sqrt{a\,\sin\left(e+f\,x\right)}\,(\cos(7\,e+7\,f\,x)+\sin(7\,e+7\,f\,x)\,1{}\mathrm{i})\,41{}\mathrm{i}}{1872\,b^2\,f}+\frac{a^6\,\cos(7\,e+7\,f\,x)\,\sqrt{a\,\sin\left(e+f\,x\right)}\,(\cos(7\,e+7\,f\,x)+\sin(7\,e+7\,f\,x)\,1{}\mathrm{i})\,1{}\mathrm{i}}{208\,b^2\,f}+\frac{a^6\,\cos(e+f\,x)\,\sqrt{a\,\sin\left(e+f\,x\right)}\,(\cos(7\,e+7\,f\,x)+\sin(7\,e+7\,f\,x)\,1{}\mathrm{i})\,1991{}\mathrm{i}}{9360\,b^2\,f}\right)\,1{}\mathrm{i}}{2\,\sin\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(13/2)/(b*tan(e + f*x))^(3/2),x)
```

```
[Out] ((cos(7*e + 7*f*x) - sin(7*e + 7*f*x)*1i)*((b*(sin(2*e + 2*f*x) - cos(2*e +
 2*f*x)*1i + 1i))/(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))^(1/2)*((a^6
*cos(3*e + 3*f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f*x) + sin(7*e + 7*f*
x)*1i)*217i)/(9360*b^2*f) - (a^6*cos(5*e + 5*f*x)*(a*sin(e + f*x))^(1/2)*(c
os(7*e + 7*f*x) + sin(7*e + 7*f*x)*1i)*41i)/(1872*b^2*f) + (a^6*cos(7*e + 7
*f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f*x) + sin(7*e + 7*f*x)*1i)*1i)/(
208*b^2*f) + (a^6*cos(e + f*x)*(a*sin(e + f*x))^(1/2)*(cos(7*e + 7*f*x) + s
in(7*e + 7*f*x)*1i)*1991i)/(9360*b^2*f))*1i)/(2*sin(e + f*x))
```

3.135 $\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{8a^4\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

```
[Out] -2/45*a^2*(a*sin(f*x+e))^(5/2)/b/f/(b*tan(f*x+e))^(1/2)+2/9*(a*sin(f*x+e))^
(9/2)/b/f/(b*tan(f*x+e))^(1/2)-8/45*a^4*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f*x
+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2676, 2678, 2669}

$$-\frac{8a^4\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(9/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-8*a^4*Sqrt[a*Sin[e + f*x]])/(45*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*Sin
[e + f*x])^(5/2))/(45*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(9/2)
)/(9*b*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
```

```
+ f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
&& EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}} + \frac{a^2\int (a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx}{9b^2}$$
$$= -\frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}} + \frac{(4a^4)\int \sqrt{a\sin(e+fx)}\,\sqrt{b\mathrm{t}}}{45b^2}$$
$$= -\frac{8a^4\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}} - \frac{2a^2(a\sin(e+fx))^{5/2}}{45bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{9/2}}{9bf\sqrt{b\tan(e+fx)}}$$

Mathematica [A]
time = 0.24, size = 57, normalized size = 0.52

$$\frac{a^4\cos^2(e+fx)(-13+5\cos(2(e+fx)))\sqrt{a\sin(e+fx)}}{45bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(9/2)/(b*Tan[e + f*x])^(3/2),x]
[Out] (a^4*Cos[e + f*x]^2*(-13 + 5*Cos[2*(e + f*x)])*Sqrt[a*Sin[e + f*x]])/(45*b*
f*Sqrt[b*Tan[e + f*x]])
```

Maple [A]
time = 0.31, size = 60, normalized size = 0.55

method	result	size
default	$\frac{2(a\sin(fx+e))^{\frac{9}{2}}\left(5\left(\cos^2(fx+e)\right)-9\right)\cos(fx+e)}{45f\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}\sin(fx+e)^3}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/45/f*(a*sin(f*x+e))^(9/2)*(5*cos(f*x+e)^2-9)*cos(f*x+e)/(b*sin(f*x+e)/cos
(f*x+e))^(3/2)/sin(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(9/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [A]
time = 0.36, size = 77, normalized size = 0.71

$$\frac{2\left(5\,a^{4}\cos\left(fx+e\right)^{5}-9\,a^{4}\cos\left(fx+e\right)^{3}\right)\sqrt{a\sin\left(fx+e\right)}\,\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}{45\,b^{2}f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] 2/45*(5*a^4*cos(f*x + e)^5 - 9*a^4*cos(f*x + e)^3)*sqrt(a*sin(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e))/(b^2*f*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(9/2)/(b*tan(f*x + e))^(3/2), x)
```

Mupad [B]
time = 5.48, size = 94, normalized size = 0.86

$$\frac{a^4\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(47\,\sin\left(2\,e+2\,f\,x\right)+16\,\sin\left(4\,e+4\,f\,x\right)-5\,\sin\left(6\,e+6\,f\,x\right)\right)}{360\,b^2\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(9/2)/(b*tan(e + f*x))^(3/2),x)
[Out] (a^4*(a*sin(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(
1/2)*(47*sin(2*e + 2*f*x) + 16*sin(4*e + 4*f*x) - 5*sin(6*e + 6*f*x)))/(360
*b^2*f*(cos(2*e + 2*f*x) - 1))
```

3.136 $\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

```
[Out] -2/5*b*(a*sin(f*x+e))^(5/2)/f/(b*tan(f*x+e))^(5/2)
```

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{2669\}$

$$-\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*b*(a*Sin[e + f*x])^(5/2))/(5*f*(b*Tan[e + f*x])^(5/2))
```

Rule 2669

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2b(a\sin(e+fx))^{5/2}}{5f(b\tan(e+fx))^{5/2}}$$

Mathematica [A]
time = 0.15, size = 45, normalized size = 1.41

$$-\frac{2a^2\cos^2(e+fx)\sqrt{a\sin(e+fx)}}{5bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*Cos[e + f*x]^2*Sqrt[a*Sin[e + f*x]])/(5*b*f*Sqrt[b*Tan[e + f*x]])
```

Maple [A]
time = 0.30, size = 48, normalized size = 1.50

method	result	size
default	$-\frac{2(a\sin(fx+e))^{\frac{5}{2}}\cos(fx+e)}{5f\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}\sin(fx+e)}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/f*(a*sin(f*x+e))^(5/2)*cos(f*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/sin(
f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(28) = 56$.
time = 0.41, size = 60, normalized size = 1.88

$$-\frac{2\sqrt{a\sin(fx+e)}\,a^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)^3}{5\,b^2f\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(a*sin(f*x + e))*a^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e
)^3/(b^2*f*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

Mupad [B]
time = 4.01, size = 81, normalized size = 2.53

$$\frac{a^2\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\left(2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{10\,b^2\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2),x)
[Out] (a^2*(a*sin(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*((b*sin
(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*b^2*f*(cos(2*e + 2*f*x) -
 1))
```

3.137 $\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=141

$$\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{a\text{ArcTan}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}-\frac{a\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right.}{b^2f\sqrt{}}$$

```
[Out] 2*(a*sin(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)-a*arctan(cos(f*x+e)^(1/2))*
cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)-a*arctanh(
cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e)
)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2675, 2681, 12, 2645, 335, 218, 212, 209}

$$-\frac{a\sqrt{\cos(e+fx)}\,\text{ArcTan}\left(\sqrt{\cos(e+fx)}\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}-\frac{a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{b^2f\sqrt{a\sin(e+fx)}}+\frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*Sqrt[a*Sin[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) - (a*ArcTan[Sqrt[Cos[e
+ f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x
]]) - (a*ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]
])/(b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2675

```
Int[Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]/((b_.)*tan[(e_.) + (f_.)*(x_)])^(3/
2), x_Symbol] :> Simp[2*(Sqrt[a*Sin[e + f*x]]/(b*f*Sqrt[b*Tan[e + f*x]])),
x] + Dist[a^2/b^2, Int[Sqrt[b*Tan[e + f*x]]/(a*Sin[e + f*x])^(3/2), x], x]
/; FreeQ[{a, b, e, f}, x]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx}{b^2}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(a^2\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\csc(e+fx)}{a\sqrt{\cos(e+fx)}}\,dx}{b^2\sqrt{a\sin(e+fx)}}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\int \frac{\csc(e+fx)}{\sqrt{\cos(e+fx)}}\,dx}{b^2\sqrt{a\sin(e+fx)}}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx\right.}{b^2f\sqrt{a\sin(e+fx)}}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{\left(2a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x,\right.}{b^2f\sqrt{a\sin(e+fx)}}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{\left(a\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\right.}{b^2f\sqrt{a\sin(e+fx)}}$$

$$= \frac{2\sqrt{a\sin(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{a\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [A]
time = 0.40, size = 88, normalized size = 0.62

$$\frac{\left(-\text{ArcTan}\left(\sqrt[4]{\cos^2(e+fx)}\right) - \tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right) + 2\sqrt[4]{\cos^2(e+fx)}\right)\sqrt{a\sin(e+fx)}}{bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] ((-ArcTan[(Cos[e + f*x]^2)^(1/4)] - ArcTanh[(Cos[e + f*x]^2)^(1/4)] + 2*(Co
s[e + f*x]^2)^(1/4))*Sqrt[a*Sin[e + f*x]])/(b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt
[b*Tan[e + f*x]])
```

Maple [A]
time = 0.34, size = 237, normalized size = 1.68

method	result

default	$-\frac{(\cos(fx+e)-1)\left(4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cos(fx+e)-\ln\left(-\frac{2\left(\cos^2(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\left(\cos^2(fx+e)\right)+2\cos(fx+e)-2}{\sin(fx+e)^2}\right.\right.}{2f\sin(fx+e)\cos(fx+e)\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)*(4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)-ln
(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+arctan(1/2/(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))
*(a*sin(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)
/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(131) = 262.
time = 0.65, size = 577, normalized size = 4.09

$$\left[\frac{2b\sqrt{-\frac{a}{b}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{a}{b}}\cos(fx+e)}{(a\cos(fx+e)+a)\sin(fx+e)}\right)\sin(fx+e)+b\sqrt{-\frac{a}{b}}\log\left(-\frac{a\cos(fx+e)^3+4\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{a}{b}}\cos(fx+e)\sin(fx+e)-5a\cos(fx+e)^2-5a\cos(fx+e)+a}{\cos(fx+e)^3+3\cos(fx+e)^2+3\cos(fx+e)+1}\right)\sin(fx+e)+8\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{4b^2f\sin(fx+e)}, \frac{2b\sqrt{\frac{a}{b}}\arctan\left(\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{a}{b}}\cos(fx+e)}{(a\cos(fx+e)-a)\sin(fx+e)}\right)\sin(fx+e)+b\sqrt{\frac{a}{b}}\log\left(\frac{4\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{a}{b}}-\left(a\cos(fx+e)^2+6a\cos(fx+e)+a\right)\sin(fx+e)}{\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sin(fx+e)}\right)\sin(fx+e)+8\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{4b^2f\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(-a/b)*arctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(
f*x + e))*sqrt(-a/b)*cos(f*x + e)/((a*cos(f*x + e) + a)*sin(f*x + e)))*sin(
f*x + e) + b*sqrt(-a/b)*log(-(a*cos(f*x + e)^3 + 4*sqrt(a*sin(f*x + e))*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(-a/b)*cos(f*x + e)*sin(f*x + e) - 5*a*c
os(f*x + e)^2 - 5*a*cos(f*x + e) + a)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 +
3*cos(f*x + e) + 1))*sin(f*x + e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e)), 1/4*(2*b*sqrt(a/b)*ar
ctan(2*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(a/b)*cos
```

```
(f*x + e)/((a*cos(f*x + e) - a)*sin(f*x + e)))*sin(f*x + e) + b*sqrt(a/b)*l
og((4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(a/b) - (a*cos(f*x + e)^2 + 6*a*cos(f*x + e) + a)*sin
(f*x + e))/((cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x +
e) + 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))
/(b^2*f*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \sin{\left(e + f x \right)}}}{\left(b \tan{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
[Out] Integral(sqrt(a*sin(e + f*x))/(b*tan(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sin(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,\sin\left(e+f\,x\right)}}{{\left(b\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2), x)
```

3.138 $\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=151

$$-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}+\frac{\mathrm{ArcTan}\left(\sqrt{\cos(e+fx)}\right)\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}}+\frac{\tanh$$

```
[Out] -1/2/b/f/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2)+1/4*arctan(cos(f*x+e)^(1
/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/b^2/f/(a*sin(f*x+e))^(1/2)+1/4
*arctanh(cos(f*x+e)^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a/b^2/f/(a
*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2677, 2681, 12, 2645, 335, 218, 212, 209}

$$\frac{\sqrt{\cos(e+fx)}\,\mathrm{ArcTan}\left(\sqrt{\cos(e+fx)}\right)\sqrt{b\tan(e+fx)}}{4ab^2f\sqrt{a\sin(e+fx)}}+\frac{\sqrt{\cos(e+fx)}\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\sqrt{\cos(e+fx)}\right)}{4ab^2f\sqrt{a\sin(e+fx)}}-\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -1/2*1/(b*f*(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) + (ArcTan[Sqrt[Cos
[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]])/(4*a*b^2*f*Sqrt[a*Sin[
e + f*x]]) + (ArcTanh[Sqrt[Cos[e + f*x]]]*Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e +
 f*x]])/(4*a*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{3/2}}}{4b^2}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\,\sqrt{\right.}}{4}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\,\sqrt{\right.}}{4}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\left(\sqrt{\cos(e+fx)}\,\sqrt{\right.}}{}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\left(\sqrt{\cos(e+fx)}\,\sqrt{\right.}}{}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\left(\sqrt{\cos(e+fx)}\,\sqrt{\right.}}{}$$

$$= -\frac{1}{2bf(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}} + \frac{\tan^{-1}\left(\sqrt{\cos(e+\right.}}{4a}$$

Mathematica [**A**]
time = 0.36, size = 103, normalized size = 0.68

$$\frac{\left(\text{ArcTan}\left(\sqrt[4]{\cos^2(e+fx)}\right)+\tanh^{-1}\left(\sqrt[4]{\cos^2(e+fx)}\right)-2\sqrt[4]{\cos^2(e+fx)}\csc^2(e+fx)\right)\sin^2(e+fx)}{4bf\sqrt[4]{\cos^2(e+fx)}\,(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((ArcTan[(Cos[e + f*x]^2)^(1/4)] + ArcTanh[(Cos[e + f*x]^2)^(1/4)] - 2*(Cos
[e + f*x]^2)^(1/4)*Csc[e + f*x]^2)*Sin[e + f*x]^2)/(4*b*f*(Cos[e + f*x]^2)^
(1/4)*(a*Sin[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(125) = 250$.
time = 0.35, size = 320, normalized size = 2.12

method	result

default	$\frac{\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)\cos(fx+e)-\cos(fx+e)\ln\left(-\frac{2\left(\cos^2(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\left(\cos^2(fx+e)\right)+2\cos(fx+e)-}{\sin(fx+e)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)-cos(f*x+
e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-arctan(1
/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-1)/sin(f*x+e)^2)-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*sin(f
*x+e)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(a*sin(f*x+e))^(3/2)/(b*sin(f*x+
e)/cos(f*x+e))^(3/2)/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(135) = 270.
time = 0.69, size = 662, normalized size = 4.38

$$\left[-\frac{2\sqrt{-ab}\left(\cos(fx+e)^2-1\right)\arctan\left(\frac{2\sqrt{-ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)+ab)\sin(fx+e)}\right)\sin(fx+e)+\sqrt{-ab}\left(\cos(fx+e)^2-1\right)\log\left(-\frac{ab\cos(fx+e)^3-5ab\cos(fx+e)^2+4\sqrt{-ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)-5ab\cos(fx+e)+ab}{\cos(fx+e)^3+3\cos(fx+e)^2+3\cos(fx+e)+1}\right)\sin(fx+e)-8\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{16\left(a^2b^2f\cos(fx+e)^2-a^2b^2f\right)\sin(fx+e)},\ -\frac{2\sqrt{ab}\left(\cos(fx+e)^2-1\right)\arctan\left(\frac{2\sqrt{ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{(ab\cos(fx+e)-ab)\sin(fx+e)}\right)\sin(fx+e)-\sqrt{ab}\left(\cos(fx+e)^2-1\right)\log\left(-\frac{4\sqrt{ab}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}+\left(ab\cos(fx+e)^2+6ab\cos(fx+e)+ab\right)\sin(fx+e)}{\left(\cos(fx+e)^2-2\cos(fx+e)+1\right)\sin(fx+e)}\right)\sin(fx+e)-8\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{16\left(a^2b^2f\cos(fx+e)^2-a^2b^2f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*(2*sqrt(-a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(-a*b)*sqrt(a*sin(f*
x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) +
 a*b)*sin(f*x + e)))*sin(f*x + e) + sqrt(-a*b)*(cos(f*x + e)^2 - 1)*log(-(a
*b*cos(f*x + e)^3 - 5*a*b*cos(f*x + e)^2 + 4*sqrt(-a*b)*sqrt(a*sin(f*x + e)
)*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - 5*a*b*cos(f
```

```
*x + e) + a*b)/(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 3*cos(f*x + e) + 1))*si
n(f*x + e) - 8*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f
*x + e))/((a^2*b^2*f*cos(f*x + e)^2 - a^2*b^2*f)*sin(f*x + e)), -1/16*(2*sq
rt(a*b)*(cos(f*x + e)^2 - 1)*arctan(2*sqrt(a*b)*sqrt(a*sin(f*x + e))*sqrt(b
*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)/((a*b*cos(f*x + e) - a*b)*sin(f*x
+ e)))*sin(f*x + e) - sqrt(a*b)*(cos(f*x + e)^2 - 1)*log(-(4*sqrt(a*b)*(cos
(f*x + e)^2 + cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*
x + e)) + (a*b*cos(f*x + e)^2 + 6*a*b*cos(f*x + e) + a*b)*sin(f*x + e))/((c
os(f*x + e)^2 - 2*cos(f*x + e) + 1)*sin(f*x + e)))*sin(f*x + e) - 8*sqrt(a*
sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/((a^2*b^2*f*c
os(f*x + e)^2 - a^2*b^2*f)*sin(f*x + e))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5010 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a\,\sin(e+f\,x))^{3/2}\,(b\tan(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2)),x)
[Out] int(1/((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^(3/2)), x)
```

3.139 $\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}+\frac{8a^6\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\ldots}{77b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -4/77*a^4*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)-2/77*a^2*(a*sin(f*x
+e))^(7/2)/b/f/(b*tan(f*x+e))^(1/2)+2/11*(a*sin(f*x+e))^(11/2)/b/f/(b*tan(f
*x+e))^(1/2)+8/77*a^6*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellip
ticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/
f/(a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2676, 2678, 2681, 2720}

$$\frac{8a^6\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{77b^2f\sqrt{a\sin(e+fx)}}-\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(11/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-4*a^4*(a*Sin[e + f*x])^(3/2))/(77*b*f*Sqrt[b*Tan[e + f*x]]) - (2*a^2*(a*S
in[e + f*x])^(7/2))/(77*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e + f*x])^(11
/2))/(11*b*f*Sqrt[b*Tan[e + f*x]]) + (8*a^6*Sqrt[Cos[e + f*x]]*EllipticF[(e
 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(77*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{11/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}+\frac{a^2\int (a\sin(e+fx))^{7/2}\sqrt{b\tan(e+fx)}\,dx}{11b^2}$$

$$= -\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}+\frac{(6a^4)\int (a\sin(e+fx))^{3/2}\sqrt{}\ \cdots}{77b^2}$$

$$= -\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}+ \cdots$$

$$= -\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}+ \cdots$$

$$= -\frac{4a^4(a\sin(e+fx))^{3/2}}{77bf\sqrt{b\tan(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{7/2}}{77bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{11/2}}{11bf\sqrt{b\tan(e+fx)}}+\frac{8a\ \cdots}{}$$

Mathematica [A]
time = 0.77, size = 118, normalized size = 0.71

$$\frac{a^5\left(\sqrt[4]{\cos^2(e+fx)}\,(-22\cos(e+fx)-17\cos(3(e+fx))+7\cos(5(e+fx)))+64\cot(e+fx)F\left(\frac{1}{2}\mathrm{ArcSin}(\sin(e+fx))\middle|2\right)\right)\sqrt{a\sin(e+fx)}\,\tan^2(e+fx)}{616f\sqrt[4]{\cos^2(e+fx)}\,(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(11/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^5*((Cos[e + f*x]^2)^(1/4)*(-22*Cos[e + f*x] - 17*Cos[3*(e + f*x)] + 7*Co
s[5*(e + f*x)]) + 64*Cot[e + f*x]*EllipticF[ArcSin[Sin[e + f*x]]/2, 2])*Sqr
t[a*Sin[e + f*x]]*Tan[e + f*x]^2)/(616*f*(Cos[e + f*x]^2)^(1/4)*(b*Tan[e +
f*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.84, size = 181, normalized size = 1.08

method	result
default	$-\frac{2\left(-7\left(\cos^{6}(fx+e)\right)+4i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)+7\left(\cos^{5}(fx+e)\right)+13\left(\cos^{4}(fx\right.\right.}{77f(\cos(fx+e)-1)\cos(fx+e)^{2}\sin(fx+e)^{3}\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77/f*(-7*cos(f*x+e)^6+4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)+7*cos(f*x+
e)^5+13*cos(f*x+e)^4-13*cos(f*x+e)^3-4*cos(f*x+e)^2+4*cos(f*x+e))*(a*sin(f*
x+e))^(11/2)/(cos(f*x+e)-1)/cos(f*x+e)^2/sin(f*x+e)^3/(b*sin(f*x+e)/cos(f*x
+e))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(11/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 150, normalized size = 0.90

$$\frac{2\left(2\sqrt{2}\sqrt{-ab}\,a^{5}\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+2\sqrt{2}\sqrt{-ab}\,a^{5}\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e))+\left(7a^{5}\cos(fx+e)^{5}-13a^{5}\cos(fx+e)^{3}+4a^{5}\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\right)}{77b^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(2*sqrt(2)*sqrt(-a*b)*a^5*weierstrassPInverse(-4, 0, cos(f*x + e) + I*
sin(f*x + e)) + 2*sqrt(2)*sqrt(-a*b)*a^5*weierstrassPInverse(-4, 0, cos(f*x
 + e) - I*sin(f*x + e)) + (7*a^5*cos(f*x + e)^5 - 13*a^5*cos(f*x + e)^3 + 4
*a^5*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/
(b^2*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(11/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(11/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(11/2)/(b*tan(f*x + e))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a\,\sin\left(e+f\,x\right)\right)}^{11/2}}{{\left(b\,\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(11/2)/(b*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a*sin(e + f*x))^(11/2)/(b*tan(e + f*x))^(3/2), x)
```

3.140 $\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}+\frac{4a^4\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{21b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -2/21*a^2*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)+2/7*(a*sin(f*x+e))^
(7/2)/b/f/(b*tan(f*x+e))^(1/2)+4/21*a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/
2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(
f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2676, 2678, 2681, 2720}

$$\frac{4a^4\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{21b^2f\sqrt{a\sin(e+fx)}}-\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}}+\frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(7/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*(a*Sin[e + f*x])^(3/2))/(21*b*f*Sqrt[b*Tan[e + f*x]]) + (2*(a*Sin[e
 + f*x])^(7/2))/(7*b*f*Sqrt[b*Tan[e + f*x]]) + (4*a^4*Sqrt[Cos[e + f*x]]*El
lipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(21*b^2*f*Sqrt[a*Sin[e + f*x]
])
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2678

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Dist[a^2*((m + n - 1)/m), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e
 + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1]
 && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\tan(e+fx))^{3/2}}\,dx = \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{a^2\int(a\sin(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx}{7b^2}$$

$$= -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{(2a^4)\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{21b^2}$$

$$= -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{\left(2a^4\sqrt{\cos(e+fx)}\sqrt{b\tan(e\right.}{21b^2\sqrt{a\sin}}$$

$$= -\frac{2a^2(a\sin(e+fx))^{3/2}}{21bf\sqrt{b\tan(e+fx)}} + \frac{2(a\sin(e+fx))^{7/2}}{7bf\sqrt{b\tan(e+fx)}} + \frac{4a^4\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+f\right.}{21b^2f\sqrt{a\sin(}}$$

Mathematica [A]
time = 0.39, size = 97, normalized size = 0.75

$$\frac{a^3\sqrt{a\sin(e+fx)}\left(8F\left(\frac{1}{2}\text{ArcSin}(\sin(e+fx))\middle|2\right)+\sqrt[4]{\cos^2(e+fx)}\,(5\sin(e+fx)-3\sin(3(e+fx)))\right)}{42bf\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(7/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (a^3*Sqrt[a*Sin[e + f*x]]*(8*EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Cos[e
+ f*x]^2)^(1/4)*(5*Sin[e + f*x] - 3*Sin[3*(e + f*x)])))/(42*b*f*(Cos[e + f*
x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.35, size = 161, normalized size = 1.24

method	result
default	$-\frac{2\left(2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)+3\left(\cos^4(fx+e)\right)-3\left(\cos^3(fx+e)\right)-2\left(\cos^2(fx+e\right.\right.}{21f(\cos(fx+e)-1)\sin(fx+e)\cos(fx+e)^2\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/f*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)+3*cos(f*x+e)^4-3*cos(f*x+e
)^3-2*cos(f*x+e)^2+2*cos(f*x+e))*(a*sin(f*x+e))^(7/2)/(cos(f*x+e)-1)/sin(f*
x+e)/cos(f*x+e)^2/(b*sin(f*x+e)/cos(f*x+e))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(7/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 135, normalized size = 1.04

$$\frac{2\left(\sqrt{2}\,\sqrt{-ab}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{2}\,\sqrt{-ab}\,a^3\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e))-\left(3a^3\cos(fx+e)^3-2a^3\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\right)}{21b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(sqrt(2)*sqrt(-a*b)*a^3*weierstrassPInverse(-4, 0, cos(f*x + e) + I*si
n(f*x + e)) + sqrt(2)*sqrt(-a*b)*a^3*weierstrassPInverse(-4, 0, cos(f*x + e
) - I*sin(f*x + e)) - (3*a^3*cos(f*x + e)^3 - 2*a^3*cos(f*x + e))*sqrt(a*si
n(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(b^2*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(7/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(7/2)/(b*tan(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,\sin\left(e+f\,x\right))^{7/2}}{(b\tan\left(e+f\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(7/2)/(b*tan(e + f*x))^(3/2), x)
```

3.141 $\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}+\frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] 2/3*(a*sin(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(1/2)+2/3*a^2*(cos(1/2*f*x+1/2*
e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*
x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2676, 2681, 2720}

$$\frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}+\frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(a*Sin[e + f*x])^(3/2))/(3*b*f*Sqrt[b*Tan[e + f*x]]) + (2*a^2*Sqrt[Cos[e
 + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(3*b^2*f*Sqrt[a*Si
n[e + f*x]])
```

Rule 2676

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*m))
, x] - Dist[a^2*((n + 1)/(b^2*m)), Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[e +
f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
&& IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a\sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx &= \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}} + \frac{a^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{3b^2} \\
&= \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}} + \frac{\left(a^2\sqrt{\cos(e+fx)}\,\sqrt{b\tan(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{3b^2\sqrt{a\sin(e+fx)}} \\
&= \frac{2(a\sin(e+fx))^{3/2}}{3bf\sqrt{b\tan(e+fx)}} + \frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{3b^2f\sqrt{a\sin(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 80, normalized size = 0.86

$$\frac{2a\sqrt{a\sin(e+fx)}\left(F\left(\frac{1}{2}\text{ArcSin}(\sin(e+fx))\middle|2\right)+\sqrt[4]{\cos^2(e+fx)}\,\sin(e+fx)\right)}{3bf\sqrt[4]{\cos^2(e+fx)}\,\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*Sin[e + f*x]]*(EllipticF[ArcSin[Sin[e + f*x]]/2, 2] + (Cos[e +
f*x]^2)^(1/4)*Sin[e + f*x]))/(3*b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f
*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.33, size = 137, normalized size = 1.47

method	result
default	$-\frac{2\sin(fx+e)\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)-(\cos^2(fx+e))+\cos(fx+e)\right)(a\sin(}{3f(\cos(fx+e)-1)\cos(fx+e)^2\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/f*sin(f*x+e)*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-cos(f*x+e)^2+cos(f
```

```
*x+e))*(a*sin(f*x+e))^(3/2)/(cos(f*x+e)-1)/cos(f*x+e)^2/(b*sin(f*x+e)/cos(f
*x+e))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 112, normalized size = 1.20

$$\frac{2\,\sqrt{a\sin\left(fx+e\right)}\,a\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)+\sqrt{2}\,\sqrt{-ab}\,a\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+\sqrt{2}\,\sqrt{-ab}\,a\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right))}{3\,b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(a*sin(f*x + e))*a*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e
) + sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*
x + e)) + sqrt(2)*sqrt(-a*b)*a*weierstrassPInverse(-4, 0, cos(f*x + e) - I*
sin(f*x + e)))/(b^2*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a \sin(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2), x)
```

3.142 $\int \frac{1}{\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{1}{bf\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}-\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}$$

```
[Out] -1/b/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)-(cos(1/2*f*x+1/2*e)^2)^(1/
2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2
)*(b*tan(f*x+e))^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2677, 2681, 2720}

$$-\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{a\sin(e+fx)}}-\frac{1}{bf\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -(1/(b*f*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])) - (Sqrt[Cos[e + f*x]]*
EllipticF[(e + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[a*Sin[e + f*x]]
)
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{bf\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}} - \frac{\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{2b^2}$$

$$= -\frac{1}{bf\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\,\sqrt{b\ta\right.}{2b^2\,\surd}$$

$$= -\frac{1}{bf\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e\right.}{b^2f\sqrt{a}}$$

Mathematica [A]
time = 0.21, size = 79, normalized size = 0.92

$$\frac{-\sqrt[4]{\cos^2(e+fx)} - F\left(\frac{1}{2}\text{ArcSin}(\sin(e+fx))\middle|2\right)\sin(e+fx)}{bf\sqrt[4]{\cos^2(e+fx)}\,\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-(Cos[e + f*x]^2)^(1/4) - EllipticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e + f*x
])/(b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 185, normalized size = 2.15

method	result
default	$-\frac{\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)\cos(fx+e)+i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+}}\right.}{f\cos(fx+e)^2\sqrt{a\sin(fx+e)}\,\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+I*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x
```

```
+e),I)*sin(f*x+e)+cos(f*x+e))*sin(f*x+e)/cos(f*x+e)^2/(a*sin(f*x+e))^(1/2)/
(b*sin(f*x+e)/cos(f*x+e))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 160, normalized size = 1.86

$$-\frac{\left(\sqrt{2}\cos\left(fx+e\right)^{2}-\sqrt{2}\right)\sqrt{-ab}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)+\left(\sqrt{2}\cos\left(fx+e\right)^{2}-\sqrt{2}\right)\sqrt{-ab}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)-2\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{2\left(ab^{2}f\cos\left(fx+e\right)^{2}-ab^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/2*((sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4,
 0, cos(f*x + e) + I*sin(f*x + e)) + (sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqr
t(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*sqrt(
a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/(a*b^2*f*co
s(f*x + e)^2 - a*b^2*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a\,\sin\left(e+f\,x\right)}\,{\left(b\,\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2)),x)
[Out] int(1/((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^(3/2)), x)
```

3.143 $\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}+\frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{\sqrt{\cos(e+fx)}\,F\left(\tfrac{1}{2}(e+\ldots\right.}{6a^2b^2f\sqrt{a\,\mathrm{s}\ldots}}$$

```
[Out] -1/3/b/f/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2)+1/6/a^2/b/f/(a*sin(f*x+e
))^(1/2)/(b*tan(f*x+e))^(1/2)-1/6*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+
1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e)
)^(1/2)/a^2/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2677, 2679, 2681, 2720}

$$-\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{6a^2b^2f\sqrt{a\sin(e+fx)}}+\frac{1}{6a^2bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -1/3*1/(b*f*(a*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) + 1/(6*a^2*b*f*Sqr
t[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e
+ f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(6*a^2*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
```

```
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{5/2}}}{6b^2}$$

$$= -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} + \frac{}{6a^2bf\sqrt{a\sin(e+f}}$$

$$= -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} + \frac{}{6a^2bf\sqrt{a\sin(e+f}}$$

$$= -\frac{1}{3bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} + \frac{}{6a^2bf\sqrt{a\sin(e+f}}$$

Mathematica [A]
time = 0.37, size = 96, normalized size = 0.74

$$\frac{\sqrt[4]{\cos^2(e+fx)}\,(1-2\csc^2(e+fx)) - F\left(\frac{1}{2}\mathrm{ArcSin}(\sin(e+fx))\middle|2\right)\sin(e+fx)}{6a^2bf\sqrt[4]{\cos^2(e+fx)}\,\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*Sin[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[e + f*x]^2)^(1/4)*(1 - 2*Csc[e + f*x]^2) - EllipticF[ArcSin[Sin[e + f
*x]]/2, 2]*Sin[e + f*x])/(6*a^2*b*f*(Cos[e + f*x]^2)^(1/4)*Sqrt[a*Sin[e + f
*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.36, size = 337, normalized size = 2.59

method	result

default	$\frac{\left(i\sin(fx+e)\left(\cos^3(fx+e)\right)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)+i\sin(fx+e)\left(\cos^2(fx+e)\right)\sqrt{\frac{1}{\cos(fx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)+I*sin(f*x+e)*cos(
f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(cos(f*x+e)-1)/sin(f*x+e),I)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos
(f*x+e)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-cos(f*x+e)^3-cos(f*x+e))*sin(
f*x+e)/(a*sin(f*x+e))^(5/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/cos(f*x+e)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 215, normalized size = 1.65

$$-\frac{\left(\sqrt{2}\cos(fx+e)^4-2\sqrt{2}\cos(fx+e)^2+\sqrt{2}\right)\sqrt{-ab}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+\left(\sqrt{2}\cos(fx+e)^4-2\sqrt{2}\cos(fx+e)^2+\sqrt{2}\right)\sqrt{-ab}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e))+2\left(\cos(fx+e)^3+\cos(fx+e)\right)\sqrt{a\sin(fx+e)}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{12\left(a^3b^2f\cos(fx+e)^4-2a^3b^2f\cos(fx+e)^2+a^3b^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/12*((sqrt(2)*cos(f*x + e)^4 - 2*sqrt(2)*cos(f*x + e)^2 + sqrt(2))*sqrt(-
a*b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + (sqrt(2)*c
os(f*x + e)^4 - 2*sqrt(2)*cos(f*x + e)^2 + sqrt(2))*sqrt(-a*b)*weierstrassP
Inverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(cos(f*x + e)^3 + cos(f*x
 + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^3*b^2*f*c
os(f*x + e)^4 - 2*a^3*b^2*f*cos(f*x + e)^2 + a^3*b^2*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*sin(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(b\,\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2)),x)
[Out] int(1/((a*sin(e + f*x))^(5/2)*(b*tan(e + f*x))^(3/2)), x)
```

3.144 $\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}+\frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}+\frac{1}{12a^4bf\sqrt{a\sin(e+fx)}}$$

```
[Out] -1/5/b/f/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(1/2)+1/30/a^2/b/f/(a*sin(f*x+
e))^(5/2)/(b*tan(f*x+e))^(1/2)+1/12/a^4/b/f/(a*sin(f*x+e))^(1/2)/(b*tan(f*x
+e))^(1/2)-1/12*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(s
in(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*tan(f*x+e))^(1/2)/a^4/b^2/f/
(a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2677, 2679, 2681, 2720}

$$-\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\tan(e+fx)}}{12a^4b^2f\sqrt{a\sin(e+fx)}}+\frac{1}{12a^4bf\sqrt{a\sin(e+fx)}\sqrt{b\tan(e+fx)}}+\frac{1}{30a^2bf(a\sin(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}-\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a*Sin[e + f*x])^(9/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -1/5*1/(b*f*(a*Sin[e + f*x])^(9/2)*Sqrt[b*Tan[e + f*x]]) + 1/(30*a^2*b*f*(a
*Sin[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) + 1/(12*a^4*b*f*Sqrt[a*Sin[e + f
*x]]*Sqrt[b*Tan[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*
Sqrt[b*Tan[e + f*x]])/(12*a^4*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2677

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Dist[(n + 1)/(b^2*(m + n + 1)), Int[(a*Sin[e + f*x])^m*(b
*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1])
```

Rule 2679

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Dist[(m + 2)/(a^2*(m + n + 1)), Int[(a*Sin[e +
f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && Lt
Q[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}} - \frac{\int \frac{\sqrt{b\tan(e+fx)}}{(a\sin(e+fx))^{9/2}}}{10b^2}$$

$$= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}} + \frac{}{30a^2bf(a\sin(e+f}$$

$$= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}} + \frac{}{30a^2bf(a\sin(e+f}$$

$$= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}} + \frac{}{30a^2bf(a\sin(e+f}$$

$$= -\frac{1}{5bf(a\sin(e+fx))^{9/2}\sqrt{b\tan(e+fx)}} + \frac{}{30a^2bf(a\sin(e+f}$$

Mathematica [A]
time = 0.43, size = 106, normalized size = 0.63

$$\frac{\sqrt[4]{\cos^2(e+fx)}\,(5+2\csc^2(e+fx)-12\csc^4(e+fx)) - 5F\left(\frac{1}{2}\text{ArcSin}(\sin(e+fx))\middle|2\right)\sin(e+fx)}{60a^4bf\sqrt[4]{\cos^2(e+fx)}\,\sqrt{a\sin(e+fx)}\,\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a*Sin[e + f*x])^(9/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((Cos[e + f*x]^2)^(1/4)*(5 + 2*Csc[e + f*x]^2 - 12*Csc[e + f*x]^4) - 5*Elli
pticF[ArcSin[Sin[e + f*x]]/2, 2]*Sin[e + f*x])/(60*a^4*b*f*(Cos[e + f*x]^2)
^(1/4)*Sqrt[a*Sin[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 487, normalized size = 2.92

method	result
default	$-\frac{\left(5i\left(\cos^5(fx+e)\right)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}\left(\frac{i(\cos(fx+e)-1)}{\sin(fx+e)},i\right)\sin(fx+e)+5i\left(\cos^4(fx+e)\right)\sqrt{\frac{1}{\cos(fx+e)+1}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/f*(5*I*cos(f*x+e)^5*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)+5*I*cos(f*x+e
)^4*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*
(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)-10*I*sin(f*x+e)*cos(f*x+e)^3*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e
)-1)/sin(f*x+e),I)-10*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)+5*
I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(c
os(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+5*I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),
I)*sin(f*x+e)-5*cos(f*x+e)^5+12*cos(f*x+e)^3+5*cos(f*x+e))*sin(f*x+e)/(a*si
n(f*x+e))^(9/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/cos(f*x+e)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((a*sin(f*x + e))^(9/2)*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 283, normalized size = 1.69

$$-\frac{5\left(\sqrt{2}\cos\left(fx+e\right)^{6}-3\sqrt{2}\cos\left(fx+e\right)^{4}+3\sqrt{2}\cos\left(fx+e\right)^{2}-\sqrt{2}\right)\sqrt{-ab}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right))+5\left(\sqrt{2}\cos\left(fx+e\right)^{6}-3\sqrt{2}\cos\left(fx+e\right)^{4}+3\sqrt{2}\cos\left(fx+e\right)^{2}-\sqrt{2}\right)\sqrt{-ab}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right))+2\left(5\cos\left(fx+e\right)^{5}-12\cos\left(fx+e\right)^{3}-5\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}}{120\left(a^{5}b^{2}f\cos\left(fx+e\right)^{6}-3a^{5}b^{2}f\cos\left(fx+e\right)^{4}+3a^{5}b^{2}f\cos\left(fx+e\right)^{2}-a^{5}b^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/120*(5*(sqrt(2)*cos(f*x + e)^6 - 3*sqrt(2)*cos(f*x + e)^4 + 3*sqrt(2)*co
s(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4, 0, cos(f*x + e)
+ I*sin(f*x + e)) + 5*(sqrt(2)*cos(f*x + e)^6 - 3*sqrt(2)*cos(f*x + e)^4 +
3*sqrt(2)*cos(f*x + e)^2 - sqrt(2))*sqrt(-a*b)*weierstrassPInverse(-4, 0, c
os(f*x + e) - I*sin(f*x + e)) + 2*(5*cos(f*x + e)^5 - 12*cos(f*x + e)^3 - 5
*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e)))/(a^5
*b^2*f*cos(f*x + e)^6 - 3*a^5*b^2*f*cos(f*x + e)^4 + 3*a^5*b^2*f*cos(f*x +
e)^2 - a^5*b^2*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))**(9/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(f*x+e))^(9/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sin(f*x + e))^(9/2)*(b*tan(f*x + e))^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,\sin\left(e+f\,x\right)\right)^{9/2}\,\left(b\,\tan\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(9/2)*(b*tan(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a*sin(e + f*x))^(9/2)*(b*tan(e + f*x))^(3/2)), x)
```

3.145 $\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}}{17df}$$

```
[Out] 6/17*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 17/12],[29/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{3/4}(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(17*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx &= \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int \frac{(b\sin(e+fx))^{11/6}}{\sqrt{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{3/2}}\\ &= \frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}}{17df}\end{aligned}$$

Mathematica [A]
time = 10.44, size = 69, normalized size = 1.08

$$\frac{3\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}\sin(2(e+fx))\sqrt{d\tan(e+fx)}}{17f\sqrt[4]{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
[Out] (3*Hypergeometric2F1[3/4, 17/12, 29/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(4
/3)*Sin[2*(e + f*x)]*Sqrt[d*Tan[e + f*x]])/(17*f*(Cos[e + f*x]^2)^(1/4))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int (b\sin(fx+e))^{\frac{4}{3}}\sqrt{d\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
[Out] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*sin(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(4/3)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\,\sin\left(e+f\,x\right))^{4/3}\,\sqrt{d\tan\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(1/2), x)
```

3.146 $\int \sqrt[3]{b\sin(e+fx)}\,\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\,(d\tan(e+fx))^{3/2}}{11df}$$

```
[Out] 6/11*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 11/12],[23/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{3/4}\sqrt[3]{b\sin(e+fx)}\,(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)}{11df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, 11/12, 23/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(11*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b\sin(e+fx)}\,\sqrt{d\tan(e+fx)}\,dx &= \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{3/2}}\\ &= \frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\,(d\,\mathrm{ta}}{11df}\end{aligned}$$

Mathematica [A]
time = 10.39, size = 69, normalized size = 1.08

$$\frac{3\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\,\sin(2(e+fx))\sqrt{d\tan(e+fx)}}{11f\sqrt[4]{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (3*Hypergeometric2F1[3/4, 11/12, 23/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(1
/3)*Sin[2*(e + f*x)]*Sqrt[d*Tan[e + f*x]])/(11*f*(Cos[e + f*x]^2)^(1/4))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sqrt{d\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\sin\left(e+fx\right)}\sqrt{d\tan\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/3)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((b*sin(e + f*x))**(1/3)*sqrt(d*tan(e + f*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(1/2),x)
```

```
[Out] int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(1/2), x)
```

3.147 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{19}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{7df\sqrt[3]{b\sin(e+fx)}}$$

```
[Out] 6/7*(cos(f*x+e)^2)^(3/4)*hypergeom([7/12, 3/4],[19/12],sin(f*x+e)^2)*(d*tan
(f*x+e))^(3/2)/d/f/(b*sin(f*x+e))^(1/3)
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{19}{12};\sin^2(e+fx)\right)}{7df\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[7/12, 3/4, 19/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(7*d*f*(b*Sin[e + f*x])^(1/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int \frac{\sqrt[6]{b\sin(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{3/2}}$$
$$= \frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{19}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{7df\sqrt[3]{b\sin(e+fx)}}$$

Mathematica [A]
time = 10.35, size = 64, normalized size = 1.00

$$\frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{19}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{7df\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(1/3),x]
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[7/12, 3/4, 19/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(7*d*f*(b*Sin[e + f*x])^(1/3))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x)
[Out] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b*sin(f*x + e)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\tan{(e+fx)}}}{\sqrt[3]{b\sin{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))/(b*sin(e + f*x))**(1/3), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,\tan\left(e+f\,x\right)}}{{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(1/3),x)
```

```
[Out] int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(1/3), x)
```

3.148 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=62

$$\frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{13}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{df(b\sin(e+fx))^{4/3}}$$

```
[Out] 6*(cos(f*x+e)^2)^(3/4)*hypergeom([1/12, 3/4],[13/12],sin(f*x+e)^2)*(d*tan(f
*x+e))^(3/2)/d/f/(b*sin(f*x+e))^(4/3)
```

Rubi [A]
time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{13}{12};\sin^2(e+fx)\right)}{df(b\sin(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[1/12, 3/4, 13/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(3/2))/(d*f*(b*Sin[e + f*x])^(4/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{\left(b\cos^{\frac{3}{2}}(e+fx)(d\tan(e+fx))^{3/2}\right)\int \frac{1}{\sqrt{\cos(e+fx)}\,(b\sin(e+fx))^{5/6}}\,dx}{d(b\sin(e+fx))^{3/2}}$$
$$= \frac{6\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{13}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{df(b\sin(e+fx))^{4/3}}$$

Mathematica [A]
time = 10.37, size = 67, normalized size = 1.08

$$\frac{3\,{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{13}{12};\sin^2(e+fx)\right)\sin(2(e+fx))\sqrt{d\tan(e+fx)}}{f\sqrt[4]{\cos^2(e+fx)}\,(b\sin(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sin[e + f*x])^(4/3),x]
[Out] (3*Hypergeometric2F1[1/12, 3/4, 13/12, Sin[e + f*x]^2]*Sin[2*(e + f*x)]*Sqr
t[d*Tan[e + f*x]])/(f*(Cos[e + f*x]^2)^(1/4)*(b*Sin[e + f*x])^(4/3))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\tan(fx+e)}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x)
[Out] int((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sin(f*x + e))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b^2*cos(f*x + e)^2 -
 b^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(b*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sin(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{{\left(b\,\sin\left(e+f\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(4/3),x)
```

```
[Out] int((d*tan(e + f*x))^(1/2)/(b*sin(e + f*x))^(4/3), x)
```

3.149 $\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{5/2}}{23df}$$

```
[Out] 6/23*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 23/12],[35/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(4/3)*(d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{5/4}(b\sin(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)}{23df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 23/12, 35/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(23*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\sin(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx &= \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int \frac{(b\sin(e+fx))^{17/6}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}\\ &= \frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}(d}{23df}\end{aligned}$$

Mathematica [**A**]
time = 50.63, size = 63, normalized size = 0.98

$$-\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\;{}_2F_1\left(\frac{1}{4},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)\right)(b\sin(e+fx))^{4/3}\sqrt{d\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 11/12, 23/12, Sin
[e + f*x]^2])*(b*Sin[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]])/f
```

Maple [**F**]
time = 0.24, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{4}{3}}\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*d*sin(f*x + e)*tan(f
*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(4/3)*(d*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(b\,\sin\left(e+f\,x\right)\right)}^{4/3}\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((b*sin(e + f*x))^(4/3)*(d*tan(e + f*x))^(3/2), x)
```

3.150 $\int \sqrt[3]{b\sin(e+fx)}\,(d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\,(d\tan(e+fx))^{5/2}}{17df}$$

```
[Out] 6/17*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 17/12],[29/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(1/3)*(d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{5/4}\sqrt[3]{b\sin(e+fx)}\,(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(17*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sqrt[3]{b\sin(e+fx)}\,(d\tan(e+fx))^{3/2}\,dx = \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int \frac{(b\sin(e+fx))^{11/6}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}$$

$$= \frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\,(d}{17df}$$

Mathematica [A]
time = 50.52, size = 63, normalized size = 0.98

$$-\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{5}{12};\frac{17}{12};\sin^2(e+fx)\right)\right)\sqrt[3]{b\sin(e+fx)}\,\sqrt{d\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
[Out] (-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/12, 17/12, Sin[
e + f*x]^2])*(b*Sin[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]])/f
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int (b\sin(fx+e))^{\frac{1}{3}}\,(d\tan(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
[Out] int((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((b*sin(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/3)*(d*tan(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\,\sin\left(e+f\,x\right))^{1/3}\,(d\tan(e+f\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(3/2),x)
```

```
[Out] int((b*sin(e + f*x))^(1/3)*(d*tan(e + f*x))^(3/2), x)
```

3.151 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{25}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{13df\sqrt[3]{b\sin(e+fx)}}$$

```
[Out] 6/13*(cos(f*x+e)^2)^(5/4)*hypergeom([13/12, 5/4],[25/12],sin(f*x+e)^2)*(d*t
an(f*x+e))^(5/2)/d/f/(b*sin(f*x+e))^(1/3)
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{25}{12};\sin^2(e+fx)\right)}{13df\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[13/12, 5/4, 25/12, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(5/2))/(13*d*f*(b*Sin[e + f*x])^(1/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int \frac{(b\sin(e+fx))^{7/6}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}$$
$$= \frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{25}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{13df\sqrt[3]{b\sin(e+fx)}}$$

Mathematica [A]
time = 50.54, size = 63, normalized size = 0.98

$$-\frac{2d\left(-1+\sqrt[4]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{12},\frac{1}{4};\frac{13}{12};\sin^2(e+fx)\right)\right)\sqrt{d\tan(e+fx)}}{f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(1/3),x]
[Out] (-2*d*(-1 + (Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/12, 1/4, 13/12, Sin[
e + f*x]^2])*Sqrt[d*Tan[e + f*x]])/(f*(b*Sin[e + f*x])^(1/3))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x)
[Out] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b*sin(
f*x + e)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\tan(e+f\,x))^{3/2}}{(b\,\sin\left(e+f\,x\right))^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(1/3),x)
```

```
[Out] int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(1/3), x)
```

3.152 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{19}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{7df(b\sin(e+fx))^{4/3}}$$

```
[Out] 6/7*(cos(f*x+e)^2)^(5/4)*hypergeom([7/12, 5/4],[19/12],sin(f*x+e)^2)*(d*tan
(f*x+e))^(5/2)/d/f/(b*sin(f*x+e))^(4/3)
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{19}{12};\sin^2(e+fx)\right)}{7df(b\sin(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[7/12, 5/4, 19/12, Sin[e + f*x]^
2]*(d*Tan[e + f*x])^(5/2))/(7*d*f*(b*Sin[e + f*x])^(4/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sin(e+fx))^{4/3}}\,dx = \frac{\left(b\cos^{\frac{5}{2}}(e+fx)(d\tan(e+fx))^{5/2}\right)\int \frac{\sqrt[6]{b\sin(e+fx)}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{d(b\sin(e+fx))^{5/2}}$$
$$= \frac{6\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{19}{12};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{7df(b\sin(e+fx))^{4/3}}$$

Mathematica [A]
time = 50.57, size = 69, normalized size = 1.08

$$-\frac{2d\left(-7+4\sqrt[4]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)\right)(b\sin(e+fx))^{2/3}\sqrt{d\tan(e+fx)}}{7b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-2*d*(-7 + 4*(Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, 7/12, 19/12, Si
n[e + f*x]^2])*(b*Sin[e + f*x])^(2/3)*Sqrt[d*Tan[e + f*x]])/(7*b^2*f)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sin(fx+e))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x)
```

```
[Out] int((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sin(f*x + e))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b^2*c
os(f*x + e)^2 - b^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sin(f*x+e))**(4/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sin(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\tan(e+f\,x))^{3/2}}{(b\,\sin\left(e+f\,x\right))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(4/3),x)
```

```
[Out] int((d*tan(e + f*x))^(3/2)/(b*sin(e + f*x))^(4/3), x)
```

3.153 $\int \sqrt{b\sin(e+fx)}\,(d\tan(e+fx))^{4/3}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}\,(d\tan(e+fx))^{7/3}}{17df}$$

```
[Out] 6/17*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 17/12],[29/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(1/2)*(d*tan(f*x+e))^(7/3)/d/f
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{7/6}\sqrt{b\sin(e+fx)}\,(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 17/12, 29/12, Sin[e + f*x]
^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(17*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{b\sin(e+fx)}\,(d\tan(e+fx))^{4/3}\,dx = \frac{\left(b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\right)\int \frac{(b\sin(e+fx))^{11/6}}{\cos^{\frac{4}{3}}(e+fx)}\,dx}{d(b\sin(e+fx))^{7/3}}$$

$$= \frac{6\cos^2(e+fx)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}\,(d\,\mathrm{ta}}{17df}$$

Mathematica [A]
time = 10.49, size = 65, normalized size = 1.02

$$-\frac{3d\left(-1+{}_2F_1\left(\frac{5}{12},\frac{5}{4};\frac{17}{12};-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\right)\sqrt{b\sin(e+fx)}\,\sqrt[3]{d\tan(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (-3*d*(-1 + Hypergeometric2F1[5/12, 5/4, 17/12, -Tan[e + f*x]^2]*(Sec[e + f
*x]^2)^(1/4))*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3))/f
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)}\,\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*d*tan(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/2)*(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,\sin\left(e+f\,x\right)}\,\left(d\tan(e+f\,x)\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(4/3),x)
```

```
[Out] int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(4/3), x)
```

3.154 $\int \sqrt{b\sin(e+fx)}\,\sqrt[3]{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}\,(d\tan(e+fx))^{4/3}}{11df}$$

```
[Out] 6/11*(cos(f*x+e)^2)^(2/3)*hypergeom([2/3, 11/12],[23/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3)/d/f
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{2/3}\sqrt{b\sin(e+fx)}\,(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)}{11df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 11/12, 23/12, Sin[e + f*x]
^2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(11*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sin(e+fx)}\,\sqrt[3]{d\tan(e+fx)}\,dx &= \frac{\left(b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\right)\int \frac{(b\sin(e+fx))^{5/6}}{\sqrt[3]{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{4/3}}\\ &= \frac{6\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{23}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}\,(d\ta}{11df}\end{aligned}$$

Mathematica [A]
time = 10.41, size = 66, normalized size = 1.03

$$\frac{6\,{}_2F_1\left(\frac{11}{12},\frac{5}{4};\frac{23}{12};-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\,\sqrt{b\sin(e+fx)}\,(d\tan(e+fx))^{4/3}}{11df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*Hypergeometric2F1[11/12, 5/4, 23/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(
1/4)*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(11*d*f)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin(fx+e)}\,(d\tan(fx+e))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin(e+fx)}\,\sqrt[3]{d\tan(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/2)*(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral(sqrt(b*sin(e + f*x))*(d*tan(e + f*x))**(1/3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b\,\sin\left(e+f\,x\right)}\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(1/3),x)
```

```
[Out] int((b*sin(e + f*x))^(1/2)*(d*tan(e + f*x))^(1/3), x)
```

3.155 $\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{6\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}\,(d\tan(e+fx))^{2/3}}{7df}$$

```
[Out] 6/7*(cos(f*x+e)^2)^(1/3)*hypergeom([1/3, 7/12],[19/12],sin(f*x+e)^2)*(b*sin
(f*x+e))^(1/2)*(d*tan(f*x+e))^(2/3)/d/f
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\sqrt[3]{\cos^2(e+fx)}\sqrt{b\sin(e+fx)}\,(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 7/12, 19/12, Sin[e + f*x]^
2]*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(7*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{b\sin(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{\left(b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\right)\int \sqrt[3]{\cos(e+fx)}\,\sqrt[6]{b\sin(e+fx)}\,dx}{d(b\sin(e+fx))^{2/3}}$$
$$= \frac{6\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}\,(d\tan(e+fx))^{2/3}}{7df}$$

Mathematica [A]
time = 10.38, size = 66, normalized size = 1.03

$$\frac{6\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{19}{12};-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\,\sqrt{b\sin(e+fx)}\,(d\tan(e+fx))^{2/3}}{7df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
[Out] (6*Hypergeometric2F1[7/12, 5/4, 19/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1
/4)*Sqrt[b*Sin[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(7*d*f)
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sin(fx+e)}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
[Out] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)/(d*tan(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sin\left(e+fx\right)}}{\sqrt[3]{d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/2)/(d*tan(f*x+e))**(1/3),x)
[Out] Integral(sqrt(b*sin(e + f*x))/(d*tan(e + f*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,\sin\left(e+f\,x\right)}}{{\left(d\tan\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3),x)
[Out] int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3), x)
```

3.156 $\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=62

$$\frac{6\,{}_2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{13}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}}{df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

```
[Out] 6*hypergeom([-1/6, 1/12],[13/12],sin(f*x+e)^2)*(b*sin(f*x+e))^(1/2)/d/f/(co
s(f*x+e)^2)^(1/6)/(d*tan(f*x+e))^(1/3)
```

Rubi [A]
time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\sqrt{b\sin(e+fx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{13}{12};\sin^2(e+fx)\right)}{df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*Hypergeometric2F1[-1/6, 1/12, 13/12, Sin[e + f*x]^2]*Sqrt[b*Sin[e + f*x]
])/(d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{\sqrt{b\sin(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{\left(b\sqrt[3]{b\sin(e+fx)}\right)\int \frac{\cos^{\frac{4}{3}}(e+fx)}{(b\sin(e+fx))^{5/6}}\,dx}{d\sqrt[3]{\cos(e+fx)}\,\sqrt[3]{d\tan(e+fx)}}$$
$$= \frac{6\,{}_2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{13}{12};\sin^2(e+fx)\right)\sqrt{b\sin(e+fx)}}{df\sqrt[6]{\cos^2(e+fx)}\,\sqrt[3]{d\tan(e+fx)}}$$

Mathematica [A]
time = 10.45, size = 64, normalized size = 1.03

$$\frac{6\,{}_2F_1\left(\frac{1}{12},\frac{5}{4};\frac{13}{12};-\tan^2(e+fx)\right)\sqrt[4]{\sec^2(e+fx)}\,\sqrt{b\sin(e+fx)}}{df\sqrt[3]{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sin[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
[Out] (6*Hypergeometric2F1[1/12, 5/4, 13/12, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1
/4)*Sqrt[b*Sin[e + f*x]])/(d*f*(d*Tan[e + f*x])^(1/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sin(fx+e)}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
[Out] int((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)/(d^2*tan(f*x + e)^2),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/2)/(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)}}{{\left(d\tan\left(e+f\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3),x)
```

```
[Out] int((b*sin(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3), x)
```

3.157 $\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{7/3}}{23df}$$

```
[Out] 6/23*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 23/12],[35/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(3/2)*(d*tan(f*x+e))^(7/3)/d/f
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{7/6}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)}{23df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 23/12, 35/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(23*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx &= \frac{\left(b\cos^{\frac{7}{3}}(e+fx)(d\tan(e+fx))^{7/3}\right)\int \frac{(b\sin(e+fx))^{17/6}}{\cos^{\frac{4}{3}}(e+fx)}\,dx}{d(b\sin(e+fx))^{7/3}}\\ &= \frac{6\cos^2(e+fx)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{35}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d}{23df}\end{aligned}$$

Mathematica [A]
time = 50.71, size = 85, normalized size = 1.33

$$\frac{3d\left(-{}_2F_1\left(\frac{11}{12},\frac{7}{4};\frac{23}{12};-\tan^2(e+fx)\right)\sec^2(e+fx)+\sqrt[4]{\sec^2(e+fx)}\right)(b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}}{f\sqrt[4]{\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (3*d*(-(Hypergeometric2F1[11/12, 7/4, 23/12, -Tan[e + f*x]^2]*Sec[e + f*x]^
2) + (Sec[e + f*x]^2)^(1/4))*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3))
/(f*(Sec[e + f*x]^2)^(1/4))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(d\tan\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*b*d*sin(f*x + e)*tan(f
*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(3/2)*(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b \sin(e + f x))^{3/2} (d \tan(e + f x))^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(4/3),x)
```

```
[Out] int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(4/3), x)
```

3.158 $\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{6\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}}{17df}$$

```
[Out] 6/17*(cos(f*x+e)^2)^(2/3)*hypergeom([2/3, 17/12],[29/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3)/d/f
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\cos^2(e+fx)^{2/3}(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)}{17df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[2/3, 17/12, 29/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(17*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (b\sin(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx &= \frac{\left(b\cos^{\frac{4}{3}}(e+fx)(d\tan(e+fx))^{4/3}\right)\int \frac{(b\sin(e+fx))^{11/6}}{\sqrt[3]{\cos(e+fx)}}\,dx}{d(b\sin(e+fx))^{4/3}}\\ &= \frac{6\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{29}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d}{17df}\end{aligned}$$

Mathematica [A]
time = 10.53, size = 72, normalized size = 1.12

$$\frac{6\cos(e+fx)\,{}_2F_1\left(\frac{17}{12},\frac{7}{4};\frac{29}{12};-\tan^2(e+fx)\right)\sec^2(e+fx)^{7/4}(b\sin(e+fx))^{5/2}\sqrt[3]{d\tan(e+fx)}}{17bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
[Out] (6*Cos[e + f*x]*Hypergeometric2F1[17/12, 7/4, 29/12, -Tan[e + f*x]^2]*(Sec[
e + f*x]^2)^(7/4)*(b*Sin[e + f*x])^(5/2)*(d*Tan[e + f*x])^(1/3))/(17*b*f)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int (b\sin(fx+e))^{\frac{3}{2}} (d\tan(fx+e))^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
[Out] int((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(1/3)*b*sin(f*x + e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(3/2)*(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARb near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\,\sin(e+f\,x))^{3/2}\,(d\tan(e+f\,x))^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(1/3),x)
```

```
[Out] int((b*sin(e + f*x))^(3/2)*(d*tan(e + f*x))^(1/3), x)
```

3.159 $\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{6\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{25}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{2/3}}{13df}$$

```
[Out] 6/13*(cos(f*x+e)^2)^(1/3)*hypergeom([1/3, 13/12],[25/12],sin(f*x+e)^2)*(b*s
in(f*x+e))^(3/2)*(d*tan(f*x+e))^(2/3)/d/f
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6\sqrt[3]{\cos^2(e+fx)}\,(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{25}{12};\sin^2(e+fx)\right)}{13df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (6*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 13/12, 25/12, Sin[e + f*x]
^2]*(b*Sin[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(13*d*f)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(b\sin(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{\left(b\cos^{\frac{2}{3}}(e+fx)(d\tan(e+fx))^{2/3}\right)\int \sqrt[3]{\cos(e+fx)}\,(b\sin(e+fx))^{7/6}\,dx}{d(b\sin(e+fx))^{2/3}}$$
$$= \frac{6\sqrt[3]{\cos^2(e+fx)}\;{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{25}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}(d\tan(e+fx))^{2/3}}{13df}$$

Mathematica [A]
time = 50.85, size = 67, normalized size = 1.05

$$\frac{2d\left(-1+{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{13}{12};-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)(b\sin(e+fx))^{3/2}}{3f(d\tan(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (2*d*(-1 + Hypergeometric2F1[1/12, 3/4, 13/12, -Tan[e + f*x]^2]*(Sec[e + f*
x]^2)^(3/4))*(b*Sin[e + f*x])^(3/2))/(3*f*(d*Tan[e + f*x])^(4/3))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sin(f*x + e)/(d*tan(
f*x + e)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(3/2)/(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(d\,\tan\left(e+f\,x\right)\right)^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3),x)
```

```
[Out] int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3), x)
```

3.160 $\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=64

$$\frac{6\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

```
[Out] 6/7*hypergeom([-1/6, 7/12],[19/12],sin(f*x+e)^2)*(b*sin(f*x+e))^(3/2)/d/f/(
cos(f*x+e)^2)^(1/6)/(d*tan(f*x+e))^(1/3)
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2657}

$$\frac{6(b\sin(e+fx))^{3/2}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (6*Hypergeometric2F1[-1/6, 7/12, 19/12, Sin[e + f*x]^2]*(b*Sin[e + f*x])^(3
/2))/(7*d*f*(Cos[e + f*x]^2)^(1/6)*(d*Tan[e + f*x])^(1/3))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(b\sin(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = \frac{\left(b\sqrt[3]{b\sin(e+fx)}\right)\int \cos^{\frac{4}{3}}(e+fx)\sqrt[6]{b\sin(e+fx)}\,dx}{d\sqrt[3]{\cos(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$
$$= \frac{6\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{19}{12};\sin^2(e+fx)\right)(b\sin(e+fx))^{3/2}}{7df\sqrt[6]{\cos^2(e+fx)}\sqrt[3]{d\tan(e+fx)}}$$

Mathematica [A]
time = 10.43, size = 70, normalized size = 1.09

$$\frac{2\left(7+2\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{19}{12};-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\right)(b\sin(e+fx))^{3/2}}{21df\sqrt[3]{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
[Out] (2*(7 + 2*Hypergeometric2F1[7/12, 3/4, 19/12, -Tan[e + f*x]^2]*(Sec[e + f*x
]^2)^(3/4))*(b*Sin[e + f*x])^(3/2))/(21*d*f*(d*Tan[e + f*x])^(1/3))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(b\sin(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
[Out] int((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sin(f*x + e)/(d^2*ta
n(f*x + e)^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(3/2)/(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,\sin\left(e+f\,x\right))^{3/2}}{(d\tan\left(e+f\,x\right))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3),x)
```

```
[Out] int((b*sin(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3), x)
```

3.161 $\int (a\sin(e+fx))^m \tan^3(e+fx)\,dx$

Optimal. Leaf size=48

$$\frac{{}_2F_1\left(2,\frac{4+m}{2};\frac{6+m}{2};\sin^2(e+fx)\right)(a\sin(e+fx))^{4+m}}{a^4f(4+m)}$$

```
[Out] hypergeom([2, 2+1/2*m],[3+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^(4+m)/a^4/f/(
4+m)
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2672, 371}

$$\frac{(a\sin(e+fx))^{m+4}\,{}_2F_1\left(2,\frac{m+4}{2};\frac{m+6}{2};\sin^2(e+fx)\right)}{a^4f(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] (Hypergeometric2F1[2, (4 + m)/2, (6 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x]
)^(4 + m))/(a^4*f*(4 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\int (a\sin(e+fx))^m\tan^3(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^{3+m}}{(a^2-x^2)^2}\,dx, x, a\sin(e+fx)\right)}{f}\\ &= \frac{{}_2F_1\left(2,\frac{4+m}{2};\frac{6+m}{2};\sin^2(e+fx)\right)(a\sin(e+fx))^{4+m}}{a^4f(4+m)}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 53, normalized size = 1.10

$$\frac{{}_2F_1\left(2,\frac{4+m}{2};1+\frac{4+m}{2};\sin^2(e+fx)\right)\sin^4(e+fx)(a\sin(e+fx))^m}{f(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] (Hypergeometric2F1[2, (4 + m)/2, 1 + (4 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x
]^4*(a*Sin[e + f*x])^m)/(f*(4 + m))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (a\sin(fx+e))^m \left(\tan^3(fx+e)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

```
[Out] int((a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(e+fx))^m \tan^3(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e)**3,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*tan(e + f*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^3\,{\left(a\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a*sin(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)^3*(a*sin(e + f*x))^m, x)
```

3.162 $\int (a\sin(e+fx))^m \tan(e+fx)\,dx$

Optimal. Leaf size=48

$$\frac{{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};\sin^2(e+fx)\right)(a\sin(e+fx))^{2+m}}{a^2f(2+m)}$$

```
[Out] hypergeom([1, 1+1/2*m],[2+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^(2+m)/a^2/f/(
2+m)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2672, 371}

$$\frac{(a\sin(e+fx))^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};\sin^2(e+fx)\right)}{a^2f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] (Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x]
)^(2 + m))/(a^2*f*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\int (a\sin(e+fx))^m\tan(e+fx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^{1+m}}{a^2-x^2}\,dx,x,a\sin(e+fx)\right)}{f}\\ &= \frac{{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};\sin^2(e+fx)\right)(a\sin(e+fx))^{2+m}}{a^2f(2+m)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 1.10

$$\frac{{}_2F_1\left(1,\frac{2+m}{2};1+\frac{2+m}{2};\sin^2(e+fx)\right)\sin^2(e+fx)(a\sin(e+fx))^m}{f(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] (Hypergeometric2F1[1, (2 + m)/2, 1 + (2 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x
]^2*(a*Sin[e + f*x])^m)/(f*(2 + m))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (a\sin(fx+e))^m \tan(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*tan(f*x+e),x)
```

```
[Out] int((a*sin(f*x+e))^m*tan(f*x+e),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(e+fx))^m \tan(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e),x)
```

```
[Out] Integral((a*sin(e + f*x))**m*tan(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)\,\left(a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a*sin(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)*(a*sin(e + f*x))^m, x)
```

3.163 $\int \cot(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=17

$$\frac{(a\sin(e+fx))^m}{fm}$$

```
[Out] (a*sin(f*x+e))^m/f/m
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2672, 30}

$$\frac{(a\sin(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a*Sin[e + f*x])^m,x]
```

```
[Out] (a*Sin[e + f*x])^m/(f*m)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)(a\sin(e+fx))^m\,dx &= \frac{\text{Subst}\left(\int x^{-1+m}\,dx, x, a\sin(e+fx)\right)}{f}\\ &= \frac{(a\sin(e+fx))^m}{fm}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{(a\sin(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a*Sin[e + f*x])^m,x]
```

```
[Out] (a*Sin[e + f*x])^m/(f*m)
```

Maple [**A**]
time = 0.34, size = 18, normalized size = 1.06

method	result
derivativedivides	$\frac{(a\sin(fx+e))^m}{fm}$
default	$\frac{(a\sin(fx+e))^m}{fm}$
risch	$\mathrm{e}^{-\frac{m\left(-i\pi\,\mathrm{csgn}\left(i\left(\mathrm{e}^{2i(fx+e)}-1\right)\right)\mathrm{csgn}(\sin(fx+e))^2-i\pi\,\mathrm{csgn}\left(i\mathrm{e}^{-i(fx+e)}\right)\mathrm{csgn}(\sin(fx+e))^2+i\pi\,\mathrm{csgn}(a\sin(fx+e))\mathrm{csgn}(ia\sin(fx+e)}{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a*sin(f*x+e))^m,x,method=_RETURNVERBOSE)
```

```
[Out] (a*sin(f*x+e))^m/f/m
```

Maxima [**A**]
time = 0.27, size = 19, normalized size = 1.12

$$\frac{a^m \sin\left(fx+e\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] a^m*sin(f*x + e)^m/(f*m)
```

Fricas [**A**]
time = 0.41, size = 18, normalized size = 1.06

$$\frac{\left(a\sin\left(fx+e\right)\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] (a*sin(f*x + e))^m/(f*m)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(e+fx\right)\right)^m \cot\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x), x)
```

Giac [A]
time = 0.44, size = 18, normalized size = 1.06

$$\frac{(a\sin(fx+e))^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] (a*sin(f*x + e))^m/(f*m)
```

Mupad [B]
time = 2.45, size = 17, normalized size = 1.00

$$\frac{(a\,\sin(e+f\,x))^m}{f\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a*sin(e + f*x))^m,x)
```

```
[Out] (a*sin(e + f*x))^m/(f*m)
```

3.164 $\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=46

$$-\frac{a^2(a\sin(e+fx))^{-2+m}}{f(2-m)}-\frac{(a\sin(e+fx))^m}{fm}$$

```
[Out] -a^2*(a*sin(f*x+e))^(-2+m)/f/(2-m)-(a*sin(f*x+e))^m/f/m
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2672, 14}

$$-\frac{a^2(a\sin(e+fx))^{m-2}}{f(2-m)}-\frac{(a\sin(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a^2*(a*Sin[e + f*x])^(-2 + m))/(f*(2 - m))) - (a*Sin[e + f*x])^m/(f*m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cot^3(e+fx)(a\sin(e+fx))^m\,dx &= \frac{\text{Subst}\left(\int x^{-3+m}\left(a^2-x^2\right)\,dx, x, a\sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a^2x^{-3+m}-x^{-1+m}\right)\,dx, x, a\sin(e+fx)\right)}{f}\\
&= -\frac{a^2(a\sin(e+fx))^{-2+m}}{f(2-m)}-\frac{(a\sin(e+fx))^m}{fm}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 37, normalized size = 0.80

$$\frac{(2-m+m\csc^2(e+fx))\,(a\sin(e+fx))^m}{f(-2+m)m}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a*Sin[e + f*x])^m,x]
```

```
[Out] ((2 - m + m*Csc[e + f*x]^2)*(a*Sin[e + f*x])^m)/(f*(-2 + m)*m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.68, size = 3063, normalized size = 66.59

method	result	size
risch	Expression too large to display	3063

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a*sin(f*x+e))^m,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-2+m)/f/(exp(2*I*(f*x+e))-1)^2/m*(m*(exp(2*I*(f*x+e))-1)^m/(exp(I*(Re(f
*x)+Re(e)))^m)*a^m/(2^m)*exp(m*Im(f*x)+m*Im(e))*exp(1/2*I*m*Pi*csgn(I*exp(2
*I*(f*x+e))-I)*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2)*exp(-1/2*I*m*Pi*csg
n(I*a)*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*csgn(sin(f*x)*cos(e)+cos(f
*x)*sin(e)))*exp(-1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*csgn
(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*a*sin(f*
x)*cos(e)+I*a*cos(f*x)*sin(e))^2)*exp(-1/2*I*m*Pi*csgn(I*a*sin(f*x)*cos(e)+
I*a*cos(f*x)*sin(e))^3)*exp(1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*si
n(e))*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e)))*exp(1/2*I*m*Pi*csgn(a*
sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^3)*exp(-1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e
)+a*cos(f*x)*sin(e))^2*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(1/2*I*m*P
i*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^3)*exp(1/2*I*m*Pi*csgn(I*exp(-I*(f*
x+e)))*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*a)*cs
gn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*exp(2*I*(f
*x+e))-I)*csgn(I*exp(-I*(f*x+e)))*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*ex
p(-1/2*I*Pi*m)*exp(4*I*f*x)*exp(4*I*e)-2*(exp(2*I*(f*x+e))-1)^m/(exp(I*(Re(
f*x)+Re(e)))^m)*a^m/(2^m)*exp(m*Im(f*x)+m*Im(e))*exp(1/2*I*m*Pi*csgn(I*exp(
2*I*(f*x+e))-I)*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2)*exp(-1/2*I*m*Pi*cs
gn(I*a)*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*csgn(sin(f*x)*cos(e)+cos(
f*x)*sin(e)))*exp(-1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*csg
n(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*a*sin(f
*x)*cos(e)+I*a*cos(f*x)*sin(e))^2)*exp(-1/2*I*m*Pi*csgn(I*a*sin(f*x)*cos(e)
+I*a*cos(f*x)*sin(e))^3)*exp(1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*s
in(e))*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e)))*exp(1/2*I*m*Pi*csgn(a
*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^3)*exp(-1/2*I*m*Pi*csgn(a*sin(f*x)*cos(
```

```
e)+a*cos(f*x)*sin(e))^2*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(1/2*I*m*
Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^3)*exp(1/2*I*m*Pi*csgn(I*exp(-I*(f
*x+e)))*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*a)*c
sgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*exp(2*I*(
f*x+e))-I)*csgn(I*exp(-I*(f*x+e)))*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*e
xp(-1/2*I*Pi*m)*exp(4*I*f*x)*exp(4*I*e)+2*m*(exp(2*I*(f*x+e))-1)^m/(exp(I*(
Re(f*x)+Re(e)))^m)*a^m/(2^m)*exp(m*Im(f*x)+m*Im(e))*exp(1/2*I*m*Pi*csgn(I*e
xp(2*I*(f*x+e))-I)*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2)*exp(-1/2*I*m*Pi
*csgn(I*a)*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*csgn(sin(f*x)*cos(e)+c
os(f*x)*sin(e)))*exp(-1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*
csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*a*si
n(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2)*exp(-1/2*I*m*Pi*csgn(I*a*sin(f*x)*cos
(e)+I*a*cos(f*x)*sin(e))^3)*exp(1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*x
)*sin(e))*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e)))*exp(1/2*I*m*Pi*csg
n(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^3)*exp(-1/2*I*m*Pi*csgn(a*sin(f*x)*c
os(e)+a*cos(f*x)*sin(e))^2*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(1/2*I
*m*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^3)*exp(1/2*I*m*Pi*csgn(I*exp(-I
*(f*x+e)))*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*a
)*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*exp(2*
I*(f*x+e))-I)*csgn(I*exp(-I*(f*x+e)))*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))
)*exp(-1/2*I*Pi*m)*exp(2*I*f*x)*exp(2*I*e)+4*(exp(2*I*(f*x+e))-1)^m/(exp(I*
(Re(f*x)+Re(e)))^m)*a^m/(2^m)*exp(m*Im(f*x)+m*Im(e))*exp(1/2*I*m*Pi*csgn(I*
exp(2*I*(f*x+e))-I)*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2)*exp(-1/2*I*m*P
i*csgn(I*a)*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*csgn(sin(f*x)*cos(e)+
cos(f*x)*sin(e)))*exp(-1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))
*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*a*s
in(f*x)*cos(e)+I*a*cos(f*x)*sin(e))^2)*exp(-1/2*I*m*Pi*csgn(I*a*sin(f*x)*co
s(e)+I*a*cos(f*x)*sin(e))^3)*exp(1/2*I*m*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*
x)*sin(e))*csgn(I*a*sin(f*x)*cos(e)+I*a*cos(f*x)*sin(e)))*exp(1/2*I*m*Pi*cs
gn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^3)*exp(-1/2*I*m*Pi*csgn(a*sin(f*x)*
cos(e)+a*cos(f*x)*sin(e))^2*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)))*exp(1/2*
I*m*Pi*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^3)*exp(1/2*I*m*Pi*csgn(I*exp(-
I*(f*x+e)))*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*
a)*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2)*exp(1/2*I*m*Pi*csgn(I*exp(2
*I*(f*x+e))-I)*csgn(I*exp(-I*(f*x+e)))*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e)
))*exp(-1/2*I*Pi*m)*exp(2*I*f*x)*exp(2*I*e)+m*(exp(2*I*(f*x+e))-1)^m/(exp(I
*(Re(f*x)+Re(e)))^m)*a^m/(2^m)*exp(1/2*m*(I*Pi*csgn(sin(f*x)*cos(e)+cos(f*x
)*sin(e))^3+I*Pi*csgn(I*exp(2*I*(f*x+e))-I)*csgn(sin(f*x)*cos(e)+cos(f*x)*s
in(e))^2+I*Pi*csgn(I*exp(-I*(f*x+e)))*csgn(sin(f*x)*cos(e)+cos(f*x)*sin(e))
^2-I*Pi*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))^2*csgn(sin(f*x)*cos(e)+co
s(f*x)*sin(e))-I*Pi*csgn(I*a)*csgn(a*sin(f*x)*cos(e)+a*cos(f*x)*sin(e))*csg
n(sin(f*x)*cos(e)+cos(f*x)*sin(e))+I*Pi*csgn(I*...
```

Maxima [A]

time = 0.28, size = 50, normalized size = 1.09

$$-\frac{\frac{a^m \sin(fx+e)^m}{m} - \frac{a^m \sin(fx+e)^m}{(m-2)\sin(fx+e)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] -(a^m*sin(f*x + e)^m/m - a^m*sin(f*x + e)^m/((m - 2)*sin(f*x + e)^2))/f
```

Fricas [A]
time = 0.35, size = 60, normalized size = 1.30

$$\frac{\left((m-2)\cos\left(fx+e\right)^2+2\right)\left(a\sin\left(fx+e\right)\right)^m}{fm^2-\left(fm^2-2\,fm\right)\cos\left(fx+e\right)^2-2\,fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] ((m - 2)*cos(f*x + e)^2 + 2)*(a*sin(f*x + e))^m/(f*m^2 - (f*m^2 - 2*f*m)*co
s(f*x + e)^2 - 2*f*m)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(e+fx\right)\right)^m\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a*sin(f*x+e))**m,x)
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^3, x)
```

Mupad [B]
time = 3.31, size = 91, normalized size = 1.98

$$-\frac{\left(a\,\sin\left(e+f\,x\right)\right)^m\,\left(m-4\sin\left(2\,e+2\,f\,x\right)^2+m\,\left(2\sin\left(2\,e+2\,f\,x\right)^2-1\right)+16\sin\left(e+f\,x\right)^2\right)}{f\,m\,\left(2\sin\left(2\,e+2\,f\,x\right)^2-8\sin\left(e+f\,x\right)^2\right)\,\left(m-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a*sin(e + f*x))^m,x)
```

```
[Out] -((a*sin(e + f*x))^m*(m - 4*sin(2*e + 2*f*x)^2 + m*(2*sin(2*e + 2*f*x)^2 -
1) + 16*sin(e + f*x)^2))/(f*m*(2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2)*(m
- 2))
```

3.165 $\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=72

$$-\frac{a^4(a\sin(e+fx))^{-4+m}}{f(4-m)}+\frac{2a^2(a\sin(e+fx))^{-2+m}}{f(2-m)}+\frac{(a\sin(e+fx))^m}{fm}$$

```
[Out] -a^4*(a*sin(f*x+e))^(-4+m)/f/(4-m)+2*a^2*(a*sin(f*x+e))^(-2+m)/f/(2-m)+(a*s
in(f*x+e))^m/f/m
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2672, 276}

$$-\frac{a^4(a\sin(e+fx))^{m-4}}{f(4-m)}+\frac{2a^2(a\sin(e+fx))^{m-2}}{f(2-m)}+\frac{(a\sin(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a^4*(a*Sin[e + f*x])^(-4 + m))/(f*(4 - m))) + (2*a^2*(a*Sin[e + f*x])^(-
2 + m))/(f*(2 - m)) + (a*Sin[e + f*x])^m/(f*m)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^5(e+fx)(a\sin(e+fx))^m\,dx &= \frac{\text{Subst}\left(\int x^{-5+m}\left(a^2-x^2\right)^2\,dx,x,a\sin(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a^4x^{-5+m}-2a^2x^{-3+m}+x^{-1+m}\right)\,dx,x,a\sin(e+fx)\right)}{f}\\ &= -\frac{a^4(a\sin(e+fx))^{-4+m}}{f(4-m)}+\frac{2a^2(a\sin(e+fx))^{-2+m}}{f(2-m)}+\frac{(a\sin(e+fx))^m}{fm}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 62, normalized size = 0.86

$$\frac{(8-6m+m^2-2(-4+m)m\csc^2(e+fx)+(-2+m)m\csc^4(e+fx))\,(a\sin(e+fx))^m}{f(-4+m)(-2+m)m}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a*Sin[e + f*x])^m,x]
```

```
[Out] ((8 - 6*m + m^2 - 2*(-4 + m)*m*Csc[e + f*x]^2 + (-2 + m)*m*Csc[e + f*x]^4)*
(a*Sin[e + f*x])^m)/(f*(-4 + m)*(-2 + m)*m)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.66, size = 7703, normalized size = 106.99

method	result	size
risch	Expression too large to display	7703

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a*sin(f*x+e))^m,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.30, size = 76, normalized size = 1.06

$$\frac{\frac{a^m\sin(fx+e)^m}{m}-\frac{2\,a^m\sin(fx+e)^m}{(m-2)\sin(fx+e)^2}+\frac{a^m\sin(fx+e)^m}{(m-4)\sin(fx+e)^4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] (a^m*sin(f*x + e)^m/m - 2*a^m*sin(f*x + e)^m/((m - 2)*sin(f*x + e)^2) + a^m
*sin(f*x + e)^m/((m - 4)*sin(f*x + e)^4))/f
```

Fricas [A]
time = 0.39, size = 117, normalized size = 1.62

$$\frac{\left((m^2-6\,m+8)\cos\left(fx+e\right)^4+4\,(m-4)\cos\left(fx+e\right)^2+8\right)\left(a\sin\left(fx+e\right)\right)^m}{(fm^3-6\,fm^2+8\,fm)\cos\left(fx+e\right)^4+fm^3-6\,fm^2-2\,(fm^3-6\,fm^2+8\,fm)\cos\left(fx+e\right)^2+8\,fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] ((m^2 - 6*m + 8)*cos(f*x + e)^4 + 4*(m - 4)*cos(f*x + e)^2 + 8)*(a*sin(f*x
+ e))^m/((f*m^3 - 6*f*m^2 + 8*f*m)*cos(f*x + e)^4 + f*m^3 - 6*f*m^2 - 2*(f*
m^3 - 6*f*m^2 + 8*f*m)*cos(f*x + e)^2 + 8*f*m)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(e+fx))^m \cot^5(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^5, x)
```

Mupad [B]
time = 7.57, size = 219, normalized size = 3.04

$$-\frac{(a\sin(e+fx))^m\left(2\sin(2e+2fx)^2+\sin(4e+4fx)\,1\mathrm{i}-1\right)\left(-\frac{2\left(2\sin(2e+2fx)^2-1\right)\left(-2\sin(2e+2fx)^2+\sin(4e+4fx)\,1\mathrm{i}+1\right)}{fm}+\frac{\left(-2\sin(2e+2fx)^2+\sin(4e+4fx)\,1\mathrm{i}+1\right)\left(6m^2-4m+48\right)}{fm\left(m^2-6m+8\right)}+\frac{2\left(2\sin(e+fx)^2-1\right)\left(-2\sin(2e+2fx)^2+\sin(4e+4fx)\,1\mathrm{i}+1\right)\left(-4m^2+8m+32\right)}{fm\left(m^2-6m+8\right)}\right)}{16\sin(e+fx)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a*sin(e + f*x))^m,x)
```

```
[Out] -((a*sin(e + f*x))^m*(sin(4*e + 4*f*x)*1i + 2*sin(2*e + 2*f*x)^2 - 1)*(((si
n(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(6*m^2 - 4*m + 48))/(f*m*(m^2
 - 6*m + 8)) - (2*(2*sin(2*e + 2*f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i - 2*sin(2
*e + 2*f*x)^2 + 1))/(f*m) + (2*(2*sin(e + f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i
- 2*sin(2*e + 2*f*x)^2 + 1)*(8*m - 4*m^2 + 32))/(f*m*(m^2 - 6*m + 8))))/(16
*sin(e + f*x)^4)
```

3.166 $\int (a\sin(e+fx))^m \tan^4(e+fx)\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{\cos^2(e+fx)}\;{}_2F_1\left(\frac{5}{2},\frac{5+m}{2};\frac{7+m}{2};\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{5+m}}{a^5f(5+m)}$$

```
[Out] hypergeom([5/2, 5/2+1/2*m],[7/2+1/2*m],sin(f*x+e)^2)*sec(f*x+e)*(a*sin(f*x+
e))^(5+m)*(cos(f*x+e)^2)^(1/2)/a^5/f/(5+m)
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2680, 2657}

$$\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)(a\sin(e+fx))^{m+5}\;{}_2F_1\left(\frac{5}{2},\frac{m+5}{2};\frac{m+7}{2};\sin^2(e+fx)\right)}{a^5f(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[5/2, (5 + m)/2, (7 + m)/2, Sin[e +
f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2680

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a\sin(e+fx))^m\tan^4(e+fx)\,dx &= \frac{\int \sec^4(e+fx)(a\sin(e+fx))^{4+m}\,dx}{a^4}\\ &= \frac{\sqrt{\cos^2(e+fx)}\;{}_2F_1\left(\frac{5}{2},\frac{5+m}{2};\frac{7+m}{2};\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+\right.}{a^5f(5+m)}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 71, normalized size = 1.04

$$\frac{\sqrt{\cos^2(e+fx)}\ {}_2F_1\left(\frac{5}{2},\frac{5+m}{2};\frac{7+m}{2};\sin^2(e+fx)\right)\sin^4(e+fx)(a\sin(e+fx))^m\tan(e+fx)}{f(5+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[5/2, (5 + m)/2, (7 + m)/2, Sin[e +
f*x]^2]*Sin[e + f*x]^4*(a*Sin[e + f*x])^m*Tan[e + f*x])/(f*(5 + m))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (a\sin(fx+e))^m \left(\tan^4(fx+e)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

```
[Out] int((a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(e+fx))^m \tan^4(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e)**4,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^4\,(a\,\sin\left(e+f\,x\right))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a*sin(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)^4*(a*sin(e + f*x))^m, x)
```

3.167 $\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{\cos^2(e+fx)}\;{}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5+m}{2};\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{3+m}}{a^3f(3+m)}$$

```
[Out] hypergeom([3/2, 3/2+1/2*m],[5/2+1/2*m],sin(f*x+e)^2)*sec(f*x+e)*(a*sin(f*x+
e))^(3+m)*(cos(f*x+e)^2)^(1/2)/a^3/f/(3+m)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2680, 2657}

$$\frac{\sqrt{\cos^2(e+fx)}\;\sec(e+fx)(a\sin(e+fx))^{m+3}\;{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{m+5}{2};\sin^2(e+fx)\right)}{a^3f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[e +
f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2680

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a\sin(e+fx))^m\tan^2(e+fx)\,dx &= \frac{\int \sec^2(e+fx)(a\sin(e+fx))^{2+m}\,dx}{a^2}\\ &= \frac{\sqrt{\cos^2(e+fx)}\;{}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5+m}{2};\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+\end{aligned}$$

$$\overline{a^3f(3+m)}$$

Mathematica [A]
time = 0.09, size = 71, normalized size = 1.04

$$\frac{\sqrt{\cos^2(e+fx)}\ {}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5+m}{2};\sin^2(e+fx)\right)\sin^2(e+fx)(a\sin(e+fx))^m\tan(e+fx)}{f(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[3/2, (3 + m)/2, (5 + m)/2, Sin[e +
f*x]^2]*Sin[e + f*x]^2*(a*Sin[e + f*x])^m*Tan[e + f*x])/(f*(3 + m))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (a\sin(fx+e))^m \left(\tan^2(fx+e)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

```
[Out] int((a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(e+fx))^m \tan^2(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*tan(f*x+e)**2,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(e+f\,x)^2\,{\left(a\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a*sin(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)^2*(a*sin(e + f*x))^m, x)
```

3.168 $\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=69

$$-\frac{a\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(e+fx)\right)(a\sin(e+fx))^{-1+m}}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

```
[Out] -a*cos(f*x+e)*hypergeom([-1/2, -1/2+1/2*m],[1/2+1/2*m],sin(f*x+e)^2)*(a*sin
(f*x+e))^(-1+m)/f/(1-m)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2680, 2657}

$$-\frac{a\cos(e+fx)(a\sin(e+fx))^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(e+fx)\right)}{f(1-m)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a*Cos[e + f*x]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[e + f*
x]^2]*(a*Sin[e + f*x])^(-1 + m))/(f*(1 - m)*Sqrt[Cos[e + f*x]^2]))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2680

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \cot^2(e+fx)(a\sin(e+fx))^m\,dx &= a^2\int \cos^2(e+fx)(a\sin(e+fx))^{-2+m}\,dx\\ &= -\frac{a\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(e+fx)\right)(a\sin(e+fx)}{f(1-m)\sqrt{\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 66, normalized size = 0.96

$$\frac{a\sqrt{\cos^2(e+fx)}\ {}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^{-1+m}}{f(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a*Sin[e + f*x])^m,x]
```

```
[Out] (a*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[
e + f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^(-1 + m))/(f*(-1 + m))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(\cot^2(fx+e)\right)(a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^2*(a*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(e+fx))^m\cot^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^2\,{\left(a\,\sin\left(e+f\,x\right)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a*sin(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)^2*(a*sin(e + f*x))^m, x)
```

3.169 $\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=71

$$-\frac{a^3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(e+fx)\right)(a\sin(e+fx))^{-3+m}}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

```
[Out] -a^3*cos(f*x+e)*hypergeom([-3/2, -3/2+1/2*m],[-1/2+1/2*m],sin(f*x+e)^2)*(a*
sin(f*x+e))^(-3+m)/f/(3-m)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2680, 2657}

$$-\frac{a^3\cos(e+fx)(a\sin(e+fx))^{m-3}\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};\frac{m-1}{2};\sin^2(e+fx)\right)}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a^3*Cos[e + f*x]*Hypergeometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[e +
 f*x]^2]*(a*Sin[e + f*x])^(-3 + m))/(f*(3 - m)*Sqrt[Cos[e + f*x]^2]))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2680

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_Sy
mbol] :> Dist[1/a^n, Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /;
 FreeQ[{a, e, f, m}, x] && IntegerQ[n] &&  !IntegerQ[m]
```

Rubi steps

$$\int \cot^4(e+fx)(a\sin(e+fx))^m\,dx = a^4\int \cos^4(e+fx)(a\sin(e+fx))^{-4+m}\,dx$$
$$= -\frac{a^3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(e+fx)\right)(a\sin(}{f(3-m)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A]
time = 0.08, size = 71, normalized size = 1.00

$$\frac{\sqrt{\cos^2(e+fx)}\,\csc^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(e+fx)\right)\sec(e+fx)(a\sin(e+fx))^m}{f(-3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a*Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Csc[e + f*x]^3*Hypergeometric2F1[-3/2, (-3 + m)/2, (-
1 + m)/2, Sin[e + f*x]^2]*Sec[e + f*x]*(a*Sin[e + f*x])^m)/(f*(-3 + m))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\cot^4\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^4*(a*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(e+fx\right)\right)^m\cot^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*cot(e + f*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*cot(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^4\,\left(a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a*sin(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)^4*(a*sin(e + f*x))^m, x)
```

3.170 $\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=79

$$\frac{2\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(5+2m);\frac{1}{4}(9+2m);\sin^2(e+fx)\right)(a\sin(e+fx))^m(b\tan(e+fx))^{5/2}}{bf(5+2m)}$$

```
[Out] 2*(cos(f*x+e)^2)^(5/4)*hypergeom([5/4, 5/4+1/2*m],[9/4+1/2*m],sin(f*x+e)^2)
*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(5/2)/b/f/(5+2*m)
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$\frac{2\cos^2(e+fx)^{5/4}(b\tan(e+fx))^{5/2}(a\sin(e+fx))^m\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(2m+5);\frac{1}{4}(2m+9);\sin^2(e+fx)\right)}{bf(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[5/4, (5 + 2*m)/4, (9 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(5/2))/(b*f*(5 + 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^{3/2}\,dx = \frac{\left(a\cos^{\frac{5}{2}}(e+fx)(b\tan(e+fx))^{5/2}\right)\int \frac{(a\sin(e+fx))^{\frac{3}{2}+m}}{\cos^{\frac{3}{2}}(e+fx)}\,dx}{b(a\sin(e+fx))^{5/2}}$$
$$= \frac{2\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(5+2m);\frac{1}{4}(9+2m);\sin^2(e+fx)\right)(a}{bf(5+2m)}$$

Mathematica [A]
time = 8.40, size = 87, normalized size = 1.10

$$\frac{2\,{}_2F_1\left(\frac{2+m}{2},\frac{1}{4}(5+2m);\frac{1}{4}(9+2m);-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m(b\tan(e+fx))^{5/2}}{bf(5+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[(2 + m)/2, (5 + 2*m)/4, (9 + 2*m)/4, -Tan[e + f*x]^2]*
(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(5/2))/(b*f*(5 +
 2*m))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(a \sin \left(f x +e \right)\right)^{m} \left(b \tan \left(f x +e \right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x)
```

```
[Out] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)*(a*sin(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m*b*tan(f*x + e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)*(a*sin(f*x + e))^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\sin\left(e+f\,x\right))^m\,{\left(b\tan(e+f\,x)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(3/2),x)
```

```
[Out] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(3/2), x)
```

3.171 $\int (a\sin(e+fx))^m \sqrt{b\tan(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{2\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);\sin^2(e+fx)\right)(a\sin(e+fx))^m(b\tan(e+fx))^{3/2}}{bf(3+2m)}$$

```
[Out] 2*(cos(f*x+e)^2)^(3/4)*hypergeom([3/4, 3/4+1/2*m],[7/4+1/2*m],sin(f*x+e)^2)
*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(3/2)/b/f/(3+2*m)
```

Rubi [A]

time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$\frac{2\cos^2(e+fx)^{3/4}(b\tan(e+fx))^{3/2}(a\sin(e+fx))^m\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}(2m+3);\frac{1}{4}(2m+7);\sin^2(e+fx)\right)}{bf(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[3/4, (3 + 2*m)/4, (7 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2))/(b*f*(3 + 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a\sin(e+fx))^m\sqrt{b\tan(e+fx)}\,dx = \frac{\left(a\cos^{\frac{3}{2}}(e+fx)(b\tan(e+fx))^{3/2}\right)\int \frac{(a\sin(e+fx))^{\frac{1}{2}+m}}{\sqrt{\cos(e+fx)}}\,dx}{b(a\sin(e+fx))^{3/2}}$$
$$= \frac{2\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);\sin^2(e+fx)\right)(a\sin}{bf(3+2m)}$$

Mathematica [A]
time = 3.44, size = 87, normalized size = 1.10

$$\frac{2\,{}_2F_1\left(\frac{2+m}{2},\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m(b\tan(e+fx))^{3/2}}{bf(3+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]],x]
[Out] (2*Hypergeometric2F1[(2 + m)/2, (3 + 2*m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*
(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(3/2))/(b*f*(3 +
 2*m))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(fx+e\right)\right)^{m}\sqrt{b\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x)
[Out] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(e+fx\right)\right)^{m}\sqrt{b\tan\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*sin(e + f*x))**m*sqrt(b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\sin\left(e+f\,x\right))^m\,\sqrt{b\tan\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(1/2),x)
```

```
[Out] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^(1/2), x)
```

3.172 $\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4}(1+2m);\frac{1}{4}(5+2m);\sin^2(e+fx)\right)(a\sin(e+fx))^m\sqrt{b\tan(e+fx)}}{bf(1+2m)}$$

```
[Out] 2*(cos(f*x+e)^2)^(1/4)*hypergeom([1/4, 1/4+1/2*m],[5/4+1/2*m],sin(f*x+e)^2)
*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(1/2)/b/f/(1+2*m)
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\sqrt{b\tan(e+fx)}(a\sin(e+fx))^m\,{}_2F_1\left(\frac{1}{4},\frac{1}{4}(2m+1);\frac{1}{4}(2m+5);\sin^2(e+fx)\right)}{bf(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + 2*m)/4, (5 + 2*m)/4,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]])/(b*f*(1 + 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\left(a\sqrt{\cos(e+fx)}\,\sqrt{b\tan(e+fx)}\right)\int \sqrt{\cos(e+fx)}\,(a\sin(e+fx))^{-\frac{1}{2}+m}\,dx}{b\sqrt{a\sin(e+fx)}}$$
$$= \frac{2\sqrt[4]{\cos^2(e+fx)}\;{}_2F_1\left(\frac{1}{4},\frac{1}{4}(1+2m);\frac{1}{4}(5+2m);\sin^2(e+fx)\right)(a\sin(e+fx))^m\sqrt{b}}{bf(1+2m)}$$

Mathematica [A]
time = 3.00, size = 87, normalized size = 1.10

$$\frac{2\,{}_2F_1\left(\frac{2+m}{2},\frac{1}{4}(1+2m);\frac{1}{4}(5+2m);-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}(a\sin(e+fx))^m\sqrt{b\tan(e+fx)}}{bf(1+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[(2 + m)/2, (1 + 2*m)/4, (5 + 2*m)/4, -Tan[e + f*x]^2]*
(Sec[e + f*x]^2)^(m/2)*(a*Sin[e + f*x])^m*Sqrt[b*Tan[e + f*x]])/(b*f*(1 + 2
*m))
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(fx+e))^m}{\sqrt{b\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x)
```

```
[Out] int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m/sqrt(b*tan(f*x + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m/(b*tan(f*x + e)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(e+fx))^m}{\sqrt{b\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m/(b*tan(f*x+e))**(1/2),x)
[Out] Integral((a*sin(e + f*x))**m/sqrt(b*tan(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m/sqrt(b*tan(f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,\sin(e+f\,x))^m}{\sqrt{b\tan(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(1/2), x)
```

3.173 $\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(-1+2m);\frac{1}{4}(3+2m);\sin^2(e+fx)\right)(a\sin(e+fx))^m}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\,\sqrt{b\tan(e+fx)}}$$

```
[Out] -2*hypergeom([-1/4, -1/4+1/2*m],[3/4+1/2*m],sin(f*x+e)^2)*(a*sin(f*x+e))^m/
b/f/(1-2*m)/(cos(f*x+e)^2)^(1/4)/(b*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$-\frac{2(a\sin(e+fx))^m\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(2m-1);\frac{1}{4}(2m+3);\sin^2(e+fx)\right)}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\,\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, (-1 + 2*m)/4, (3 + 2*m)/4, Sin[e + f*x]^2]*(a*S
in[e + f*x])^m)/(b*f*(1 - 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[b*Tan[e + f*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a\sin(e+fx))^m}{(b\tan(e+fx))^{3/2}}\,dx &= \frac{\left(a\sqrt{a\sin(e+fx)}\right)\int \cos^{\frac{3}{2}}(e+fx)(a\sin(e+fx))^{-\frac{3}{2}+m}\,dx}{b\sqrt{\cos(e+fx)}\,\sqrt{b\tan(e+fx)}}\\ &= -\frac{2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(-1+2m);\frac{1}{4}(3+2m);\sin^2(e+fx)\right)(a\sin(e+fx))^m}{bf(1-2m)\sqrt[4]{\cos^2(e+fx)}\,\sqrt{b\tan(e+fx)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 224 vs. 2(79) = 158.
time = 5.55, size = 224, normalized size = 2.84

$$\frac{\sec^4(e+fx)\sec^2(e+fx)^{\frac{1}{2}(-4+m)}(a\sin(e+fx))^m\left({}_2F_1\left(\frac{m}{2},\frac{1}{4}(-1+2m);\frac{1}{4}(3+2m);-\tan^2(e+fx)\right)+\frac{\cos(2(e+fx))\sec^2(e+fx)\left(-\left((3+2m)\,{}_2F_1\left(\frac{m}{2},\frac{1}{4}(-1+2m);\frac{1}{4}(3+2m);-\tan^2(e+fx)\right)\right)+2(-1+2m)\,{}_2F_1\left(\frac{2+m}{2},\frac{1}{4}(3+2m);\frac{1}{4}(7+2m);-\tan^2(e+fx)\right)\tan^2(e+fx)\right)}{(3+2m)(-1+\tan^2(e+fx))}\right)}{bf(-1+2m)\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^4*(Sec[e + f*x]^2)^((-4 + m)/2)*(a*Sin[e + f*x])^m*(Hypergeom
etric2F1[m/2, (-1 + 2*m)/4, (3 + 2*m)/4, -Tan[e + f*x]^2] + (Cos[2*(e + f*x
)]*Sec[e + f*x]^2*(-((3 + 2*m)*Hypergeometric2F1[m/2, (-1 + 2*m)/4, (3 + 2*
m)/4, -Tan[e + f*x]^2]) + 2*(-1 + 2*m)*Hypergeometric2F1[(2 + m)/2, (3 + 2*
m)/4, (7 + 2*m)/4, -Tan[e + f*x]^2]*Tan[e + f*x]^2))/((3 + 2*m)*(-1 + Tan[e
 + f*x]^2))))/(b*f*(-1 + 2*m)*Sqrt[b*Tan[e + f*x]])
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(a\sin(fx+e))^m}{(b\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x)
```

```
[Out] int((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m/(b*tan(f*x + e))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*tan(f*x + e))*(a*sin(f*x + e))^m/(b^2*tan(f*x + e)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sin{(e+fx)})^m}{(b\tan{(e+fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m/(b*tan(f*x+e))**(3/2),x)
[Out] Integral((a*sin(e + f*x))**m/(b*tan(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m/(b*tan(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,\sin{(e+f\,x)})^m}{(b\tan{(e+f\,x)})^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^m/(b*tan(e + f*x))^(3/2), x)
```

3.174 $\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=83

$$\frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);\sin^2(e+fx)\right)(a\sin(e+fx))^m(b\tan(e+fx))^{1+n}}{bf(1+m+n)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/2+1/2*n, 1/2+1/2*m+1/2*n],[3/2+1/2*
m+1/2*n],sin(f*x+e)^2)*(a*sin(f*x+e))^m*(b*tan(f*x+e))^(1+n)/b/f/(1+m+n)
```

Rubi [A]

time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2682, 2657}

$$\frac{\cos^2(e+fx)^{\frac{n+1}{2}}(a\sin(e+fx))^m(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(e+fx)\right)}{bf(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)/2, (
3 + m + n)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + m + n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a\sin(e+fx))^m(b\tan(e+fx))^n\,dx = \frac{\left(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(}{b}$$

$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);\sin^2(e+f}{bf(1+m+n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 2.07, size = 260, normalized size = 3.13

$$\frac{(3+m+n)F_1\left(\frac{1}{2}(1+m+n);n,1+m;\frac{1}{2}(3+m+n);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)(a\sin(e+fx))^m(b\tan(e+fx))^n}{f(1+m+n)\left((3+m+n)F_1\left(\frac{1}{2}(1+m+n);n,1+m;\frac{1}{2}(3+m+n);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left((1+m)F_1\left(\frac{1}{2}(3+m+n);n,2+m;\frac{1}{2}(5+m+n);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-nF_1\left(\frac{1}{2}(3+m+n);1+n,1+m;\frac{1}{2}(5+m+n);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Sin[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((3 + m + n)*AppellF1[(1 + m + n)/2, n, 1 + m, (3 + m + n)/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]*(a*Sin[e + f*x])^m*(b*Tan[e + f*x]
)^n)/(f*(1 + m + n)*((3 + m + n)*AppellF1[(1 + m + n)/2, n, 1 + m, (3 + m +
 n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((1 + m)*AppellF1[(3 +
m + n)/2, n, 2 + m, (5 + m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 - n*AppellF1[(3 + m + n)/2, 1 + n, 1 + m, (5 + m + n)/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (a\sin(fx+e))^m (b\tan(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin{(e+fx)})^m\,(b\tan{(e+fx)})^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*(b*tan(f*x+e))**n,x)
[Out] Integral((a*sin(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(b\,\mathrm{tan}\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^n,x)
[Out] int((a*sin(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.175 $\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=50

$$\frac{{}_2F_1\left(3,\frac{5+n}{2};\frac{7+n}{2};-\tan^2(e+fx)\right)(b\tan(e+fx))^{5+n}}{b^5f(5+n)}$$

```
[Out] hypergeom([3, 5/2+1/2*n],[7/2+1/2*n],-tan(f*x+e)^2)*(b*tan(f*x+e))^(5+n)/b^
5/f/(5+n)
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2671, 371}

$$\frac{(b\tan(e+fx))^{n+5}\,{}_2F_1\left(3,\frac{n+5}{2};\frac{n+7}{2};-\tan^2(e+fx)\right)}{b^5f(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[3, (5 + n)/2, (7 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x
])^(5 + n))/(b^5*f*(5 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sin^4(e+fx)(b\tan(e+fx))^n\,dx &= \frac{b\text{Subst}\left(\int \frac{x^{4+n}}{(b^2+x^2)^3}\,dx,x,b\tan(e+fx)\right)}{f}\\ &= \frac{{}_2F_1\left(3,\frac{5+n}{2};\frac{7+n}{2};-\tan^2(e+fx)\right)(b\tan(e+fx))^{5+n}}{b^5f(5+n)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 4.99, size = 916, normalized size = 18.32

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

```
[Out] (64*(3 + n)*(AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] - 2*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + AppellF1[(1 + n)/2, n, 5, (3 + n)/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^7*Sin[(e + f*x)/2]^5*(b*Tan[e
 + f*x])^n)/(f*(1 + n)*((3 + n)*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) + (3 + n)*AppellF1[(1
 + n)/2, n, 5, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos
[e + f*x]) + 2*(-5*AppellF1[(3 + n)/2, n, 6, (5 + n)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[(3 + n)/2, 1 + n
, 5, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 6*AppellF1[(1 +
n)/2, n, 4, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*
x)/2]^2 - 2*n*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 3*AppellF1[(3 + n)/2, n, 4, (5 + n)/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(-1 + Cos[e + f*x]) - 8*AppellF1
[(3 + n)/2, n, 5, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(-1 +
 Cos[e + f*x]) + 5*AppellF1[(3 + n)/2, n, 6, (5 + n)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[e + f*x] - n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x] + 2*n*AppellF1[(
3 + n)/2, 1 + n, 4, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos
[e + f*x] - n*AppellF1[(3 + n)/2, 1 + n, 5, (5 + n)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*Cos[e + f*x])))
```

Maple [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \left(\sin^{4}\left(f x+e\right)\right) \left(b \tan \left(f x+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

```
[Out] int(sin(f*x+e)^4*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*tan(f*x + e))^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(e+fx))^n \sin^4(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((b*tan(e + f*x))**n*sin(e + f*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin(e+f\,x)^4\, (b\tan(e+f\,x))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(b*tan(e + f*x))^n,x)
```

```
[Out] int(sin(e + f*x)^4*(b*tan(e + f*x))^n, x)
```

3.176 $\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=50

$$\frac{{}_2F_1\left(2,\frac{3+n}{2};\frac{5+n}{2};-\tan^2(e+fx)\right)(b\tan(e+fx))^{3+n}}{b^3f(3+n)}$$

```
[Out] hypergeom([2, 3/2+1/2*n],[5/2+1/2*n],-tan(f*x+e)^2)*(b*tan(f*x+e))^(3+n)/b^
3/f/(3+n)
```

Rubi [A]
time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2671, 371}

$$\frac{(b\tan(e+fx))^{n+3}\,{}_2F_1\left(2,\frac{n+3}{2};\frac{n+5}{2};-\tan^2(e+fx)\right)}{b^3f(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x
])^(3 + n))/(b^3*f*(3 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \sin^2(e+fx)(b\tan(e+fx))^n\,dx &= \frac{b\text{Subst}\left(\int \frac{x^{2+n}}{(b^2+x^2)^2}\,dx,x,b\tan(e+fx)\right)}{f}\\ &= \frac{{}_2F_1\left(2,\frac{3+n}{2};\frac{5+n}{2};-\tan^2(e+fx)\right)(b\tan(e+fx))^{3+n}}{b^3f(3+n)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 2.28, size = 450, normalized size = 9.00

$$\frac{16(3+n)\left(F_1\left(\frac{1+n}{2};n,2;\frac{3+n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-F_1\left(\frac{1+n}{2};n,3;\frac{3+n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\cos^5\left(\frac{1}{2}(e+fx)\right)\sin^3\left(\frac{1}{2}(e+fx)\right)(b\tan(e+fx))^n}{f(1+n)\left(-2(3+n)F_1\left(\frac{1+n}{2};n,3;\frac{3+n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)+2\left(2F_1\left(\frac{3+n}{2};n,3;\frac{5+n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-3F_1\left(\frac{3+n}{2};n,4;\frac{5+n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+n\left(-F_1\left(\frac{3+n}{2};1+n,2;\frac{5+n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+F_1\left(\frac{3+n}{2};1+n,3;\frac{5+n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)(-1+\cos(e+fx))+(3+n)F_1\left(\frac{1+n}{2};n,2;\frac{3+n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1+\cos(e+fx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

```
[Out] (16*(3 + n)*(AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] - AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2])*Cos[(e + f*x)/2]^5*Sin[(e + f*x)/2]^3*(b*Tan[e + f*x]
)^n)/(f*(1 + n)*(-2*(3 + n)*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(2*AppellF1[(3 + n)/2
, n, 3, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 3*AppellF1[(3
 + n)/2, n, 4, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*(-Ap
pellF1[(3 + n)/2, 1 + n, 2, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]))*(-1 + Cos[e + f*x]) + (3 + n)*AppellF1[(1 + n)/2, n, 2, (3
+ n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

Maple [F]
time = 0.49, size = 0, normalized size = 0.00

$$\int \left(\sin^2\left(fx+e\right)\right)\left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

```
[Out] int(sin(f*x+e)^2*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan{(e+fx)})^n \sin^2{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*sin(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin{(e+f\,x)}^2\,(b\tan(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(b*tan(e + f*x))^n,x)
[Out] int(sin(e + f*x)^2*(b*tan(e + f*x))^n, x)
```

3.177 $\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=25

$$-\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] -b*(b*tan(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2671, 30}

$$-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((b*(b*Tan[e + f*x])^(-1 + n))/(f*(1 - n)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)(b\tan(e+fx))^n\,dx &= \frac{b\text{Subst}\left(\int x^{-2+n}\,dx, x, b\tan(e+fx)\right)}{f}\\ &= -\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 22, normalized size = 0.88

$$\frac{b(b\tan(e+fx))^{-1+n}}{f(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(b*Tan[e + f*x])^n,x]
```

```
[Out] (b*(b*Tan[e + f*x])^(-1 + n))/(f*(-1 + n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.76, size = 1782, normalized size = 71.28

method	result	size
risch	Expression too large to display	1782

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

```
[Out] I/(-1+n)/f/(exp(2*I*(f*x+e))-1)*(1/((exp(2*I*(f*x+e))+1)^n)*(exp(2*I*(f*x+e
))-1)^n*b^n*exp(1/2*I*n*Pi*csgn(I/(exp(2*I*(f*x+e))+1)*b*exp(2*I*(f*x+e))-I
/(exp(2*I*(f*x+e))+1)*b)*csgn(1/(exp(2*I*(f*x+e))+1)*b*exp(2*I*(f*x+e))-b/(
exp(2*I*(f*x+e))+1))^2)*exp(-1/2*I*n*Pi*csgn(1/(exp(2*I*(f*x+e))+1)*b*exp(2
*I*(f*x+e))-b/(exp(2*I*(f*x+e))+1))^3)*exp(1/2*I*n*Pi*csgn(1/(exp(2*I*(f*x+
e))+1)*b*exp(2*I*(f*x+e))-b/(exp(2*I*(f*x+e))+1))^2)*exp(-1/2*I*n*Pi*csgn(I
/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(2
*I*(f*x+e))-I)*csgn(I/(exp(2*I*(f*x+e))+1)))*exp(-1/2*I*n*Pi*csgn(I/(exp(2*
I*(f*x+e))+1)*b*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)*b)*csgn(1/(exp(2*I*
(f*x+e))+1)*b*exp(2*I*(f*x+e))-b/(exp(2*I*(f*x+e))+1)))*exp(-1/2*I*Pi*n)*ex
p(1/2*I*n*Pi*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e
))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1)))*exp(1/2*I*n*Pi*csgn(I/(exp(2*I*(f*x+
e))+1)*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1))*csgn(I/(exp(2*I*(f*x+e))+1)
*b*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)*b)^2)*exp(-1/2*I*n*Pi*csgn(I/(ex
p(2*I*(f*x+e))+1)*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1))*csgn(I/(exp(2*I*
(f*x+e))+1)*b*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)*b)*csgn(I*b))*exp(-1/
2*I*n*Pi*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1
))^3)*exp(1/2*I*n*Pi*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e))-I/(exp(2*
I*(f*x+e))+1))^2*csgn(I*exp(2*I*(f*x+e))-I))*exp(-1/2*I*n*Pi*csgn(I/(exp(2*
I*(f*x+e))+1)*b*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)*b)^3)*exp(1/2*I*n*P
i*csgn(I/(exp(2*I*(f*x+e))+1)*b*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)*b)^
2*csgn(I*b))*exp(2*I*f*x)*exp(2*I*e)+1/((exp(2*I*(f*x+e))+1)^n)*(exp(2*I*(f
*x+e))-1)^n*b^n*exp(-1/2*I*Pi*n*(csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e
))-I/(exp(2*I*(f*x+e))+1))^3-csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e))-I
/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(2*I*(f*x+e))-I)-csgn(I/(exp(2*I*(f*x+e)
)+1)*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1))*csgn(I/(exp(2*I*(f*x+e))+1)*b
*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)*b)^2+csgn(I/(exp(2*I*(f*x+e))+1)*e
xp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1))*csgn(I/(exp(2*I*(f*x+e))+1)*b*exp(2
*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)*b)*csgn(I*b)-csgn(I/(exp(2*I*(f*x+e))+1)
*b*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)*b)^2*csgn(I*b)-csgn(I/(exp(2*I*(
f*x+e))+1)*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+
```

```
e))+1))+csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)
)*csgn(I*exp(2*I*(f*x+e))-I)*csgn(I/(exp(2*I*(f*x+e))+1))+csgn(I/(exp(2*I*(
f*x+e))+1)*b*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)*b)^3-csgn(I/(exp(2*I*(
f*x+e))+1)*b*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)*b)*csgn(1/(exp(2*I*(f*
x+e))+1)*b*exp(2*I*(f*x+e))-b/(exp(2*I*(f*x+e))+1))^2+csgn(1/(exp(2*I*(f*x+
e))+1)*b*exp(2*I*(f*x+e))-b/(exp(2*I*(f*x+e))+1))^3+csgn(I/(exp(2*I*(f*x+e)
)+1)*b*exp(2*I*(f*x+e))-I/(exp(2*I*(f*x+e))+1)*b)*csgn(1/(exp(2*I*(f*x+e))+
1)*b*exp(2*I*(f*x+e))-b/(exp(2*I*(f*x+e))+1))-csgn(1/(exp(2*I*(f*x+e))+1)*b
*exp(2*I*(f*x+e))-b/(exp(2*I*(f*x+e))+1))^2+1)))
```

Maxima [**A**]
time = 0.30, size = 30, normalized size = 1.20

$$\frac{b^n \tan(fx+e)^n}{f(n-1)\tan(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] b^n*tan(f*x + e)^n/(f*(n - 1)*tan(f*x + e))
```

Fricas [**A**]
time = 0.39, size = 46, normalized size = 1.84

$$\frac{\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^n \cos(fx+e)}{(fn-f)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] (b*sin(f*x + e)/cos(f*x + e))^n*cos(f*x + e)/((f*n - f)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(e+fx))^n \csc^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^2, x)
```

Mupad [B]
time = 2.62, size = 53, normalized size = 2.12

$$-\frac{\sin\left(2\,e+2\,f\,x\right)\,{\left(\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{2\,{\cos\left(e+f\,x\right)}^2}\right)}^n}{2\,f\,\left({\cos\left(e+f\,x\right)}^2-1\right)\,\left(n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^n/sin(e + f*x)^2,x)
```

```
[Out] -(sin(2*e + 2*f*x)*((b*sin(2*e + 2*f*x))/(2*cos(e + f*x)^2))^n)/(2*f*(cos(e
 + f*x)^2 - 1)*(n - 1))
```

3.178 $\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=53

$$-\frac{b^3(b\tan(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] -b^3*(b*tan(f*x+e))^(-3+n)/f/(3-n)-b*(b*tan(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2671, 14}

$$-\frac{b^3(b\tan(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((b^3*(b*Tan[e + f*x])^(-3 + n))/(f*(3 - n))) - (b*(b*Tan[e + f*x])^(-1 +
n))/(f*(1 - n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(e+fx)(b\tan(e+fx))^n\,dx &= \frac{b\text{Subst}\left(\int x^{-4+n}\left(b^2+x^2\right)\,dx,x,b\tan(e+fx)\right)}{f}\\ &= \frac{b\text{Subst}\left(\int \left(b^2x^{-4+n}+x^{-2+n}\right)\,dx,x,b\tan(e+fx)\right)}{f}\\ &= -\frac{b^3(b\tan(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 46, normalized size = 0.87

$$\frac{b(-2+n+\cos(2(e+fx)))\csc^2(e+fx)(b\tan(e+fx))^{-1+n}}{f(-3+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(b*Tan[e + f*x])^n,x]
```

```
[Out] (b*(-2 + n + Cos[2*(e + f*x)])*Csc[e + f*x]^2*(b*Tan[e + f*x])^(-1 + n))/(f
*(-3 + n)*(-1 + n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 5437, normalized size = 102.58

method	result	size
risch	Expression too large to display	5437

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.30, size = 59, normalized size = 1.11

$$\frac{\frac{b^n\tan(fx+e)^n}{(n-1)\tan(fx+e)}+\frac{b^n\tan(fx+e)^n}{(n-3)\tan(fx+e)^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] (b^n*tan(f*x + e)^n/((n - 1)*tan(f*x + e)) + b^n*tan(f*x + e)^n/((n - 3)*ta
n(f*x + e)^3))/f
```

Fricas [A]
time = 0.38, size = 92, normalized size = 1.74

$$\frac{\left(2\cos\left(fx+e\right)^3+(n-3)\cos\left(fx+e\right)\right)\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^n}{\left(fn^2-\left(fn^2-4\,fn+3\,f\right)\cos\left(fx+e\right)^2-4\,fn+3\,f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] (2*cos(f*x + e)^3 + (n - 3)*cos(f*x + e))*(b*sin(f*x + e)/cos(f*x + e))^n/(
(f*n^2 - (f*n^2 - 4*f*n + 3*f)*cos(f*x + e)^2 - 4*f*n + 3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(e+fx))^n \csc^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^4, x)
```

Mupad [B]
time = 3.74, size = 138, normalized size = 2.60

$$-\frac{2\left(-\frac{b\sin(2e+2fx)}{2\sin(e+fx)^2-2}\right)^n(9\sin(2e+2fx)-6\sin(4e+4fx)+\sin(6e+6fx)-4n\sin(2e+2fx)+2n\sin(4e+4fx))}{f\left(30\sin(e+fx)^2-12\sin(2e+2fx)^2+2\sin(3e+3fx)^2\right)(n^2-4n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^n/sin(e + f*x)^4,x)
```

```
[Out] -(2*(-(b*sin(2*e + 2*f*x))/(2*sin(e + f*x)^2 - 2))^n*(9*sin(2*e + 2*f*x) -
6*sin(4*e + 4*f*x) + sin(6*e + 6*f*x) - 4*n*sin(2*e + 2*f*x) + 2*n*sin(4*e
+ 4*f*x)))/(f*(2*sin(3*e + 3*f*x)^2 - 12*sin(2*e + 2*f*x)^2 + 30*sin(e + f*
x)^2)*(n^2 - 4*n + 3))
```

3.179 $\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=80

$$-\frac{b^5(b\tan(e+fx))^{-5+n}}{f(5-n)}-\frac{2b^3(b\tan(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}$$

```
[Out] -b^5*(b*tan(f*x+e))^(-5+n)/f/(5-n)-2*b^3*(b*tan(f*x+e))^(-3+n)/f/(3-n)-b*(b
*tan(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A]

time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2671, 276}

$$-\frac{b^5(b\tan(e+fx))^{n-5}}{f(5-n)}-\frac{2b^3(b\tan(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\tan(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((b^5*(b*Tan[e + f*x])^(-5 + n))/(f*(5 - n))) - (2*b^3*(b*Tan[e + f*x])^(-
3 + n))/(f*(3 - n)) - (b*(b*Tan[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \csc^6(e+fx)(b\tan(e+fx))^n\,dx &= \frac{b\text{Subst}\left(\int x^{-6+n}\left(b^2+x^2\right)^2\,dx,x,b\tan(e+fx)\right)}{f}\\ &= \frac{b\text{Subst}\left(\int \left(b^4x^{-6+n}+2b^2x^{-4+n}+x^{-2+n}\right)\,dx,x,b\tan(e+fx)\right)}{f}\\ &= -\frac{b^5(b\tan(e+fx))^{-5+n}}{f(5-n)}-\frac{2b^3(b\tan(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\tan(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 69, normalized size = 0.86

$$\frac{b(8-6n+n^2+2(-3+n)\cos(2(e+fx))+\cos(4(e+fx)))\csc^4(e+fx)(b\tan(e+fx))^{-1+n}}{f(-5+n)(-3+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*(b*Tan[e + f*x])^n,x]
```

```
[Out] (b*(8 - 6*n + n^2 + 2*(-3 + n)*Cos[2*(e + f*x)] + Cos[4*(e + f*x)])*Csc[e +
 f*x]^4*(b*Tan[e + f*x])^(-1 + n))/(f*(-5 + n)*(-3 + n)*(-1 + n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.52, size = 10922, normalized size = 136.52

method	result	size
risch	Expression too large to display	10922

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(b*tan(f*x+e))^n,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.32, size = 87, normalized size = 1.09

$$\frac{\frac{b^n\tan(fx+e)^n}{(n-1)\tan(fx+e)}+\frac{2\,b^n\tan(fx+e)^n}{(n-3)\tan(fx+e)^3}+\frac{b^n\tan(fx+e)^n}{(n-5)\tan(fx+e)^5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] (b^n*tan(f*x + e)^n/((n - 1)*tan(f*x + e)) + 2*b^n*tan(f*x + e)^n/((n - 3)*
tan(f*x + e)^3) + b^n*tan(f*x + e)^n/((n - 5)*tan(f*x + e)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(75) = 150.
time = 0.39, size = 152, normalized size = 1.90

$$\frac{\left(8\cos\left(fx+e\right)^5+4\left(n-5\right)\cos\left(fx+e\right)^3+\left(n^2-8\,n+15\right)\cos\left(fx+e\right)\right)\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^n}{\left(\left(fn^3-9\,fn^2+23\,fn-15\,f\right)\cos\left(fx+e\right)^4+fn^3-9\,fn^2-2\left(fn^3-9\,fn^2+23\,fn-15\,f\right)\cos\left(fx+e\right)^2+23\,fn-15\,f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] (8*cos(f*x + e)^5 + 4*(n - 5)*cos(f*x + e)^3 + (n^2 - 8*n + 15)*cos(f*x + e
))*(b*sin(f*x + e)/cos(f*x + e))^n/((((f*n^3 - 9*f*n^2 + 23*f*n - 15*f)*cos(
f*x + e)^4 + f*n^3 - 9*f*n^2 - 2*(f*n^3 - 9*f*n^2 + 23*f*n - 15*f)*cos(f*x
+ e)^2 + 23*f*n - 15*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(b*tan(f*x+e))**n,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^n}{\sin\left(e+f\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^n/sin(e + f*x)^6,x)
[Out] int((b*tan(e + f*x))^n/sin(e + f*x)^6, x)
```

3.180 $\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=78

$$\frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{4+n}{2};\frac{6+n}{2};\sin^2(e+fx)\right)\sin^3(e+fx)(b\tan(e+fx))^{1+n}}{bf(4+n)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([2+1/2*n, 1/2+1/2*n],[3+1/2*n],sin(f*x
+e)^2)*sin(f*x+e)^3*(b*tan(f*x+e))^(1+n)/b/f/(4+n)
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2682, 2657}

$$\frac{\sin^3(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+4}{2};\frac{n+6}{2};\sin^2(e+fx)\right)}{bf(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (6 +
n)/2, Sin[e + f*x]^2]*Sin[e + f*x]^3*(b*Tan[e + f*x])^(1 + n))/(b*f*(4 + n)
)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sin^3(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(\cos^{1+n}(e+fx)\sin^{-1-n}(e+fx)(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e+fx)\,\mathrm{si}}{b}$$

$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{4+n}{2};\frac{6+n}{2};\sin^2(e+fx)\right)\sin^3(e+fx)(b\tan(e-}{bf(4+n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 3.08, size = 456, normalized size = 5.85

$$\frac{4(4+n)\left(F_1\left(1+\frac{n}{2};n,3;2+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-F_1\left(1+\frac{n}{2};n,4;2+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)\sin\left(\frac{1}{2}(e+fx)\right)\sin^3(e+fx)(b\tan(e+fx))^n}{f(2+n)\left(-2(4+n)F_1\left(1+\frac{n}{2};n,4;2+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)+2\left(3F_1\left(2+\frac{n}{2};n,4;3+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4F_1\left(2+\frac{n}{2};n,5;3+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+n\left(-F_1\left(2+\frac{n}{2};1+n,3;3+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+F_1\left(2+\frac{n}{2};1+n,4;3+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)(-1+\cos(e+fx))+(4+n)F_1\left(1+\frac{n}{2};n,3;2+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1+\cos(e+fx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] (4*(4 + n)*(AppellF1[1 + n/2, n, 3, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] - AppellF1[1 + n/2, n, 4, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2])*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*Sin[e + f*x]^3*(b*Tan[e +
f*x])^n)/(f*(2 + n)*(-2*(4 + n)*AppellF1[1 + n/2, n, 4, 2 + n/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(3*AppellF1[2 + n/2,
n, 4, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[2 + n/
2, n, 5, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*(-AppellF1[2
 + n/2, 1 + n, 3, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + Appel
lF1[2 + n/2, 1 + n, 4, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*
(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 3, 2 + n/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

Maple [F]
time = 0.50, size = 0, normalized size = 0.00

$$\int \left(\sin^3\left(fx+e\right)\right)\left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

```
[Out] int(sin(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*tan(f*x + e))^n*sin(f*x + e), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,{\left(b\,\tan(e+f\,x)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(b*tan(e + f*x))^n,x)
```

```
[Out] int(sin(e + f*x)^3*(b*tan(e + f*x))^n, x)
```

3.181 $\int \sin(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=76

$$\frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(e+fx)\right)\sin(e+fx)(b\tan(e+fx))^{1+n}}{bf(2+n)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[2+1/2*n],sin(f*x
+e)^2)*sin(f*x+e)*(b*tan(f*x+e))^(1+n)/b/f/(2+n)
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2682, 2657}

$$\frac{\sin(e+fx)\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(e+fx)\right)}{bf(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (4 +
n)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*Tan[e + f*x])^(1 + n))/(b*f*(2 + n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sin(e+fx)(b\tan(e+fx))^n\,dx = \frac{\left(\cos^{1+n}(e+fx)\sin^{-1-n}(e+fx)(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e+fx)\,\mathrm{si}}{b}$$

$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(e+fx)\right)\sin(e+fx)(b\tan(e+}{bf(2+n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.20, size = 252, normalized size = 3.32

$$\frac{8(4+n)F_1\left(1+\frac{n}{2};n,2;2+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sin^2\left(\frac{1}{2}(e+fx)\right)(b\tan(e+fx))^n}{f(2+n)\left(2\left(2F_1\left(2+\frac{n}{2};n,3;3+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-nF_1\left(2+\frac{n}{2};1+n,2;3+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(-1+\cos(e+fx))+(4+n)F_1\left(1+\frac{n}{2};n,2;2+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1+\cos(e+fx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]*(b*Tan[e + f*x])^n,x]
```

```
[Out] (8*(4 + n)*AppellF1[1 + n/2, n, 2, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Cos[(e + f*x)/2]^4*Sin[(e + f*x)/2]^2*(b*Tan[e + f*x])^n)/(f*(2 +
 n)*(2*(2*AppellF1[2 + n/2, n, 3, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] - n*AppellF1[2 + n/2, 1 + n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 2, 2 +
n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sin\left(fx+e\right)\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(b*tan(f*x+e))^n,x)
```

```
[Out] int(sin(f*x+e)*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e))^n*sin(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan (e+fx))^n \sin (e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*sin(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*sin(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin \left(e+f\,x\right)\,{\left(b\,\mathrm{tan}\left(e+f\,x\right)\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(b*tan(e + f*x))^n,x)
[Out] int(sin(e + f*x)*(b*tan(e + f*x))^n, x)
```

3.182 $\int \csc(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=78

$$-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1-n}{2},\frac{2-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

```
[Out] -cos(f*x+e)*hypergeom([1-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*tan
(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

Rubi [A]

time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2681, 2656}

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,{}_2F_1\left(\frac{1-n}{2},\frac{2-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (2 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\int \csc(e+fx)(b\tan(e+fx))^n\,dx = \left(\cos^n(e+fx)\sin^{-n}(e+fx)(b\tan(e+fx))^n\right)\int \cos^{-n}(e+fx)\sin^{-1+n}($$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1-n}{2},\frac{2-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e}{f(1-n)}$$

Mathematica [A]
time = 0.24, size = 64, normalized size = 0.82

$$\frac{{}_2F_1\left(\frac{n}{2},n;1+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^n(b\tan(e+fx))^n}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(b*Tan[e + f*x])^n,x]
[Out] (Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(
e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(f*n)
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \csc(fx+e)\,(b\tan(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*tan(f*x+e))^n,x)
[Out] int(csc(f*x+e)*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e))^n*csc(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(e+fx))^n\csc(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^n}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^n/sin(e + f*x),x)
```

```
[Out] int((b*tan(e + f*x))^n/sin(e + f*x), x)
```

3.183 $\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=78

$$-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1-n}{2},\frac{4-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

```
[Out] -cos(f*x+e)*hypergeom([2-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*tan
(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2681, 2656}

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,{}_2F_1\left(\frac{1-n}{2},\frac{4-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (4 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\int \csc^3(e+fx)(b\tan(e+fx))^n\,dx = \left(\cos^n(e+fx)\sin^{-n}(e+fx)(b\tan(e+fx))^n\right)\int \cos^{-n}(e+fx)\sin^{-3+}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1-n}{2},\frac{4-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan}{f(1-n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 15.62, size = 1242, normalized size = 15.92

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Cot[(e + f*x)/2]^2*Hypergeometric2F1[-1 + n/2, n, n/2, Tan[(e + f*x)/2]^2]
*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(f*(-8 + 4*n)) + (
(4 + n)*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sin[e + f*x]^2*(b*Tan[e + f*x])^n)/(4*f*(2 + n)*(2*(AppellF1[2 + n/2
, n, 2, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[2 +
n/2, 1 + n, 1, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos
[e + f*x]) + (4 + n)*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]))) + (Hypergeometric2F1[1 + n/2, n, 2
 + n/2, Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*
x)/2]^2*(b*Tan[e + f*x])^n)/(f*(8 + 4*n)) + (Cot[(e + f*x)/2]*(Cos[e + f*x]
*Sec[(e + f*x)/2]^2)^n*((2 + n)*Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e +
 f*x)/2]^2] - n*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^n*(b*Tan[e + f*x])^n)/(8*f*
n*(2 + n)*(((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^2*((2 + n)*Hyp
ergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x)/2]^2] - n*AppellF1[1 + n/2, n
, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*
Tan[e + f*x]^(-1 + n))/(2*(2 + n)) + ((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1
 + n)*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2])*((2 + n)*Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x
)/2]^2] - n*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^n)/(2*(2 + n)) + ((Cos[e + f*x]
*Sec[(e + f*x)/2]^2)^n*(-(n*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - n*Tan[(e
+ f*x)/2]^2*(-(((1 + n/2)*AppellF1[2 + n/2, n, 2, 3 + n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + n/2)) +
((1 + n/2)*n*AppellF1[2 + n/2, 1 + n, 1, 3 + n/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + n/2)) + (n*(2 + n
)*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(-Hypergeometric2F1[n/2, n, 1 + n/2, Ta
n[(e + f*x)/2]^2] + (1 - Tan[(e + f*x)/2]^2)^(-n)))/2)*Tan[e + f*x]^n)/(2*n
*(2 + n))))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(\csc^{3}\left(f x +e \right)\right) \left(b \tan \left(f x +e \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*tan(f*x+e))^n,x)
[Out] int(csc(f*x+e)^3*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan(e+fx))^n \csc^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^n}{\sin(e+f\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^n/sin(e + f*x)^3,x)
[Out] int((b*tan(e + f*x))^n/sin(e + f*x)^3, x)
```

3.184 $\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=78

$$-\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1-n}{2},\frac{6-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n}{f(1-n)}$$

```
[Out] -cos(f*x+e)*hypergeom([3-1/2*n, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*tan
(f*x+e))^n/f/(1-n)/((sin(f*x+e)^2)^(1/2*n))
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2681, 2656}

$$-\frac{\cos(e+fx)\sin^2(e+fx)^{-n/2}(b\tan(e+fx))^n\,{}_2F_1\left(\frac{1-n}{2},\frac{6-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((Cos[e + f*x]*Hypergeometric2F1[(1 - n)/2, (6 - n)/2, (3 - n)/2, Cos[e +
f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - n)*(Sin[e + f*x]^2)^(n/2)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2681

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-1
)]) || IntegersQ[m - 1/2, n - 1/2])
```

Rubi steps

$$\int \csc^5(e+fx)(b\tan(e+fx))^n\,dx = \left(\cos^n(e+fx)\sin^{-n}(e+fx)(b\tan(e+fx))^n\right)\int \cos^{-n}(e+fx)\sin^{-5+}$$

$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1-n}{2},\frac{6-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{-n/2}(b\tan}{f(1-n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 17.55, size = 1516, normalized size = 19.44

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^5*(b*Tan[e + f*x])^n,x]
```

```
[Out] (3*Cot[(e + f*x)/2]^2*Hypergeometric2F1[-1 + n/2, n, n/2, Tan[(e + f*x)/2]^
2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*Tan[e + f*x])^n)/(16*f*(-2 + n))
+ (Cot[(e + f*x)/2]^2*((-2 + n)*Cot[(e + f*x)/2]^2*Hypergeometric2F1[-2 + n
/2, n, -1 + n/2, Tan[(e + f*x)/2]^2] + (-4 + n)*Hypergeometric2F1[-1 + n/2,
 n, n/2, Tan[(e + f*x)/2]^2])*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(b*Tan[e
+ f*x])^n)/(16*f*(-4 + n)*(-2 + n)) + (3*(4 + n)*AppellF1[1 + n/2, n, 1, 2
+ n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x]^2*(b*Tan[e + f
*x])^n)/(16*f*(2 + n)*(2*(AppellF1[2 + n/2, n, 2, 3 + n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[2 + n/2, 1 + n, 1, 3 + n/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (4 + n)*AppellF1[1 +
 n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e +
f*x]))) + (3*Hypergeometric2F1[1 + n/2, n, 2 + n/2, Tan[(e + f*x)/2]^2]*(Co
s[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*(b*Tan[e + f*x])^n)/(16
*f*(2 + n)) + ((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*((4 +
 n)*Hypergeometric2F1[1 + n/2, n, 2 + n/2, Tan[(e + f*x)/2]^2] + (2 + n)*Hy
pergeometric2F1[2 + n/2, n, 3 + n/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2
)*(b*Tan[e + f*x])^n)/(16*f*(2 + n)*(4 + n)) + (9*Cot[(e + f*x)/2]*(Cos[e +
 f*x]*Sec[(e + f*x)/2]^2)^n*((2 + n)*Hypergeometric2F1[n/2, n, 1 + n/2, Tan
[(e + f*x)/2]^2] - n*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^n*(b*Tan[e + f*x])^n)/
(128*f*n*(2 + n)*((3*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^2*((2
 + n)*Hypergeometric2F1[n/2, n, 1 + n/2, Tan[(e + f*x)/2]^2] - n*AppellF1[1
 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*
x)/2]^2)*Tan[e + f*x]^(-1 + n))/(8*(2 + n)) + (3*(Cos[e + f*x]*Sec[(e + f*x
)/2]^2)^(-1 + n)*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2])*((2 + n)*Hypergeometric2F1[n/2, n, 1 + n/2, T
an[(e + f*x)/2]^2] - n*AppellF1[1 + n/2, n, 1, 2 + n/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)*Tan[e + f*x]^n)/(8*(2 + n)) + (3*
(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(-(n*AppellF1[1 + n/2, n, 1, 2 + n/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
) - n*Tan[(e + f*x)/2]^2*(-(((1 + n/2)*AppellF1[2 + n/2, n, 2, 3 + n/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/
(2 + n/2)) + ((1 + n/2)*n*AppellF1[2 + n/2, 1 + n, 1, 3 + n/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + n/2)
) + (n*(2 + n)*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(-Hypergeometric2F1[n/2, n
```

```
, 1 + n/2, Tan[(e + f*x)/2]^2] + (1 - Tan[(e + f*x)/2]^2)^(-n)))/2)*Tan[e +
 f*x]^n)/(8*n*(2 + n))))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(\csc^5\left(fx+e\right)\right)\left(b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(b*tan(f*x+e))^n,x)
[Out] int(csc(f*x+e)^5*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(e+fx\right)\right)^n\csc^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(b*tan(f*x+e))**n,x)
[Out] Integral((b*tan(e + f*x))**n*csc(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*csc(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^n}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^n/sin(e + f*x)^5,x)
```

```
[Out] int((b*tan(e + f*x))^n/sin(e + f*x)^5, x)
```

3.185 $\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=89

$$\frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\sin^2(e+fx)\right)(a\sin(e+fx))^{3/2}(b\tan(e+fx))^{1+n}}{bf(5+2n)}$$

```
[Out] 2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([5/4+1/2*n, 1/2+1/2*n],[9/4+1/2*n],s
in(f*x+e)^2)*(a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^(1+n)/b/f/(5+2*n)
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$\frac{2(a\sin(e+fx))^{3/2}\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\sin^2(e+fx)\right)}{bf(2n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^n,x]
```

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (5 + 2*n)/4, (
9 + 2*n)/4, Sin[e + f*x]^2]*(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^(1 + n)
)/(b*f*(5 + 2*n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a\sin(e+fx))^{3/2}(b\tan(e+fx))^n\,dx &= \frac{\left(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}\\ &= \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\sin^2(e+fx)\right)(a}{bf(5+2n)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 33.33, size = 297, normalized size = 3.34

$$\frac{8(9+2n)F_1\left(\frac{5}{4}+\frac{n}{2};n,\frac{5}{2};\frac{9}{4}+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)\sin\left(\frac{1}{2}(e+fx)\right)(a\sin(e+fx))^{3/2}(b\tan(e+fx))^n}{f(5+2n)\left(2(9+2n)F_1\left(\frac{5}{4}+\frac{n}{2};n,\frac{5}{2};\frac{9}{4}+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)+2\left(5F_1\left(\frac{9}{4}+\frac{n}{2};n,\frac{7}{2};\frac{13}{4}+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2nF_1\left(\frac{9}{4}+\frac{n}{2};1+n,\frac{5}{2};\frac{13}{4}+\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)(-1+\cos(e+fx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)*(b*Tan[e + f*x])^n,x]
```

```
[Out] (8*(9 + 2*n)*AppellF1[5/4 + n/2, n, 5/2, 9/4 + n/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sin[(e + f*x)/2]*(a*Sin[e + f*x])^(3/2
)*(b*Tan[e + f*x])^n)/(f*(5 + 2*n)*(2*(9 + 2*n)*AppellF1[5/4 + n/2, n, 5/2,
 9/4 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2
*(5*AppellF1[9/4 + n/2, n, 7/2, 13/4 + n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] - 2*n*AppellF1[9/4 + n/2, 1 + n, 5/2, 13/4 + n/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n*a*sin(f*x + e), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*tan(f*x+e))**n,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(3/2)*(b*tan(f*x + e))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\sin (e+f\,x))^{3/2}\,(b\tan(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^n,x)
[Out] int((a*sin(e + f*x))^(3/2)*(b*tan(e + f*x))^n, x)
```

3.186 $\int \sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=89

$$\frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{1+n}}{bf(3+2n)}$$

```
[Out] 2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([3/4+1/2*n, 1/2+1/2*n],[7/4+1/2*n],s
in(f*x+e)^2)*(a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^(1+n)/b/f/(3+2*n)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$\frac{2\sqrt{a\sin(e+fx)}\,\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\sin^2(e+fx)\right)}{bf(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^n,x]
```

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4, (
7 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(3 + 2*n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int \sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^n\,dx = \frac{\left(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e}{b}$$

$$= \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\sin^2(e+fx)\right)\sqrt{a}}{bf(3+2n)}$$

Mathematica [A]
time = 11.77, size = 91, normalized size = 1.02

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(-1+n)}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}\,\sin(2(e+fx))(b\tan(e+fx))^n}{f(3+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (3 + 2*n)/4, (7
 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*Sin[2*(e + f*x)]*(b*Tan[e +
 f*x])^n)/(f*(3 + 2*n))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin(fx+e)}\,(b\tan(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral(sqrt(a*sin(e + f*x))*(b*tan(e + f*x))**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a\,\sin\left(e+f\,x\right)}\,{\left(b\tan(e+f\,x)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^n,x)
```

```
[Out] int((a*sin(e + f*x))^(1/2)*(b*tan(e + f*x))^n, x)
```

3.187 $\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1+2n)\sqrt{a\sin(e+fx)}}$$

```
[Out] 2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([1/4+1/2*n, 1/2+1/2*n],[5/4+1/2*n],s
in(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1+2*n)/(a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$\frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\sin^2(e+fx)\right)}{bf(2n+1)\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^n/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4, (
5 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 + 2*n)*Sqrt[a
*Sin[e + f*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^n}{\sqrt{a\sin(e+fx)}}\,dx = \frac{(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n})\int\cos^{-n}(e+fx)(a\sin(e+}{b}$$
$$= \frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1+2n)\sqrt{a\sin(e+fx)}}$$

Mathematica [A]
time = 11.42, size = 89, normalized size = 1.00

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(-1+n)}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\sin^2(e+fx)\right)\sin(2(e+fx))(b\tan(e+fx))^n}{(f+2fn)\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^n/Sqrt[a*Sin[e + f*x]],x]
[Out] ((Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + 2*n)/4, (5
 + 2*n)/4, Sin[e + f*x]^2]*Sin[2*(e + f*x)]*(b*Tan[e + f*x])^n)/((f + 2*f*n
)*Sqrt[a*Sin[e + f*x]])
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(b\tan(fx+e))^n}{\sqrt{a\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x)
[Out] int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^n/sqrt(a*sin(f*x + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n/(a*sin(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\tan{(e+fx)})^n}{\sqrt{a\sin{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**n/(a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((b*tan(e + f*x))**n/sqrt(a*sin(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n/sqrt(a*sin(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^n}{\sqrt{a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(1/2), x)
```

3.188 $\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

```
[Out] -2*(cos(f*x+e)^2)^(1/2+1/2*n)*hypergeom([-1/4+1/2*n, 1/2+1/2*n],[3/4+1/2*n]
,sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-2*n)/(a*sin(f*x+e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2682, 2657}

$$-\frac{2\cos^2(e+fx)^{\frac{n+1}{2}}(b\tan(e+fx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\sin^2(e+fx)\right)}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^n/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*(Cos[e + f*x]^2)^((1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (-1 + 2*n)/4,
 (3 + 2*n)/4, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - 2*n)*(a*S
in[e + f*x])^(3/2))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^n}{(a\sin(e+fx))^{3/2}}\,dx = \frac{\left(a\cos^{1+n}(e+fx)(a\sin(e+fx))^{-1-n}(b\tan(e+fx))^{1+n}\right)\int \cos^{-n}(e+fx)(a\sin(\cdots}{b}$$
$$= -\frac{2\cos^2(e+fx)^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\sin^2(e+fx)\right)(b\tan(e+fx\cdots}{bf(1-2n)(a\sin(e+fx))^{3/2}}$$

Mathematica [A]
time = 12.13, size = 90, normalized size = 1.01

$$\frac{2b\cos^2(e+fx)^{\frac{1}{2}(-1+n)}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\sin^2(e+fx)\right)\sqrt{a\sin(e+fx)}\,(b\tan(e+fx))^{-1+n}}{a^2f(-1+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^n/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*b*(Cos[e + f*x]^2)^((-1 + n)/2)*Hypergeometric2F1[(1 + n)/2, (-1 + 2*n)/
4, (3 + 2*n)/4, Sin[e + f*x]^2]*Sqrt[a*Sin[e + f*x]]*(b*Tan[e + f*x])^(-1 +
 n))/(a^2*f*(-1 + 2*n))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(b\tan(fx+e))^n}{(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x)
```

```
[Out] int((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n/(a*sin(f*x + e))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e))*(b*tan(f*x + e))^n/(a^2*cos(f*x + e)^2 - a^2
), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\tan{(e+fx)})^n}{(a\sin{(e+fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**n/(a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x))**n/(a*sin(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^n/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n/(a*sin(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^n}{(a\,\sin(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(3/2),x)
```

```
[Out] int((b*tan(e + f*x))^n/(a*sin(e + f*x))^(3/2), x)
```

3.189 $\int (a\cos(e+fx))^m (b\tan(e+fx))^n\,dx$

Optimal. Leaf size=86

$$\frac{(a\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+n)}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1-m+n);\frac{3+n}{2};\sin^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1+n)}$$

```
[Out] (a*cos(f*x+e))^m*(cos(f*x+e)^2)^(1/2-1/2*m+1/2*n)*hypergeom([1/2+1/2*n, 1/2
-1/2*m+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1+n)
```

Rubi [A]

time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2683, 2697}

$$\frac{(a\cos(e+fx))^m (b\tan(e+fx))^{n+1} \cos^2(e+fx)^{\frac{1}{2}(-m+n+1)}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(-m+n+1);\frac{n+3}{2};\sin^2(e+fx)\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((a*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m + n)/2)*Hypergeometric2F1[(1 +
 n)/2, (1 - m + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + n))
```

Rule 2683

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cos[e + f*x])^FracPart[m]*(Sec[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sec[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] && !IntegerQ[m/2]
```

Rubi steps

$$\int (a\cos(e+fx))^m (b\tan(e+fx))^n\,dx = \left((a\cos(e+fx))^m \left(\frac{\sec(e+fx)}{a}\right)^m\right)\int \left(\frac{\sec(e+fx)}{a}\right)^{-m} (b\tan$$

$$= \frac{(a\cos(e+fx))^m \cos^2(e+fx)^{\frac{1}{2}(1-m+n)}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1-m+n);\frac{3+}{2}\right.}{bf(1+n)}$$

Mathematica [A]
time = 0.54, size = 81, normalized size = 0.94

$$\frac{(a\cos(e+fx))^m\,{}_2F_1\left(\frac{2+m}{2},\frac{1+n}{2};\frac{3+n}{2};-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}\tan(e+fx)(b\tan(e+fx))^n}{f(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((a*Cos[e + f*x])^m*Hypergeometric2F1[(2 + m)/2, (1 + n)/2, (3 + n)/2, -Tan
[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]*(b*Tan[e + f*x])^n)/(f*(1
+ n))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (a\cos(fx+e))^m (b\tan(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(e+fx))^m (b\tan(e+fx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((a*cos(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\cos\left(e+f\,x\right))^m\,(b\tan(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m*(b*tan(e + f*x))^n,x)
```

```
[Out] int((a*cos(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.190 $\int (a\tan(e+fx))^m (b\tan(e+fx))^n\,dx$

Optimal. Leaf size=63

$$\frac{{}_2F_1\left(1,\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);-\tan^2(e+fx)\right)(a\tan(e+fx))^{1+m}(b\tan(e+fx))^n}{af(1+m+n)}$$

```
[Out] hypergeom([1, 1/2+1/2*m+1/2*n],[3/2+1/2*m+1/2*n],-tan(f*x+e)^2)*(a*tan(f*x+
e))^(1+m)*(b*tan(f*x+e))^n/a/f/(1+m+n)
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {20, 3557, 371}

$$\frac{(a\tan(e+fx))^{m+1}(b\tan(e+fx))^n\,{}_2F_1\left(1,\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);-\tan^2(e+fx)\right)}{af(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + n)/2, (3 + m + n)/2, -Tan[e + f*x]^2]*(a*Tan
[e + f*x])^(1 + m)*(b*Tan[e + f*x])^n)/(a*f*(1 + m + n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\int (a\tan(e+fx))^m(b\tan(e+fx))^n\,dx = \left((a\tan(e+fx))^{-n}(b\tan(e+fx))^n\right)\int (a\tan(e+fx))^{m+n}\,dx$$

$$= \frac{(a(a\tan(e+fx))^{-n}(b\tan(e+fx))^n)\,\text{Subst}\Big(\int \frac{x^{m+n}}{a^2+x^2}\,dx, x, a\tan(}{f}$$

$$= \frac{{}_2F_1\big(1,\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);-\tan^2(e+fx)\big)\,(a\tan(e+f}{af(1+m+n)}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 1.05

$$\frac{{}_2F_1\big(1,\frac{1}{2}(1+m+n);1+\frac{1}{2}(1+m+n);-\tan^2(e+fx)\big)\tan(e+fx)(a\tan(e+fx))^m(b\tan(e+fx))^n}{f(1+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m + n)/2, 1 + (1 + m + n)/2, -Tan[e + f*x]^2]*Ta
n[e + f*x]*(a*Tan[e + f*x])^m*(b*Tan[e + f*x])^n)/(f*(1 + m + n))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(a\tan\left(fx+e\right)\right)^{m}\left(b\tan\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\tan{(e+fx)})^m\,(b\tan{(e+fx)})^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tan(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((a*tan(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*tan(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*tan(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a\,\mathrm{tan}\left(e+f\,x\right)\right)}^m\,{\left(b\,\mathrm{tan}\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*tan(e + f*x))^m*(b*tan(e + f*x))^n,x)
```

```
[Out] int((a*tan(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.191 $\int \sqrt{d\cot(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=232

$$\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{\sqrt{d}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{2d^3}{5f(d\cot(e+fx))^{5/2}}$$

```
[Out] 2/5*d^3/f/(d*cot(f*x+e))^(5/2)+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^
(1/2))*d^(1/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))
*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))
^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(
f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)-2*d/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}+\frac{2d^3}{5f(d\cot(e+fx))^{5/2}}-\frac{2d}{f\sqrt{d\cot(e+fx)}}-\frac{\sqrt{d}\,\log\left(\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}+\frac{\sqrt{d}\,\log\left(\sqrt{d}\,\cot(e+fx)+\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d^3)/(5*f*(d*Cot[e + f*x])^(5/2)) - (2*d)/(f*Sqrt[d*Cot[e + f*x]]) - (Sq
rt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(
2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d
*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{d\cot(e+fx)}\tan^4(e+fx)\,dx &= d^4\int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx\\
&= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - d^2\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx\\
&= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} + \int \sqrt{d\cot(e+fx)}\,dx\\
&= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x,\right.}{f}\\
&= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{(2d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx,\right.}{f}\\
&= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x,\right.}{f}\\
&= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{}}{-d-\sqrt{2}}\right.}{}\\
&= \frac{2d^3}{5f(d\cot(e+fx))^{5/2}} - \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\,\mathrm{c}\right.}{}\\
&= \frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}}{}\right.}{\sqrt{2}f}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 45, normalized size = 0.19

$$
\frac{2\sqrt{d\cot(e+fx)}\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(e+fx)\right)\tan^3(e+fx)}{5f}
$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^4,x]

[Out] (2*Sqrt[d*Cot[e + f*x]]*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[e + f*x]^2]*Tan[e + f*x]^3)/(5*f)

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 7.37, size = 728, normalized size = 3.14

method	result
default	$-\frac{(\cos(fx+e)-1)\left(-5i(\cos^2(fx+e))\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\ \sin(fx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)

[Out] -1/10/f*(cos(f*x+e)-1)*(-5*I*cos(f*x+e)^2*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+5*I*cos(f*x+e)^2*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-10*cos(f*x+e)^2*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+5*cos(f*x+e)^2*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+5*cos(f*x+e)^2*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+12*cos(f*x+e)^3*2^(1/2)-12*cos(f*x+e)^2*2^(1/2)-2*cos(f*x+e)*2^(1/2)+2*2^(1/2))*(cos(f*x+e)+1)^2*(d*cos(f*x+e)/sin(f*x+e))^(1/2)/cos(f*x+e)^3/sin(f*x+e)^3*2^(1/2)

Maxima [A]

time = 0.50, size = 215, normalized size = 0.93

$$-\frac{d^{5}\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right)}{d^{4}}-\frac{8\left(d^{2}-\frac{5d^{2}}{\tan(fx+e)^{2}}\right)}{d^{4}\left(\frac{d}{\tan(fx+e)}\right)^{\frac{5}{2}}}\right)}{20f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] -1/20*d^5*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*
sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^2 - 5
*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(187) = 374$.
time = 0.40, size = 643, normalized size = 2.77

$$\frac{20\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}-\sqrt{2}f\sqrt{\frac{\sqrt{2}df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\sin(fx+e)+d^2f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)+d^3\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}+d^2}{d^2}\right)\cos(fx+e)^3+20\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}-\sqrt{2}f\sqrt{-\frac{\sqrt{2}df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\sin(fx+e)-d^2f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)-d^3\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}-d^2}{d^2}\right)\cos(fx+e)^3+5\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\cos(fx+e)^3\log\left(\frac{\sqrt{2}df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\sin(fx+e)+d^2f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)+d^3\cos(fx+e)}{\sin(fx+e)}\right)-5\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\cos(fx+e)^3\log\left(-\frac{\sqrt{2}df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\sin(fx+e)-d^2f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)-d^3\cos(fx+e)}{\sin(fx+e)}\right)-8\left(6\cos(fx+e)^2-1\right)\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)}{20f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/20*(20*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*cos(f*x + e)
/sin(f*x + e))*(d^2/f^4)^(1/4) - sqrt(2)*f*sqrt((sqrt(2)*d*f^3*sqrt(d*cos(f
*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) + d^2*f^2*sqrt(d^2/f^4)*
sin(f*x + e) + d^3*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(1/4) + d^2)/d^2)*
cos(f*x + e)^3 + 20*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*c
os(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4) - sqrt(2)*f*sqrt(-(sqrt(2)*d*f^3*
sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) - d^2*f^2*sq
rt(d^2/f^4)*sin(f*x + e) - d^3*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(1/4)
- d^2)/d^2)*cos(f*x + e)^3 + 5*sqrt(2)*f*(d^2/f^4)^(1/4)*cos(f*x + e)^3*log
((sqrt(2)*d*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x +
 e) + d^2*f^2*sqrt(d^2/f^4)*sin(f*x + e) + d^3*cos(f*x + e))/sin(f*x + e))
- 5*sqrt(2)*f*(d^2/f^4)^(1/4)*cos(f*x + e)^3*log(-(sqrt(2)*d*f^3*sqrt(d*cos
(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) - d^2*f^2*sqrt(d^2/f^4
```

```
)*sin(f*x + e) - d^3*cos(f*x + e))/sin(f*x + e)) - 8*(6*cos(f*x + e)^2 - 1)
*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cot(e+fx)}\,\tan^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**4,x)
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^4, x)
```

Mupad [B]
time = 2.58, size = 97, normalized size = 0.42

$$\frac{\frac{2d^3}{5}-\frac{2d^3}{\tan(e+fx)^2}}{f\left(\frac{d}{\tan(e+fx)}\right)^{5/2}}-\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f}+\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(d*cot(e + f*x))^(1/2),x)
[Out] ((2*d^3)/5 - (2*d^3)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1/
4)*d^(1/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(1/
4)*d^(1/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.192 $\int \sqrt{d \cot(e+fx)} \tan^3(e+fx)\,dx$

Optimal. Leaf size=214

$$-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{2d^2}{3f(d\cot(e+fx))^{3/}}$$

```
[Out] 2/3*d^2/f/(d*cot(f*x+e))^(3/2)-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^
(1/2))*d^(1/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))
*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))
^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(
f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}+\frac{2d^2}{3f(d\cot(e+fx))^{3/2}}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^3,x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^2)/(3*f*(d*Cot[e + f*x])^(3/2)) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Co
t[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sq
rt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \sqrt{d\cot(e+fx)}\tan^3(e+fx)\,dx &= d^3\int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx \\
&= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} - d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx \\
&= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx,x,d\cot(e+fx)\right)}{f} \\
&= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f} \\
&= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} + \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx,x,\sqrt{d\cot(e+fx)}\right)}{f} + \\
&= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} - \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx,x,\sqrt{d\cot(}\right.}{2\sqrt{2}f} \\
&= \frac{2d^2}{3f(d\cot(e+fx))^{3/2}} - \frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d}\right.}{2\sqrt{2}f} \\
&= -\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{}}{}\right.}{\sqrt{2}f}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 45, normalized size = 0.21

$$\frac{2\sqrt{d\cot(e+fx)}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(e+fx)\right)\tan^2(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^3,x]
```

```
[Out] (2*Sqrt[d*Cot[e + f*x]]*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[e + f*x]^2]*Ta
n[e + f*x]^2)/(3*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.42, size = 548, normalized size = 2.56

method	result
default	$\frac{(\cos(fx+e)-1)\left(3i\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(cos(f*x+e)-1)*(3*I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*
2^(1/2))*cos(f*x+e)-3*I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^
(1/2))*cos(f*x+e)-3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*cos(f*x+e)-3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*c
os(f*x+e)+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*(cos(f*x+e)+1)^2*(d*cos(f*x+e)/si
n(f*x+e))^(1/2)/sin(f*x+e)^2/cos(f*x+e)^2*2^(1/2)
```

Maxima [A]
time = 0.51, size = 197, normalized size = 0.92

$$\frac{d^4\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}\right)}{d^2}+\frac{8}{d^2\left(\frac{d}{\tan(fx+e)}\right)^{\frac{3}{2}}}\right)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/12*d^4*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
```

```
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2*(d/t
an(f*x + e))^(3/2)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(170) = 340$.
time = 0.38, size = 604, normalized size = 2.82

$$-\frac{12\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac14}\arctan\left(-\frac{\sqrt{2}f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac34}-\sqrt{2}f^3\sqrt{\frac{f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)+\sqrt{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac14}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac34}+d^2}{d^2}\right)\cos(fx+e)^2+12\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac14}\arctan\left(-\frac{\sqrt{2}f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac34}-\sqrt{2}f^3\sqrt{\frac{f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)-\sqrt{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac14}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac34}-d^2}{d^2}\right)\cos(fx+e)^2-3\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac14}\cos(fx+e)^2\log\left(\frac{f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)+\sqrt{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac14}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}\right)+3\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac14}\cos(fx+e)^2\log\left(\frac{f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)-\sqrt{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac14}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}\right)+8\left(\cos(fx+e)^2-1\right)\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}}{12f\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*f^3*sqrt(d*cos(f*x + e
)/sin(f*x + e))*(d^2/f^4)^(3/4) - sqrt(2)*f^3*sqrt((f^2*sqrt(d^2/f^4)*sin(f
*x + e) + sqrt(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*sin(f
*x + e) + d*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(3/4) + d^2)/d^2)*cos(f*x
 + e)^2 + 12*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*f^3*sqrt(d*cos(f*x
+ e)/sin(f*x + e))*(d^2/f^4)^(3/4) - sqrt(2)*f^3*sqrt((f^2*sqrt(d^2/f^4)*si
n(f*x + e) - sqrt(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*si
n(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(3/4) - d^2)/d^2)*cos(
f*x + e)^2 - 3*sqrt(2)*f*(d^2/f^4)^(1/4)*cos(f*x + e)^2*log((f^2*sqrt(d^2/f
^4)*sin(f*x + e) + sqrt(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1
/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) + 3*sqrt(2)*f*(d^2/f^4)^(1
/4)*cos(f*x + e)^2*log((f^2*sqrt(d^2/f^4)*sin(f*x + e) - sqrt(2)*f*sqrt(d*c
os(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*sin(f*x + e) + d*cos(f*x + e))/si
n(f*x + e)) + 8*(cos(f*x + e)^2 - 1)*sqrt(d*cos(f*x + e)/sin(f*x + e)))/(f*
cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\cot(e+fx)}\,\tan^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^3, x)
```

Mupad [B]
time = 2.47, size = 83, normalized size = 0.39

$$\frac{2\,d^2}{3\,f\,{\left(\frac{d}{\tan\left(e+f\,x\right)}\right)}^{3/2}}-\frac{{\left(-1\right)}^{1/4}\,\sqrt{d}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)\,1{}\mathrm{i}}{f}-\frac{{\left(-1\right)}^{1/4}\,\sqrt{d}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)\,1{}\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] (2*d^2)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)
*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(
1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.193 $\int \sqrt{d\cot(e+fx)}\,\tan^2(e+fx)\,dx$

Optimal. Leaf size=210

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{2d}{f\sqrt{d\cot(e+fx)}}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)-1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/
2)+2*d/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}+\frac{2d}{f\sqrt{d\cot(e+fx)}}+\frac{\sqrt{d}\,\log\left(\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}-\frac{\sqrt{d}\,\log\left(\sqrt{d}\,\cot(e+fx)+\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d)/(f*Sqrt[d*Cot[e + f*x]]) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e +
f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d]
+ Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rubi steps

$$\int \sqrt{d\cot(e+fx)}\tan^2(e+fx)\,dx = d^2\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$
$$= \frac{2d}{f\sqrt{d\cot(e+fx)}} - \int \sqrt{d\cot(e+fx)}\,dx$$
$$= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$
$$= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{(2d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= \frac{2d}{f\sqrt{d\cot(e+fx)}} - \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{dS}{}$$
$$= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\cot(e}{2\sqrt{2}f}$$
$$= \frac{2d}{f\sqrt{d\cot(e+fx)}} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\,\text{co}}{2\sqrt{2}f}$$
$$= -\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{}}{}\right.}{\sqrt{2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 36, normalized size = 0.17

$$\frac{2d\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(e+fx)\right)}{f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] (2*d*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[e + f*x]^2])/(f*Sqrt[d*Cot[e + f*
x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.37, size = 660, normalized size = 3.14

method	result
default	$\frac{\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\,(\cos(fx+e)+1)^2(\cos(fx+e)-1)\left(i\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{-1+\cos(fx+e)-\sin(f}{\sin(fx+e)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(d*cos(f*x+e)/sin(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)*(I*((
cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*((cos(
f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+((cos(f*x+e)
-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+((cos(f*x+e)-1)/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*((cos(f*x+e)-1)/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))/cos(f*x
+e)/sin(f*x+e)^3*2^(1/2)
```

Maxima [A]
time = 0.50, size = 196, normalized size = 0.93

$$\frac{d^3\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}}{d^2}+\frac{8}{d^2\sqrt{\frac{d}{\tan(fx+e)}}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
+ e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt
```

```
(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sqrt(d/
tan(f*x + e))))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(168) = 336$.
time = 0.44, size = 621, normalized size = 2.96

$$-\frac{4\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}-\sqrt{2}f\sqrt{\frac{\sqrt{2}df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\sin(fx+e)+d^2f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)+d^3\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}+d^2}{d^2}\right)\cos(fx+e)+4\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}-\sqrt{2}f\sqrt{-\frac{\sqrt{2}df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\sin(fx+e)-d^2f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)-d^3\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}-d^2}{d^2}\right)\cos(fx+e)+\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\cos(fx+e)\log\left(\frac{\sqrt{2}df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\sin(fx+e)+d^2f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)+d^3\cos(fx+e)}{\sin(fx+e)}\right)-\sqrt{2}f\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\cos(fx+e)\log\left(-\frac{\sqrt{2}df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\sin(fx+e)-d^2f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)-d^3\cos(fx+e)}{\sin(fx+e)}\right)-8\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)}{4f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*cos(f*x + e)/
sin(f*x + e))*(d^2/f^4)^(1/4) - sqrt(2)*f*sqrt((sqrt(2)*d*f^3*sqrt(d*cos(f*
x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) + d^2*f^2*sqrt(d^2/f^4)*s
in(f*x + e) + d^3*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(1/4) + d^2)/d^2)*c
os(f*x + e) + 4*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*cos(f
*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4) - sqrt(2)*f*sqrt(-(sqrt(2)*d*f^3*sqrt
(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) - d^2*f^2*sqrt(d
^2/f^4)*sin(f*x + e) - d^3*cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(1/4) - d^
2)/d^2)*cos(f*x + e) + sqrt(2)*f*(d^2/f^4)^(1/4)*cos(f*x + e)*log((sqrt(2)*
d*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) + d^2*
f^2*sqrt(d^2/f^4)*sin(f*x + e) + d^3*cos(f*x + e))/sin(f*x + e)) - sqrt(2)*
f*(d^2/f^4)^(1/4)*cos(f*x + e)*log(-(sqrt(2)*d*f^3*sqrt(d*cos(f*x + e)/sin(
f*x + e))*(d^2/f^4)^(3/4)*sin(f*x + e) - d^2*f^2*sqrt(d^2/f^4)*sin(f*x + e)
 - d^3*cos(f*x + e))/sin(f*x + e)) - 8*sqrt(d*cos(f*x + e)/sin(f*x + e))*si
n(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cot(e+fx)}\,\tan^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e)^2, x)
```

Mupad [B]
time = 2.48, size = 80, normalized size = 0.38

$$\frac{2\,d}{f\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}+\frac{{\left(-1\right)}^{1/4}\,\sqrt{d}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)}{f}-\frac{{\left(-1\right)}^{1/4}\,\sqrt{d}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] (2*d)/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d/
tan(e + f*x))^(1/2))/d^(1/2)))/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*(d
/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.194 $\int \sqrt{d\cot(e+fx)}\,\tan(e+fx)\,dx$

Optimal. Leaf size=192

$$\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)-1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2
)
```

Rubi [A]
time = 0.10, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {16, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}-\frac{\sqrt{d}\,\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x],x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{d\cot(e+fx)}\tan(e+fx)\,dx &= d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx \\
&= -\frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f} \\
&= -\frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&= -\frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{\right.}}{f} \\
&= \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,f} + \frac{\sqrt{d}\,\text{Subst}\left(\right.}{} \\
&= \frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,f} - \frac{\sqrt{d}\,\log}{} \\
&= \frac{\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f} - \frac{\sqrt{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d\,\text{c}}}{}\right.}{\sqrt{2}\,f}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 132, normalized size = 0.69

$$
\frac{d\sqrt{\cot(e+fx)}\left(2\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}\right)+\log\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)-\log\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)\right)}{2\sqrt{2}\,f\sqrt{d\cot(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cot[e + f*x]]*Tan[e + f*x],x]
```

```
[Out] (d*Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[
1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[
e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2]
*f*Sqrt[d*Cot[e + f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 292, normalized size = 1.52

method	result

default	$-\frac{\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\,(\cos(fx+e)-1)\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\,\left(i\,\text{Ellipti}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(1/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(d*cos(f*x+e)/sin(f*x+e))^(1/2)*(cos(f*x+e)-1)*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((c
os(f*x+e)-1)/sin(f*x+e))^(1/2)*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/sin(f*x+e)^2/c
os(f*x+e)*(cos(f*x+e)+1)^2*2^(1/2)
```

Maxima [A]
time = 0.53, size = 173, normalized size = 0.90

$$-\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/4*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
+ e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt
(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(151) = 302$.
time = 0.38, size = 515, normalized size = 2.68

$$\sqrt{2}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}-\sqrt{2}f^3\sqrt{\frac{f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)+\sqrt{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}+d^2}{d^2}\right)+\sqrt{2}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}-\sqrt{2}f^3\sqrt{\frac{f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)-\sqrt{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}-d^2}{d^2}\right)-\frac{1}{4}\sqrt{2}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(\frac{f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)+\sqrt{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}\right)+\frac{1}{4}\sqrt{2}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(\frac{f^2\sqrt{\frac{d^2}{f^4}}\sin(fx+e)-\sqrt{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*f^3*sqrt(d*cos(f*x + e)/sin(f*x +
e))*(d^2/f^4)^(3/4) - sqrt(2)*f^3*sqrt((f^2*sqrt(d^2/f^4)*sin(f*x + e) + sq
rt(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*sin(f*x + e) + d*
cos(f*x + e))/sin(f*x + e))*(d^2/f^4)^(3/4) + d^2)/d^2) + sqrt(2)*(d^2/f^4)
^(1/4)*arctan(-(sqrt(2)*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(d^2/f^4)^(3/
4) - sqrt(2)*f^3*sqrt((f^2*sqrt(d^2/f^4)*sin(f*x + e) - sqrt(2)*f*sqrt(d*co
s(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin
(f*x + e))*(d^2/f^4)^(3/4) - d^2)/d^2) - 1/4*sqrt(2)*(d^2/f^4)^(1/4)*log((f
^2*sqrt(d^2/f^4)*sin(f*x + e) + sqrt(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))
*(d^2/f^4)^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) + 1/4*sqrt(2)
*(d^2/f^4)^(1/4)*log((f^2*sqrt(d^2/f^4)*sin(f*x + e) - sqrt(2)*f*sqrt(d*cos
(f*x + e)/sin(f*x + e))*(d^2/f^4)^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin(
f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \cot{\left(e + f x \right)}} \tan{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*tan(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*tan(f*x + e), x)
```

Mupad [B]
time = 0.21, size = 61, normalized size = 0.32

$$\frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)\,1{}\mathrm{i}}{f}+\frac{(-1)^{1/4}\,\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)\,1{}\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
 + ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1
i)/f
```

3.195 $\int \sqrt{d\cot(e+fx)}\,dx$

Optimal. Leaf size=192

$$\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)+1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}-\frac{\sqrt{d}\,\log\left(\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}+\frac{\sqrt{d}\,\log\left(\sqrt{d}\,\cot(e+fx)+\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \sqrt{d\cot(e+fx)}\,dx = -\frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{(2d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right.}{f}$$

$$= -\frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,f} - \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}}{-d+\sqrt{2}}\right.}{}$$

$$= -\frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,f} + \frac{\sqrt{d}\,\log\left(\sqrt{d}+\right.}{}$$

$$= \frac{\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f} - \frac{\sqrt{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 40, normalized size = 0.21

$$-\frac{2(d\cot(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cot[e + f*x]],x]
[Out] (-2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2])
/(3*d*f)
```

Maple [A]
time = 0.22, size = 136, normalized size = 0.71

method	result
derivativedivides	$-\frac{d\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+\right.\right.}{4f(d^2)^{\frac{1}{4}}}$

default	$-\frac{d\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)\right.}{4f(d^2)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*d/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(
1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^
(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*
arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1))
```

Maxima [A]
time = 0.51, size = 171, normalized size = 0.89

$$-\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d
/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(d)
*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cot(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e)), x)
```

Mupad [B]
time = 2.55, size = 50, normalized size = 0.26

$$-\frac{(-1)^{1/4}\sqrt{d}\left(\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot\left(e+f\,x\right)}}{\sqrt{d}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot\left(e+f\,x\right)}}{\sqrt{d}}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^(1/2),x)
```

```
[Out] -((-1)^(1/4)*d^(1/2)*(atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)) - a
tanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))))/f
```

3.196 $\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx$

Optimal. Leaf size=209

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{2\sqrt{d\cot(e+fx)}}{f}-$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)+1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))*d^(1/2)/f*2^(1/
2)-2*(d*cot(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}-\frac{2\sqrt{d\cot(e+fx)}}{f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*Sqrt[d*Cot[e + f*x]])/f - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x]
 - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (Sqrt[d]*Log[Sqrt[d] + Sq
rt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \cot(e+fx)\sqrt{d\cot(e+fx)}\,dx &= \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d} \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} - d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f} \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} + \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d\text{Sul}}{} \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\cot(e+}\right.}{2\sqrt{2}\,f} \\
&= -\frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(\right.}{2\sqrt{2}\,f} \\
&= -\frac{\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f} + \frac{\sqrt{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d\,\text{c}}}{\sqrt{}}\right.}{\sqrt{2}\,f}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 162, normalized size = 0.78

$$
-\frac{(d\cot(e+fx))^{3/2}\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}\right)+8\sqrt{\cot(e+fx)}+\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)-\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)\right)}{4df\cot^{\frac{3}{2}}(e+fx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -1/4*((d*Cot[e + f*x])^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x
]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 8*Sqrt[Cot[e + f*x
]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]] - Sqrt[2]*L
og[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(d*f*Cot[e + f*x]^(3/2)
)
```

Maple [A]
time = 0.07, size = 149, normalized size = 0.71

method	result
derivativedivides	$\frac{-2\sqrt{d\cot(fx+e)}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\sqrt{2}\right.}{4}}{f}$
default	$\frac{-2\sqrt{d\cot(fx+e)}+\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\sqrt{2}\right.}{4}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*(d*cot(f*x+e))^(1/2)+1/4*(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^2
)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)
*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d
*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
))
```

Maxima [A]
time = 0.51, size = 185, normalized size = 0.89

$$\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-8\sqrt{\frac{d}{\tan(fx+e)}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x + e)))/
f
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cot(e+fx)}\cot(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*cot(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*cot(f*x + e), x)
```

Mupad [B]
time = 2.57, size = 74, normalized size = 0.35

$$-\frac{2\sqrt{d\cot(e+fx)}}{f}-\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)1\mathrm{i}}{f}-\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)1\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] - (2*(d*cot(e + f*x))^(1/2))/f - ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*co
t(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*(
d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.197 $\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx$

Optimal. Leaf size=214

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{2(d\cot(e+fx))^{3/2}}{3df}$$

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/d/f-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(
1/2))*d^(1/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))*
d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^
(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f
*x+e))^(1/2))*d^(1/2)/f*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}-\frac{2(d\cot(e+fx))^{3/2}}{3df}+\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{\sqrt{d}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*(d*Cot[e + f*x])^(3/2))/(3*d*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[
e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt
[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{d\cot(e+fx)}\,dx = \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d^2}$$
$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} - \int \sqrt{d\cot(e+fx)}\,dx$$
$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$
$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{(2d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} - \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \cdots$$
$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(\cdots}{2\sqrt{2}f}$$
$$= -\frac{2(d\cot(e+fx))^{3/2}}{3df} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d}\cdots}{2\sqrt{2}f}$$
$$= -\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\cdots}}{}\right.}{\sqrt{2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.06, size = 42, normalized size = 0.20

$$\frac{2(d\cot(e+fx))^{3/2}\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (2*(d*Cot[e + f*x])^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x
]^2]))/(3*d*f)
```

Maple [A]
time = 0.12, size = 156, normalized size = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^{2}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}{d\cot(fx+e)+(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot}}{(d^{2})}\right.\right.}{8(d^{2})^{\frac{1}{4}}}\right.}{fd}$
default	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^{2}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}{d\cot(fx+e)+(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot}}{(d^{2})}\right.\right.}{8(d^{2})^{\frac{1}{4}}}\right.}{fd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d*(1/3*(d*cot(f*x+e))^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x
+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^
2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^
(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^
(1/2)+1)))
```

Maxima [A]

time = 0.50, size = 194, normalized size = 0.91

$$\frac{3\,d^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right)-8\left(\frac{d}{\tan(fx+e)}\right)^{\frac{3}{2}}}{12\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f*x +
e))^(3/2))/(d*f)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cot(e+fx)}\,\cot^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*cot(f*x + e)^2, x)
```

Mupad [B]
time = 2.60, size = 76, normalized size = 0.36

$$\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f}-\frac{2\,(d\cot(e+f\,x))^{3/2}}{3\,d\,f}-\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/f -
(2*(d*cot(e + f*x))^(3/2))/(3*d*f) - ((-1)^(1/4)*d^(1/2)*atanh(((-1)^(1/4)*
(d*cot(e + f*x))^(1/2))/d^(1/2)))/f
```

3.198 $\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx$

Optimal. Leaf size=231

$$\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{\sqrt{d}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{2\sqrt{d\cot(e+fx)}}{f}-2($$

```
[Out] -2/5*(d*cot(f*x+e))^(5/2)/d^2/f+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))*d^(1/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2)
)*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e)
)^(1/2))*d^(1/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot
(f*x+e))^(1/2))*d^(1/2)/f*2^(1/2)+2*(d*cot(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.14, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}-\frac{2(d\cot(e+fx))^{5/2}}{5d^2f}+\frac{2\sqrt{d\cot(e+fx)}}{f}+\frac{\sqrt{d}\,\log\left(\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}-\frac{\sqrt{d}\,\log\left(\sqrt{d}\,\cot(e+fx)+\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*Sqrt[d*Cot[e + f*x]])/f - (2*(d*Cot[e + f*x])^(5/2))/(5*d^2*f) + (Sqrt[d
]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sq
rt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot
[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \cot^3(e+fx)\sqrt{d\cot(e+fx)}\,dx &= \frac{\int (d\cot(e+fx))^{7/2}\,dx}{d^3} \\
&= -\frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d} \\
&= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + d\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx \\
&= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, \ldots\right.}{f} \\
&= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \ldots\right.}{f} \\
&= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} - \frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d}\ldots\right.}{f} \\
&= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2\ldots}{-d-\sqrt{2}\sqrt{d}\,x\ldots}\right.}{2\ldots} \\
&= \frac{2\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^2f} + \frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\,\cot(e\ldots\right.}{2\ldots} \\
&= \frac{\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f} - \frac{\sqrt{d}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot\ldots}}{\sqrt{\ldots}}\right.}{\sqrt{2}\,f}
\end{aligned}$$

Mathematica [A]
time = 0.47, size = 172, normalized size = 0.74

$$\frac{\sqrt{d\cot(e+fx)}\left(10\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-10\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+40\sqrt{\cot(e+fx)}-8\cot^{\frac{5}{2}}(e+fx)+5\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+fx)\right)-5\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+fx)\right)\right)}{20f\sqrt{\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (Sqrt[d*Cot[e + f*x]]*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] -
10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 40*Sqrt[Cot[e + f*x]] -
 8*Cot[e + f*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[
e + f*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/
(20*f*Sqrt[Cot[e + f*x]])
```

Maple [A]
time = 0.11, size = 171, normalized size = 0.74

method	result
derivativedivides	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^2\sqrt{d\cot(fx+e)}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}}\right.\right.}{}\right)}{f\,d^2}$
default	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^2\sqrt{d\cot(fx+e)}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}}\right.\right.}{}\right)}{f\,d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d^2*(1/5*(d*cot(f*x+e))^(5/2)-d^2*(d*cot(f*x+e))^(1/2)+1/8*d^2*(d^2)^(
1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^
2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2
)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/
(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)))
```

Maxima [A]
time = 0.55, size = 207, normalized size = 0.90

$$-\frac{10\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+10\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+5\sqrt{2}\,d^{\frac{5}{2}}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-5\sqrt{2}\,d^{\frac{5}{2}}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-40\,d^2\sqrt{\frac{d}{\tan(fx+e)}}+8\left(\frac{d}{\tan(fx+e)}\right)^{\frac{5}{2}}}{20\,d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/ta
n(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log(-sq
rt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt(d/ta
n(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d^2*f)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cot(e+fx)}\,\cot^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*cot(e + f*x))*cot(e + f*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cot(f*x + e))*cot(f*x + e)^3, x)
```

Mupad [**B**]
time = 2.87, size = 90, normalized size = 0.39

$$\frac{2\sqrt{d\cot(e+fx)}}{f}-\frac{2\,(d\cot(e+fx))^{5/2}}{5\,d^2\,f}+\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}+\frac{(-1)^{1/4}\sqrt{d}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}\,1\mathrm{i}}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(d*cot(e + f*x))^(1/2),x)
```

```
[Out] (2*(d*cot(e + f*x))^(1/2))/f - (2*(d*cot(e + f*x))^(5/2))/(5*d^2*f) + ((-1)
^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f + ((
-1)^(1/4)*d^(1/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/f
```

3.199 $\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx$

Optimal. Leaf size=234

$$\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{2d^4}{5f(d\cot(e+fx))^{5/2}}$$

```
[Out] 2/5*d^4/f/(d*cot(f*x+e))^(5/2)+1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^
(1/2)/d^(1/2))/f*2^(1/2)-1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/
d^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot
(f*x+e))^(1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)
*(d*cot(f*x+e))^(1/2))/f*2^(1/2)-2*d^2/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{2d^4}{5f(d\cot(e+fx))^{5/2}}-\frac{2d^2}{f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d^4)/(5*f*(d*Cot[e + f*x])^(5/2)) - (2*d^2)/(f*Sqrt[d*Cot[e + f*x]]) - (
d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])
/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt
[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (d\cot(e+fx))^{3/2}\tan^5(e+fx)\,dx = d^5\int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx$$
$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - d^3\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$
$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + d\int \sqrt{d\cot(e+fx)}\,d$$
$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^2\mathrm{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x\right.}{f}$$
$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{(2d^2)\,\mathrm{Subst}\left(\int \frac{x^2}{d^2+x^4}\,d\right.}{}$$
$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^2\mathrm{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x\right.}{f}$$
$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^{3/2}\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{}}{-d-\sqrt{2}}\right.}{}$$
$$= \frac{2d^4}{5f(d\cot(e+fx))^{5/2}} - \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\,\mathrm{c}\right.}{}$$
$$= \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f} - \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}}{}\right.}{\sqrt{2}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 45, normalized size = 0.19

$$\frac{2(d\cot(e+fx))^{3/2}\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(e+fx)\right)\tan^4(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] (2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[e + f*x]^2]
*Tan[e + f*x]^4)/(5*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.41, size = 728, normalized size = 3.11

method	result
default	$\frac{(\cos(fx+e)-1)\left(5i(\cos^2(fx+e))\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\sin(fx+e)\,\mathrm{E}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/f*(cos(f*x+e)-1)*(5*I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*cos(f*x+e)^2-5*I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(f*x+e)*((cos(f*x+e)-1)/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2+10*cos(f*x+e)^2*((cos(f*x+e)-1
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-5*cos(f*x+e)^2*((cos(f*x+e)-1)/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*cos(f*x+e)^2*((cos(f
*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-12*cos(f*x+e)
^3*2^(1/2)+12*cos(f*x+e)^2*2^(1/2)+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*(d*cos(f
*x+e)/sin(f*x+e))^(3/2)*(cos(f*x+e)+1)^2/sin(f*x+e)^2/cos(f*x+e)^4*2^(1/2)
```

Maxima [A]
time = 0.53, size = 215, normalized size = 0.92

$$-\frac{d^6\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right)}{d^4}-\frac{8\left(d^2-\frac{5d^2}{\tan(fx+e)^2}\right)}{d^4\left(\frac{d}{\tan(fx+e)}\right)^{\frac{5}{2}}}\right)}{20f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/20*d^6*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*
sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^2 - 5
*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(189) = 378$.
time = 0.44, size = 654, normalized size = 2.79

$$\frac{20\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f \arctan\left(-\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} d^4 f \sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}} + d^6 - \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f \sqrt{\frac{d^9\cos(fx+e) + \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} d^4 f^3 \sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e) + \sqrt{\frac{d^6}{f^4}} d^6 f^2 \sin(fx+e)}{\sin(fx+e)}}}{d^6}\right)\cos(fx+e)^3 + 20\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f \arctan\left(-\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} d^4 f \sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}} - d^6 - \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f \sqrt{\frac{d^9\cos(fx+e) - \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} d^4 f^3 \sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e) + \sqrt{\frac{d^6}{f^4}} d^6 f^2 \sin(fx+e)}{\sin(fx+e)}}}{d^6}\right)\cos(fx+e)^3 + 5\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\cos(fx+e)^3 \log\left(\frac{d^9\cos(fx+e) + \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} d^4 f^3 \sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e) + \sqrt{\frac{d^6}{f^4}} d^6 f^2 \sin(fx+e)}{\sin(fx+e)}\right) - 5\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\cos(fx+e)^3 \log\left(\frac{d^9\cos(fx+e) - \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} d^4 f^3 \sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e) + \sqrt{\frac{d^6}{f^4}} d^6 f^2 \sin(fx+e)}{\sin(fx+e)}\right) - 8\left(6d\cos(fx+e)^2 - d\right)\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)}{20 f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] 1/20*(20*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan(-(sqrt(2)*(d^6/f^4)^(1/4)*d^4*f*s
qrt(d*cos(f*x + e)/sin(f*x + e)) + d^6 - sqrt(2)*(d^6/f^4)^(1/4)*f*sqrt((d^
9*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*
x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)))/d
^6)*cos(f*x + e)^3 + 20*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan(-(sqrt(2)*(d^6/f^4
)^(1/4)*d^4*f*sqrt(d*cos(f*x + e)/sin(f*x + e)) - d^6 - sqrt(2)*(d^6/f^4)^(
1/4)*f*sqrt((d^9*cos(f*x + e) - sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(
f*x + e)/sin(f*x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/s
in(f*x + e)))/d^6)*cos(f*x + e)^3 + 5*sqrt(2)*(d^6/f^4)^(1/4)*f*cos(f*x + e
)^3*log((d^9*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x
+ e)/sin(f*x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f
*x + e)) - 5*sqrt(2)*(d^6/f^4)^(1/4)*f*cos(f*x + e)^3*log((d^9*cos(f*x + e)
 - sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*
x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)) - 8*(6*d*cos(f*x
 + e)^2 - d)*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e))/(f*cos(f*x + e
)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}} \tan^5\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^5, x)
```

Mupad [B]
time = 2.58, size = 97, normalized size = 0.41

$$\frac{\frac{2\,d^4}{5}-\frac{2\,d^4}{{\mathrm{tan}\left(e+f\,x\right)}^2}}{f\,{\left(\frac{d}{\mathrm{tan}\left(e+f\,x\right)}\right)}^{5/2}}-\frac{{\left(-1\right)}^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\mathrm{tan}\left(e+f\,x\right)}}}{\sqrt{d}}\right)}{f}+\frac{{\left(-1\right)}^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\mathrm{tan}\left(e+f\,x\right)}}}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((2*d^4)/5 - (2*d^4)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1/
4)*d^(3/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(1/
4)*d^(3/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.200 $\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=214

$$-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}$$

```
[Out] 2/3*d^3/f/(d*cot(f*x+e))^(3/2)-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^
(1/2)/d^(1/2))/f*2^(1/2)+1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/
d^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot
(f*x+e))^(1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)
*(d*cot(f*x+e))^(1/2))/f*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{2d^3}{3f(d\cot(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^3)/(3*f*(d*Cot[e + f*x])^(3/2)) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Co
t[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sq
rt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

IntegerQ[n]

Rubi steps

$$
\begin{aligned}
\int (d\cot(e+fx))^{3/2}\tan^4(e+fx)\,dx &= d^4\int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} - d^2\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} + \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} + \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right.}{f}\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} + \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} +\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} - \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\cot}\right.}{2\sqrt{2}\,f}\\
&= \frac{2d^3}{3f(d\cot(e+fx))^{3/2}} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d}\right.}{2\sqrt{2}\,f}\\
&= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{}}{}\right.}{\sqrt{2}\,f}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 45, normalized size = 0.21

$$
\frac{2(d\cot(e+fx))^{3/2}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(e+fx)\right)\tan^3(e+fx)}{3f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] (2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[e + f*x]^2]*
Tan[e + f*x]^3)/(3*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.39, size = 548, normalized size = 2.56

method	result
default	$-\frac{(\cos(fx+e)-1)\left(3i\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+c}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(cos(f*x+e)-1)*(3*I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)*cos(f*x+e)-3*I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*cos(f*x+e)+3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*El
lipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*cos(f*x+e)+3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*
cos(f*x+e)-2*cos(f*x+e)*2^(1/2)+2*2^(1/2))*(cos(f*x+e)+1)^2*(d*cos(f*x+e)/s
in(f*x+e))^(3/2)/sin(f*x+e)/cos(f*x+e)^3*2^(1/2)
```

Maxima [A]
time = 0.51, size = 197, normalized size = 0.92

$$\frac{d^5\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}\right)}{d^2}+\frac{8}{d^2\left(\frac{d}{\tan(fx+e)}\right)^{\frac{3}{2}}}\right)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 1/12*d^5*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
```

```
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2*(d/t
an(f*x + e))^(3/2)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(170) = 340$.
time = 0.38, size = 623, normalized size = 2.91

$$-\frac{12\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\arctan\left(-\frac{d^6+\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}-\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} f^3\sqrt{\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)+d^3\cos(fx+e)+\sqrt{\frac{d^6}{f^4}}f^2\sin(fx+e)}{\sin(fx+e)}}}{d^6}\right)\cos(fx+e)^2+12\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\arctan\left(\frac{d^6-\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}+\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} f^3\sqrt{-\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)-d^3\cos(fx+e)-\sqrt{\frac{d^6}{f^4}}f^2\sin(fx+e)}{\sin(fx+e)}}}{d^6}\right)\cos(fx+e)^2-3\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\cos(fx+e)^2\log\left(\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)+d^3\cos(fx+e)+\sqrt{\frac{d^6}{f^4}}f^2\sin(fx+e)}{\sin(fx+e)}\right)+3\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f\cos(fx+e)^2\log\left(-\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)-d^3\cos(fx+e)-\sqrt{\frac{d^6}{f^4}}f^2\sin(fx+e)}{\sin(fx+e)}\right)+8\left(d\cos(fx+e)^2-d\right)\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}}{12f\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan(-(d^6 + sqrt(2)*(d^6/f^4)^(3/4)*
d*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e)) - sqrt(2)*(d^6/f^4)^(3/4)*f^3*sqrt(
(sqrt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e)
 + d^3*cos(f*x + e) + sqrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e)))/d^6)*c
os(f*x + e)^2 + 12*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan((d^6 - sqrt(2)*(d^6/f^4
)^(3/4)*d*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e)) + sqrt(2)*(d^6/f^4)^(3/4)*f
^3*sqrt(-(sqrt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin
(f*x + e) - d^3*cos(f*x + e) - sqrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e)
))/d^6)*cos(f*x + e)^2 - 3*sqrt(2)*(d^6/f^4)^(1/4)*f*cos(f*x + e)^2*log((sq
rt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) +
d^3*cos(f*x + e) + sqrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e)) + 3*sqrt(2
)*(d^6/f^4)^(1/4)*f*cos(f*x + e)^2*log(-(sqrt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d
*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) - d^3*cos(f*x + e) - sqrt(d^6/f^4)
*f^2*sin(f*x + e))/sin(f*x + e)) + 8*(d*cos(f*x + e)^2 - d)*sqrt(d*cos(f*x
+ e)/sin(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^{\frac{3}{2}}\tan^4(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^4, x)
```

Mupad [B]
time = 2.52, size = 83, normalized size = 0.39

$$\frac{2\,d^3}{3\,f\,{\left(\frac{d}{\mathrm{tan}\left(e+f\,x\right)}\right)}^{3/2}}-\frac{{\left(-1\right)}^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\mathrm{tan}\left(e+f\,x\right)}}}{\sqrt{d}}\right)\,1{}\mathrm{i}}{f}-\frac{{\left(-1\right)}^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\mathrm{tan}\left(e+f\,x\right)}}}{\sqrt{d}}\right)\,1{}\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(d*cot(e + f*x))^(3/2),x)
```

```
[Out] (2*d^3)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)
*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(
1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.201 $\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx$

Optimal. Leaf size=212

$$-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{2d^2}{f\,\sqrt{d\cot(e+fx)}}+$$

```
[Out] -1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/4*d^(3/2)
*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)-1/4*
d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/
2)+2*d^2/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}+\frac{d^{3/2}\log\left(\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}-\frac{d^{3/2}\log\left(\sqrt{d}\,\cot(e+fx)+\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}+\frac{2d^2}{f\,\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (2*d^2)/(f*Sqrt[d*Cot[e + f*x]]) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e
+ f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d
] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int (d\cot(e+fx))^{3/2}\tan^3(e+fx)\,dx &= d^3\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - d\int \sqrt{d\cot(e+fx)}\,dx \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^2\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f} \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} - \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \cdots \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\cot(e \cdots}{2\sqrt{2}f} \\
&= \frac{2d^2}{f\sqrt{d\cot(e+fx)}} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot \cdots}{2\sqrt{2}f} \\
&= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\cdots}}{\cdots}\right)}{\sqrt{2}f}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 38, normalized size = 0.18

$$\frac{2d^2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(e+fx)\right)}{f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^3,x]
[Out] (2*d^2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[e + f*x]^2])/(f*Sqrt[d*Cot[e +
f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.39, size = 660, normalized size = 3.11

method	result
default	$\frac{\left(i\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\,\sin(fx+e)\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(fx+e}{\sin(f}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*Ell
ipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2
))-I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+(
(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+((cos(f
*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*((cos(f*x+e
)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*cos(f*x+e)*2^(1/2)-2*2^(1/
2))*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)*(d*cos(f*x+e)/sin(f*x+e))^(3/2)/sin(f*x
+e)^2/cos(f*x+e)^2*2^(1/2)
```

Maxima [A]
time = 0.52, size = 196, normalized size = 0.92

$$\frac{d^4\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}}{d^2}+\frac{8}{d^2\sqrt{\frac{d}{\tan(fx+e)}}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/4*d^4*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
+ e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt
```

```
(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sqrt(d/
tan(f*x + e))))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(170) = 340$.
time = 0.41, size = 630, normalized size = 2.97

$$-\frac{4\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f \arctan\left(-\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} d^4 f \sqrt{\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}} + d^6 - \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f \sqrt{\frac{d^9\cos\left(fx+e\right) + \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} d^4 f^3 \sqrt{\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right) + \sqrt{\frac{d^6}{f^4}} d^6 f^2 \sin\left(fx+e\right)}{\sin\left(fx+e\right)}}}{d^6}\right)\cos\left(fx+e\right) + 4\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f \arctan\left(-\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} d^4 f \sqrt{\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}} - d^6 - \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f \sqrt{\frac{d^9\cos\left(fx+e\right) - \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} d^4 f^3 \sqrt{\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right) + \sqrt{\frac{d^6}{f^4}} d^6 f^2 \sin\left(fx+e\right)}{\sin\left(fx+e\right)}}}{d^6}\right)\cos\left(fx+e\right) + \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f \cos\left(fx+e\right)\log\left(\frac{d^9\cos\left(fx+e\right) + \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} d^4 f^3 \sqrt{\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right) + \sqrt{\frac{d^6}{f^4}} d^6 f^2 \sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right) - \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}} f \cos\left(fx+e\right)\log\left(\frac{d^9\cos\left(fx+e\right) - \sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}} d^4 f^3 \sqrt{\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right) + \sqrt{\frac{d^6}{f^4}} d^6 f^2 \sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right) - 8d\sqrt{\frac{d\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\sin\left(fx+e\right)}{4f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan(-(sqrt(2)*(d^6/f^4)^(1/4)*d^4*f*sq
rt(d*cos(f*x + e)/sin(f*x + e)) + d^6 - sqrt(2)*(d^6/f^4)^(1/4)*f*sqrt((d^9
*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x
 + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)))/d^
6)*cos(f*x + e) + 4*sqrt(2)*(d^6/f^4)^(1/4)*f*arctan(-(sqrt(2)*(d^6/f^4)^(1
/4)*d^4*f*sqrt(d*cos(f*x + e)/sin(f*x + e)) - d^6 - sqrt(2)*(d^6/f^4)^(1/4)
*f*sqrt((d^9*cos(f*x + e) - sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x
+ e)/sin(f*x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f
*x + e)))/d^6)*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(1/4)*f*cos(f*x + e)*log((d
^9*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f
*x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)) -
 sqrt(2)*(d^6/f^4)^(1/4)*f*cos(f*x + e)*log((d^9*cos(f*x + e) - sqrt(2)*(d^
6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) + sqrt(
d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)) - 8*d*sqrt(d*cos(f*x + e)/sin(
f*x + e))*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\cot\left(e+fx\right)\right)^{\frac{3}{2}}\tan^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^3, x)
```

Mupad [B]
time = 2.49, size = 82, normalized size = 0.39

$$\frac{2\,d^2}{f\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}+\frac{{\left(-1\right)}^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)}{f}-\frac{{\left(-1\right)}^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(d*cot(e + f*x))^(3/2),x)
```

```
[Out] (2*d^2)/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(
d/tan(e + f*x))^(1/2))/d^(1/2)))/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*
(d/tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.202 $\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=192

$$\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\,\mathrm{c}\right.}{}$$

```
[Out] 1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/2*d^
(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/4*d^(3/2)*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)-1/4*d
^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2
)
```

Rubi [A]
time = 0.11, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {16, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}+\frac{d^{3/2}\log\left(\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}-\frac{d^{3/2}\log\left(\sqrt{d}\,\cot(e+fx)+\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
```

```
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (d\cot(e+fx))^{3/2}\tan^2(e+fx)\,dx = d^2\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= -\frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d^2\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx,\right.}{f}$$

$$= \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,f} + \frac{d^{3/2}\text{Subst}}{}$$

$$= \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,f} - \frac{d^{3/2}\text{lo}}{}$$

$$= \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f} - \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d}}{}\right.}{\sqrt{2}\,f}$$

Mathematica [A]
time = 0.03, size = 134, normalized size = 0.70

$$\frac{d^2\sqrt{\cot(e+fx)}\left(2\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}\right)+\log\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)-\log\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)\right)}{2\sqrt{2}\,f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (d^2*Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTa
n[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Co
t[e + f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[
2]*f*Sqrt[d*Cot[e + f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 292, normalized size = 1.52

method	result

default	$-\frac{(\cos(fx+e)+1)^2\left(i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)+1)^2*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((cos(f*x+e)-1)/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*(cos(f*x+e)-1)*(d*cos(f*x+e)/sin(f*x+e))^(3/
2)/sin(f*x+e)/cos(f*x+e)^2*2^(1/2)
```

Maxima [**A**]
time = 0.51, size = 173, normalized size = 0.90

$$-\frac{d^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -1/4*d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
+ e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt
(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(151) = 302$.
time = 0.38, size = 530, normalized size = 2.76

$$\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{d^6+\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}}df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}-\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\sqrt{\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)+d^3\cos(fx+e)+\sqrt{\frac{d^6}{f^4}}f^2\sin(fx+e)}{\sin(fx+e)}}}{d^6}\right)+\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}\arctan\left(\frac{d^6-\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}}df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}+\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}}f^3\sqrt{-\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)-d^3\cos(fx+e)-\sqrt{\frac{d^6}{f^4}}f^2\sin(fx+e)}{\sin(fx+e)}}}{d^6}\right)-\frac{1}{4}\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)+d^3\cos(fx+e)+\sqrt{\frac{d^6}{f^4}}f^2\sin(fx+e)}{\sin(fx+e)}\right)+\frac{1}{4}\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(-\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)-d^3\cos(fx+e)-\sqrt{\frac{d^6}{f^4}}f^2\sin(fx+e)}{\sin(fx+e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] sqrt(2)*(d^6/f^4)^(1/4)*arctan(-(d^6 + sqrt(2)*(d^6/f^4)^(3/4)*d*f^3*sqrt(d
*cos(f*x + e)/sin(f*x + e)) - sqrt(2)*(d^6/f^4)^(3/4)*f^3*sqrt((sqrt(2)*(d^
6/f^4)^(1/4)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) + d^3*cos(f
*x + e) + sqrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e)))/d^6) + sqrt(2)*(d^
6/f^4)^(1/4)*arctan((d^6 - sqrt(2)*(d^6/f^4)^(3/4)*d*f^3*sqrt(d*cos(f*x + e
)/sin(f*x + e)) + sqrt(2)*(d^6/f^4)^(3/4)*f^3*sqrt(-(sqrt(2)*(d^6/f^4)^(1/4
)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) - d^3*cos(f*x + e) - s
qrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e)))/d^6) - 1/4*sqrt(2)*(d^6/f^4)^
(1/4)*log((sqrt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*si
n(f*x + e) + d^3*cos(f*x + e) + sqrt(d^6/f^4)*f^2*sin(f*x + e))/sin(f*x + e
)) + 1/4*sqrt(2)*(d^6/f^4)^(1/4)*log(-(sqrt(2)*(d^6/f^4)^(1/4)*d*f*sqrt(d*c
os(f*x + e)/sin(f*x + e))*sin(f*x + e) - d^3*cos(f*x + e) - sqrt(d^6/f^4)*f
^2*sin(f*x + e))/sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^{\frac{3}{2}}\tan^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e)**2,x)
[Out] Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e)^2, x)
```

Mupad [B]
time = 2.50, size = 61, normalized size = 0.32

$$\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}+\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+f\,x)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1i)/f
 + ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))*1
i)/f
```

3.203 $\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx$

Optimal. Leaf size=192

$$\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\ \text{c}\right.}{}$$

```
[Out] 1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/2*d^
(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/4*d^(3/2)*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)+1/4*d
^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2
)
```

Rubi [A]
time = 0.10, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {16, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x],x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]
)/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqr
t[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (d\cot(e+fx))^{3/2}\tan(e+fx)\,dx = d\int \sqrt{d\cot(e+fx)}\,dx$$
$$= -\frac{d^2\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$
$$= -\frac{(2d^2)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{d^2\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x,\right.}{f}$$
$$= -\frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} - \frac{d^{3/2}\text{Subs}}{}$$
$$= -\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{3/2}\,\text{l}}{}$$
$$= \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\text{c}}}{}\right.}{\sqrt{2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 37, normalized size = 0.19

$$-\frac{2(d\cot(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2)*Tan[e + f*x],x]
```

```
[Out] (-2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2])
/(3*f)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 324, normalized size = 1.69

method	result

default	$-\frac{(\cos(fx+e)+1)^2\left(i\,\text{EllipticPi}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\text{EllipticPi}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2)*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)+1)^2*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*EllipticF((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)))*((cos(f*x+e)-1)/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*(cos(f*x+e)-1)*(d*cos(f*x+e)/sin(f*x+e))^(3/2)/
sin(f*x+e)/cos(f*x+e)^2*2^(1/2)
```

Maxima [A]
time = 0.56, size = 173, normalized size = 0.90

$$-\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/4*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
+ e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt
(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(151) = 302$.
time = 0.40, size = 553, normalized size = 2.88

$$\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}d^4f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}+d^6-\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\sqrt{\frac{d^9\cos(fx+e)+\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}}d^4f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)+\sqrt{\frac{d^6}{f^4}}d^6f^2\sin(fx+e)}{\sin(fx+e)}}}{d^6}\right)+\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}d^4f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}-d^6-\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}f\sqrt{\frac{d^9\cos(fx+e)-\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}}d^4f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)+\sqrt{\frac{d^6}{f^4}}d^6f^2\sin(fx+e)}{\sin(fx+e)}}}{d^6}\right)+\frac{1}{4}\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(\frac{d^9\cos(fx+e)+\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}}d^4f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)+\sqrt{\frac{d^6}{f^4}}d^6f^2\sin(fx+e)}{\sin(fx+e)}\right)-\frac{1}{4}\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{1}{4}}\log\left(\frac{d^9\cos(fx+e)-\sqrt{2}\left(\frac{d^6}{f^4}\right)^{\frac{3}{4}}d^4f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)+\sqrt{\frac{d^6}{f^4}}d^6f^2\sin(fx+e)}{\sin(fx+e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(d^6/f^4)^(1/4)*arctan(-(sqrt(2)*(d^6/f^4)^(1/4)*d^4*f*sqrt(d*cos(f
*x + e)/sin(f*x + e)) + d^6 - sqrt(2)*(d^6/f^4)^(1/4)*f*sqrt((d^9*cos(f*x +
 e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin
(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)))/d^6) + sqrt(
2)*(d^6/f^4)^(1/4)*arctan(-(sqrt(2)*(d^6/f^4)^(1/4)*d^4*f*sqrt(d*cos(f*x +
e)/sin(f*x + e)) - d^6 - sqrt(2)*(d^6/f^4)^(1/4)*f*sqrt((d^9*cos(f*x + e) -
 sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x
+ e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e)))/d^6) + 1/4*sqrt(2
)*(d^6/f^4)^(1/4)*log((d^9*cos(f*x + e) + sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*s
qrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f
*x + e))/sin(f*x + e)) - 1/4*sqrt(2)*(d^6/f^4)^(1/4)*log((d^9*cos(f*x + e)
- sqrt(2)*(d^6/f^4)^(3/4)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x
 + e) + sqrt(d^6/f^4)*d^6*f^2*sin(f*x + e))/sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d \cot (e+fx))^{\frac{3}{2}} \tan (e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2)*tan(f*x+e),x)
[Out] Integral((d*cot(e + f*x))**(3/2)*tan(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2)*tan(f*x+e),x, algorithm="giac")
[Out] integrate((d*cot(f*x + e))^(3/2)*tan(f*x + e), x)
```

Mupad [B]
time = 2.53, size = 54, normalized size = 0.28

$$-\frac{(-1)^{1/4}\, d^{3/2} \left(\operatorname{atan}\left(\frac{(-1)^{1/4} \sqrt{\frac{d}{\tan (e+f\,x)}}}{\sqrt{d}}\right)-\operatorname{atanh}\left(\frac{(-1)^{1/4} \sqrt{\frac{d}{\tan (e+f\,x)}}}{\sqrt{d}}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(d*cot(e + f*x))^(3/2),x)
[Out] -((-1)^(1/4)*d^(3/2)*(atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)) - a
tanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2))))/f
```

3.204 $\int (d\cot(e+fx))^{3/2}\,dx$

Optimal. Leaf size=210

$$-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}-$$

```
[Out] -1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+1/2*d
^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/4*d^(3/2)
*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)+1/4*
d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/
2)-2*d*(d*cot(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}-\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*d*Sqrt[d*Cot[e + f*x]])/f - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*
x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) + (d^(3/2)*Log[Sqrt[d] +
Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (d\cot(e+fx))^{3/2}\,dx = -\frac{2d\sqrt{d\cot(e+fx)}}{f} - d^2\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= -\frac{2d\sqrt{d\cot(e+fx)}}{f} + \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{2d\sqrt{d\cot(e+fx)}}{f} + \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{2d\sqrt{d\cot(e+fx)}}{f} + \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{d^2\text{Subst}\left(\int \frac{d}{d^2}\right.}{}$$

$$= -\frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,f} -$$

$$= -\frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,f}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right.}{\sqrt{2}\,f}$$

Mathematica [A]
time = 0.09, size = 159, normalized size = 0.76

$$-\frac{(d\cot(e+fx))^{3/2}\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+8\sqrt{\cot(e+fx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+fx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+fx)\right)\right)}{4f\cot^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -1/4*((d*Cot[e + f*x])^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x
]]] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 8*Sqrt[Cot[e + f*x
]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]] - Sqrt[2]*L
og[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(f*Cot[e + f*x]^(3/2))
```

Maple [A]
time = 0.08, size = 149, normalized size = 0.71

method	result

derivativedivides	$-\frac{2d\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\cdots\right)\right)}{8}\right)}{f}$
default	$-\frac{2d\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\cdots\right)\right)}{8}\right)}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*d*((d*cot(f*x+e))^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^2
)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)
*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d
*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
))
```

Maxima [A]
time = 0.54, size = 186, normalized size = 0.89

$$\frac{\left(2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-8\sqrt{\frac{d}{\tan(fx+e)}}\right)d}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x + e)))*
d/f
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2), x)
```

Mupad [B]
time = 2.65, size = 75, normalized size = 0.36

$$-\frac{2\,d\,\sqrt{d\cot(e+f\,x)}}{f}-\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}-\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cot(e + f*x))^(3/2),x)
```

```
[Out] - (2*d*(d*cot(e + f*x))^(1/2))/f - ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*
cot(e + f*x))^(1/2))/d^(1/2))*1i)/f - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)
*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f
```

3.205 $\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx$

Optimal. Leaf size=211

$$-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{2(d\cot(e+fx))^{3/2}}{3f}$$

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/f-1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/
2)/d^(1/2))/f*2^(1/2)+1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(
1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*
x+e))^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d
*cot(f*x+e))^(1/2))/f*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {16, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{2(d\cot(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 - (2*(d*Cot[e + f*x])^(3/2))/(3*f) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e
+ f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f) - (d^(3/2)*Log[Sqrt[d
] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

`IntegerQ[n]`

Rubi steps

$$
\begin{aligned}
\int \cot(e+fx)(d\cot(e+fx))^{3/2}\,dx &= \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d} \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3f} - d\int \sqrt{d\cot(e+fx)}\,dx \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{d^2\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f} \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3f} - \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\cot(\right.}{2\sqrt{2}\,f} \\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3f} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d}\right.}{2\sqrt{2}\,f} \\
&= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d}}{}\right.}{\sqrt{2}\,f}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 39, normalized size = 0.18

$$
\frac{2(d\cot(e+fx))^{3/2}\left(-1+\,_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)\right)}{3f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(d*Cot[e + f*x])^(3/2),x]
[Out] (2*(d*Cot[e + f*x])^(3/2)*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x
]^2]))/(3*f)
```

Maple [A]
time = 0.06, size = 152, normalized size = 0.72

method	result
derivativedivides	$\frac{-\frac{2(d\cot(fx+e))^{\frac{3}{2}}}{3}+\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4(d^2)^{\frac{1}{4}}}}{f}$
default	$\frac{-\frac{2(d\cot(fx+e))^{\frac{3}{2}}}{3}+\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4(d^2)^{\frac{1}{4}}}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2/3*(d*cot(f*x+e))^(3/2)+1/4*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e
)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)
^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1
/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1
/2)+1)))
```

Maxima [A]
time = 0.51, size = 191, normalized size = 0.91

$$\frac{3d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right)-8\left(\frac{d}{\tan(fx+e)}\right)^{\frac{3}{2}}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f*x +
e))^(3/2))/f
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cot (e+fx))^{\frac{3}{2}}\cot (e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2)*cot(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*cot(f*x + e), x)
```

Mupad [B]
time = 2.63, size = 73, normalized size = 0.35

$$\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot (e+f\,x)}}{\sqrt{d}}\right)}{f}-\frac{2\,(d\cot(e+f\,x))^{3/2}}{3\,f}-\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot (e+f\,x)}}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/f -
(2*(d*cot(e + f*x))^(3/2))/(3*f) - ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*(d
*cot(e + f*x))^(1/2))/d^(1/2)))/f
```

3.206 $\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx$

Optimal. Leaf size=232

$$\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}+\frac{2d\sqrt{d\cot(e+fx)}}{f}-2$$

```
[Out] -2/5*(d*cot(f*x+e))^(5/2)/d/f+1/2*d^(3/2)*arctan(1-2^(1/2)*(d*cot(f*x+e))^(
1/2)/d^(1/2))/f*2^(1/2)-1/2*d^(3/2)*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))/f*2^(1/2)+1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(
f*x+e))^(1/2))/f*2^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*
(d*cot(f*x+e))^(1/2))/f*2^(1/2)+2*d*(d*cot(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.14, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f}-\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}f}+\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{d^{3/2}\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}f}-\frac{2(d\cot(e+fx))^{5/2}}{5df}+\frac{2d\sqrt{d\cot(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) +
(2*d*Sqrt[d*Cot[e + f*x]])/f - (2*(d*Cot[e + f*x])^(5/2))/(5*d*f) + (d^(3/2
)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]])/(2*Sq
rt[2]*f) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot
[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \cot^2(e+fx)(d\cot(e+fx))^{3/2}\,dx = \frac{\int (d\cot(e+fx))^{7/2}\,dx}{d^2}$$

$$= -\frac{2(d\cot(e+fx))^{5/2}}{5df} - \int (d\cot(e+fx))^{3/2}\,dx$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + d^2\int \frac{1}{\sqrt{d\cot(e+fx)}}\,d$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} - \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,d\right.}{f}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} - \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx,\right.}{f}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} - \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x,\right.}{f}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\,\sqrt{d}}{-d-\sqrt{2}\,\sqrt{d}}\right.}{}$$

$$= \frac{2d\sqrt{d\cot(e+fx)}}{f} - \frac{2(d\cot(e+fx))^{5/2}}{5df} + \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\,\cot\right.}{}$$

$$= \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f} - \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d\cot}}{\sqrt{}}\right.}{\sqrt{2}\,f}$$

Mathematica [A]
time = 0.25, size = 172, normalized size = 0.74

$$\frac{(d\cot(e+fx))^{3/2}\left(10\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}\right)-10\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}\right)+40\sqrt{\cot(e+fx)}-8\cot^{\frac{5}{2}}(e+fx)+5\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)-5\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)\right)}{20f\cot^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ((d*Cot[e + f*x])^(3/2)*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]
- 10*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 40*Sqrt[Cot[e + f*x]]
 - 8*Cot[e + f*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Co
t[e + f*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]])
)/(20*f*Cot[e + f*x]^(3/2))
```

Maple [A]
time = 0.10, size = 171, normalized size = 0.74

method	result
derivativedivides	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^2\sqrt{d\cot(fx+e)}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}-}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}-}\right.\right.}{}\right)}{fd}$
default	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^2\sqrt{d\cot(fx+e)}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}-}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}-}\right.\right.}{}\right)}{fd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d*(1/5*(d*cot(f*x+e))^(5/2)-d^2*(d*cot(f*x+e))^(1/2)+1/8*d^2*(d^2)^(1/
4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)
^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))
)+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d
^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)))
```

Maxima [A]
time = 0.51, size = 207, normalized size = 0.89

$$-\frac{10\,\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{d}+2\,\sqrt{\frac{d}{\tan\left(fx+e\right)}}\right)}{2\,\sqrt{d}}\right)+10\,\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{d}-2\,\sqrt{\frac{d}{\tan\left(fx+e\right)}}\right)}{2\,\sqrt{d}}\right)+5\,\sqrt{2}\,d^{\frac{5}{2}}\log\left(\sqrt{2}\,\sqrt{d}\,\sqrt{\frac{d}{\tan\left(fx+e\right)}}+d+\frac{d}{\tan(fx+e)}\right)-5\,\sqrt{2}\,d^{\frac{5}{2}}\log\left(-\sqrt{2}\,\sqrt{d}\,\sqrt{\frac{d}{\tan\left(fx+e\right)}}+d+\frac{d}{\tan(fx+e)}\right)-40\,d^2\sqrt{\frac{d}{\tan\left(fx+e\right)}}+8\left(\frac{d}{\tan(fx+e)}\right)^{\frac{5}{2}}}{20\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/ta
n(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log(-sq
rt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt(d/ta
n(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d*f)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cot(e+fx))^{\frac{3}{2}}\cot^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*cot(e + f*x))**(3/2)*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(3/2)*cot(f*x + e)^2, x)
```

Mupad [B]
time = 2.94, size = 91, normalized size = 0.39

$$\frac{2\,d\,\sqrt{d\cot(e+f\,x)}}{f}-\frac{2\,(d\cot(e+f\,x))^{5/2}}{5\,d\,f}+\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{f}+\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}\,1\mathrm{i}}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(d*cot(e + f*x))^(3/2),x)
```

```
[Out] (2*d*(d*cot(e + f*x))^(1/2))/f - (2*(d*cot(e + f*x))^(5/2))/(5*d*f) + ((-1)
^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/f + ((
-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/f
```

3.207 $\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=231

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}+\frac{2d^2}{5f(d\cot(e+fx))^{5/2}}-\frac{}{f\sqrt{d\,c}}$$

```
[Out] 2/5*d^2/f/(d*cot(f*x+e))^(5/2)+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^
(1/2))/f*2^(1/2)/d^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))
/f*2^(1/2)/d^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))
^(1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(
f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)-2/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\,f}+\frac{2d^2}{5f(d\cot(e+fx))^{5/2}}-\frac{2}{f\sqrt{d\cot(e+fx)}}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}\,f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + (2*d
^2)/(5*f*(d*Cot[e + f*x])^(5/2)) - 2/(f*Sqrt[d*Cot[e + f*x]]) - Log[Sqrt[d]
 + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*
f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*
Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = d^3\int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - d\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d}$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+f}\right.}{f}$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx}\right.}{f}$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}x+x^2}\,dx, x, \sqrt{d}\right.}{2f}$$

$$= \frac{2d^2}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\right.}{2\sqrt{2}\sqrt{d}f}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f} + \frac{}{5f(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.13, size = 40, normalized size = 0.17

$$\frac{2d^2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(e+fx)\right)}{5f(d\cot(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (2*d^2*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[e + f*x]^2])/(5*f*(d*Cot[e + f
*x])^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.42, size = 728, normalized size = 3.15

method	result
default	$\frac{(\cos(fx+e)-1)\left(5i(\cos^2(fx+e))\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\ \sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\ \sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\ \sin(fx+e)\,\mathrm{E}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/f*(cos(f*x+e)-1)*(5*I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*cos(f*x+e)^2-5*I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(f*x+e)*((cos(f*x+e)-1)/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2+10*cos(f*x+e)^2*((cos(f*x+e)-1
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-5*cos(f*x+e)^2*((cos(f*x+e)-1)/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*cos(f*x+e)^2*((cos(f
*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-12*cos(f*x+e)
^3*2^(1/2)+12*cos(f*x+e)^2*2^(1/2)+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*(cos(f*x
+e)+1)^2/sin(f*x+e)^4/cos(f*x+e)^2/(d*cos(f*x+e)/sin(f*x+e))^(1/2)*2^(1/2)
```

Maxima [A]

time = 0.51, size = 215, normalized size = 0.93

$$-\frac{d^{4}\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right)}{d^{4}}-\frac{8\left(d^{2}-\frac{5d^{2}}{\tan(fx+e)^{2}}\right)}{d^{4}\left(\frac{d}{\tan(fx+e)}\right)^{\frac{5}{2}}}\right)}{20f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*d^4*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*
sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^2 - 5
*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 632 vs. 2(186) = 372.
time = 0.38, size = 632, normalized size = 2.74

$$\frac{20\sqrt{2}\,df\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{1}{4}}+\sqrt{2}f\sqrt{\frac{\sqrt{2}d^{2}f^{3}\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{3}{4}}\sin(fx+e)+d^{2}f^{2}\sqrt{\frac{1}{d^{2}f^{4}}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{1}{4}}-1\right)\cos(fx+e)^{3}+20\sqrt{2}\,df\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{1}{4}}+\sqrt{2}f\sqrt{-\frac{\sqrt{2}d^{2}f^{3}\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{3}{4}}\sin(fx+e)-d^{2}f^{2}\sqrt{\frac{1}{d^{2}f^{4}}}\sin(fx+e)-d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{1}{4}}+1\right)\cos(fx+e)^{3}+5\sqrt{2}\,df\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{1}{4}}\cos(fx+e)^{3}\log\left(\frac{\sqrt{2}d^{2}f^{3}\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{3}{4}}\sin(fx+e)+d^{2}f^{2}\sqrt{\frac{1}{d^{2}f^{4}}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}\right)-5\sqrt{2}\,df\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{1}{4}}\cos(fx+e)^{3}\log\left(-\frac{\sqrt{2}d^{2}f^{3}\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{2}f^{4}}\right)^{\frac{3}{4}}\sin(fx+e)-d^{2}f^{2}\sqrt{\frac{1}{d^{2}f^{4}}}\sin(fx+e)-d\cos(fx+e)}{\sin(fx+e)}\right)-8\left(6\cos(fx+e)^{2}-1\right)\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)}{20\,df\cos(fx+e)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*(20*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan(-sqrt(2)*f*sqrt(d*cos(f*x +
 e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4) + sqrt(2)*f*sqrt((sqrt(2)*d^2*f^3*sqr
t(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4)*sin(f*x + e) + d^2*f^2*s
qrt(1/(d^2*f^4))*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*(1/(d^2*f^4))
^(1/4) - 1)*cos(f*x + e)^3 + 20*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan(-sqr
t(2)*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4) + sqrt(2)*f*sq
rt(-(sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4)*
sin(f*x + e) - d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - d*cos(f*x + e))/sin
(f*x + e))*(1/(d^2*f^4))^(1/4) + 1)*cos(f*x + e)^3 + 5*sqrt(2)*d*f*(1/(d^2*
f^4))^(1/4)*cos(f*x + e)^3*log((sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x
 + e))*(1/(d^2*f^4))^(3/4)*sin(f*x + e) + d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x
 + e) + d*cos(f*x + e))/sin(f*x + e)) - 5*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*c
os(f*x + e)^3*log(-(sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d
```

```
^2*f^4))^(3/4)*sin(f*x + e) - d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - d*co
s(f*x + e))/sin(f*x + e)) - 8*(6*cos(f*x + e)^2 - 1)*sqrt(d*cos(f*x + e)/si
n(f*x + e))*sin(f*x + e))/(d*f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(d*cot(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^3/sqrt(d*cot(f*x + e)), x)
```

Mupad [B]
time = 2.57, size = 97, normalized size = 0.42

$$\frac{\frac{2d^2}{5}-\frac{2d^2}{\tan(e+fx)^2}}{f\left(\frac{d}{\tan(e+fx)}\right)^{5/2}}-\frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}+\frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(d*cot(e + f*x))^(1/2),x)
```

```
[Out] ((2*d^2)/5 - (2*d^2)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1/
4)*atan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f) + ((-1)^(
1/4)*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

3.208 $\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=212

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,\sqrt{d}\,f}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,\sqrt{d}\,f}+\frac{2d}{3f(d\cot(e+fx))^{3/2}}-\frac{\log\left(\right.}{}$$

```
[Out] 2/3*d/f/(d*cot(f*x+e))^(3/2)-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1
/2))/f*2^(1/2)/d^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f
*2^(1/2)/d^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(
1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*
x+e))^(1/2))/f*2^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,\sqrt{d}\,f}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,\sqrt{d}\,f}+\frac{2d}{3f(d\cot(e+fx))^{3/2}}-\frac{\log\left(\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,\sqrt{d}\,f}+\frac{\log\left(\sqrt{d}\,\cot(e+fx)+\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + (
2*d)/(3*f*(d*Cot[e + f*x])^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sq
rt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*C
ot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = d^2\int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} - \int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{(2d)\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\right.}{}$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}\,x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \text{Subs}$$

$$= \frac{2d}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}\,f} +$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} + \frac{}{3f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.10, size = 38, normalized size = 0.18

$$\frac{2d\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(e+fx)\right)}{3f(d\cot(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (2*d*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[e + f*x]^2])/(3*f*(d*Cot[e + f*x]
)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.38, size = 548, normalized size = 2.58

method	result
default	$\frac{(\cos(fx+e)+1)^2(\cos(fx+e)-1)\left(3i\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\ \mathrm{EllipticPi}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)*(3*I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),1/2-1/2*I,1/2*2^(1/2))*cos(f*x+e)-3*I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)-3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*cos(f*x+e)-3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*cos(f*x+e)+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))/sin(f*x+e)^3/(d*
cos(f*x+e)/sin(f*x+e))^(1/2)/cos(f*x+e)*2^(1/2)
```

Maxima [A]
time = 0.51, size = 197, normalized size = 0.93

$$\frac{d^3\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}\right)}{d^2}+\frac{8}{d^2\left(\frac{d}{\tan(fx+e)}\right)^{\frac{3}{2}}}\right)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*d^3*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
```

```
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2*(d/t
an(f*x + e))^(3/2)))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(168) = 336$.
time = 0.39, size = 615, normalized size = 2.90

$$-\frac{12\sqrt{2}\,df\left(\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{2}\,df^3\sqrt{\frac{d^2f^2\sqrt{\frac{1}{d^2f^4}}\sin(fx+e)+\sqrt{2}\,df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac{3}{4}}-1\right)\cos(fx+e)^2+12\sqrt{2}\,df\left(\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,df^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+\sqrt{2}\,df^3\sqrt{\frac{d^2f^2\sqrt{\frac{1}{d^2f^4}}\sin(fx+e)-\sqrt{2}\,df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac{3}{4}}+1\right)\cos(fx+e)^2-3\sqrt{2}\,df\left(\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\cos(fx+e)^2\log\left(\frac{d^2f^2\sqrt{\frac{1}{d^2f^4}}\sin(fx+e)+\sqrt{2}\,df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}\right)+3\sqrt{2}\,df\left(\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\cos(fx+e)^2\log\left(\frac{d^2f^2\sqrt{\frac{1}{d^2f^4}}\sin(fx+e)-\sqrt{2}\,df\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}\right)+8\left(\cos(fx+e)^2-1\right)\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}}{12\,df\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan(-sqrt(2)*d*f^3*sqrt(d*cos(
f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4) + sqrt(2)*d*f^3*sqrt((d^2*f^2*sq
rt(1/(d^2*f^4))*sin(f*x + e) + sqrt(2)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e)
)*(1/(d^2*f^4))^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*(1/(d^2*
f^4))^(3/4) - 1)*cos(f*x + e)^2 + 12*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan
(-sqrt(2)*d*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4) + sqr
t(2)*d*f^3*sqrt((d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - sqrt(2)*d*f*sqrt(
d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4)*sin(f*x + e) + d*cos(f*x +
 e))/sin(f*x + e))*(1/(d^2*f^4))^(3/4) + 1)*cos(f*x + e)^2 - 3*sqrt(2)*d*f*
(1/(d^2*f^4))^(1/4)*cos(f*x + e)^2*log((d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x +
 e) + sqrt(2)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4)*sin
(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) + 3*sqrt(2)*d*f*(1/(d^2*f^4))^(1/
4)*cos(f*x + e)^2*log((d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - sqrt(2)*d*f
*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4)*sin(f*x + e) + d*cos
(f*x + e))/sin(f*x + e)) + 8*(cos(f*x + e)^2 - 1)*sqrt(d*cos(f*x + e)/sin(f
*x + e)))/(d*f*cos(f*x + e)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(d*cot(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(d*cot(f*x + e)), x)
```

Mupad [B]
time = 2.51, size = 81, normalized size = 0.38

$$\frac{2\,d}{3\,f\,{\left(\frac{d}{\tan\left(e+f\,x\right)}\right)}^{3/2}}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)\,1{}\mathrm{i}}{\sqrt{d}\,f}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)\,1{}\mathrm{i}}{\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(d*cot(e + f*x))^(1/2),x)
```

```
[Out] (2*d)/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e
+ f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d/
tan(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f)
```

3.209 $\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=209

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}+\frac{2}{f\sqrt{d\cot(e+fx)}}+\frac{\log\left(\sqrt{\right.}}{}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)-1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/
2)+2/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\,f}+\frac{2}{f\sqrt{d\cot(e+fx)}}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}\,f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + 2
/(f*Sqrt[d*Cot[e + f*x]]) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sq
rt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f
*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = d\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d}$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx\right.}{}$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}\,x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{\text{Subst}\left(\right.}{}$$
$$= \frac{2}{f\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}\,f} - \frac{\log}{}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} + \frac{}{f\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 35, normalized size = 0.17

$$\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(e+fx)\right)}{f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[e + f*x]^2])/(f*Sqrt[d*Cot[e + f*x]
])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.38, size = 652, normalized size = 3.12

method	result
default	$\frac{(\cos(fx+e)+1)^2(\cos(fx+e)-1)\left(i\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\sin(fx+e)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)*(I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))+((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),1/2-1/2*I,1/2*2^(1/2))+((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+
1/2*I,1/2*2^(1/2))-2*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*si
n(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))/sin(f*x+e)^4/(d*cos(f*x+e)/sin(f*x+e))^
(1/2)*2^(1/2)
```

Maxima [A]
time = 0.54, size = 196, normalized size = 0.94

$$\frac{d^2\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}}{d^2}+\frac{8}{d^2\sqrt{\frac{d}{\tan(fx+e)}}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*d^2*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x
+ e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt
(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(
d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sqrt(d/
tan(f*x + e))))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(167) = 334$.
time = 0.40, size = 610, normalized size = 2.92

$$-\frac{4\sqrt{2}\,df\left(\frac{1}{d^2f^4}\right)^{\frac14}\arctan\left(-\sqrt{2}\,f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac14}+\sqrt{2}\,f\sqrt{\frac{\sqrt{2}\,d^2f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac34}\sin(fx+e)+d^2f^2\sqrt{\frac{1}{d^2f^4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac14}-1\right)\cos(fx+e)+4\sqrt{2}\,df\left(\frac{1}{d^2f^4}\right)^{\frac14}\arctan\left(-\sqrt{2}\,f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac14}+\sqrt{2}\,f\sqrt{-\frac{\sqrt{2}\,d^2f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac34}\sin(fx+e)-d^2f^2\sqrt{\frac{1}{d^2f^4}}\sin(fx+e)-d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac14}+1\right)\cos(fx+e)+\sqrt{2}\,df\left(\frac{1}{d^2f^4}\right)^{\frac14}\cos(fx+e)\log\left(\frac{\sqrt{2}\,d^2f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac34}\sin(fx+e)+d^2f^2\sqrt{\frac{1}{d^2f^4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}\right)-\sqrt{2}\,df\left(\frac{1}{d^2f^4}\right)^{\frac14}\cos(fx+e)\log\left(-\frac{\sqrt{2}\,d^2f^3\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^2f^4}\right)^{\frac34}\sin(fx+e)-d^2f^2\sqrt{\frac{1}{d^2f^4}}\sin(fx+e)-d\cos(fx+e)}{\sin(fx+e)}\right)-8\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\sin(fx+e)}{4\,df\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan(-sqrt(2)*f*sqrt(d*cos(f*x +
e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4) + sqrt(2)*f*sqrt((sqrt(2)*d^2*f^3*sqrt
(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4)*sin(f*x + e) + d^2*f^2*sq
rt(1/(d^2*f^4))*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*(1/(d^2*f^4))^
(1/4) - 1)*cos(f*x + e) + 4*sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*arctan(-sqrt(2)
*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(1/4) + sqrt(2)*f*sqrt(-
(sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4)*sin(
f*x + e) - d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - d*cos(f*x + e))/sin(f*x
 + e))*(1/(d^2*f^4))^(1/4) + 1)*cos(f*x + e) + sqrt(2)*d*f*(1/(d^2*f^4))^(1
/4)*cos(f*x + e)*log((sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/
(d^2*f^4))^(3/4)*sin(f*x + e) + d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) + d*
cos(f*x + e))/sin(f*x + e)) - sqrt(2)*d*f*(1/(d^2*f^4))^(1/4)*cos(f*x + e)*
log(-(sqrt(2)*d^2*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^2*f^4))^(3/4)
*sin(f*x + e) - d^2*f^2*sqrt(1/(d^2*f^4))*sin(f*x + e) - d*cos(f*x + e))/si
n(f*x + e)) - 8*sqrt(d*cos(f*x + e)/sin(f*x + e))*sin(f*x + e))/(d*f*cos(f*
x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(d*cot(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)/sqrt(d*cot(f*x + e)), x)
```

Mupad [B]
time = 0.19, size = 79, normalized size = 0.38

$$\frac{2}{f\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}+\frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{{\left(-1\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-1\right)}^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(d*cot(e + f*x))^(1/2),x)
```

```
[Out] 2/(f*(d/tan(e + f*x))^(1/2)) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e + f*x)
)^(1/2))/d^(1/2)))/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d/tan(e + f
*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

3.210 $\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=192

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}+\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)\right.}{2\sqrt{2}\sqrt{}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)-1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\,f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}\,f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) + Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*S
qrt[d]*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx = -\frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$
$$= -\frac{(2d)\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right.}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}\,x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}\,x+x^2}\,d\right.}{2f}$$
$$= \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}\,f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+\right.}{2\sqrt{}}$$
$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} + \frac{\log\left(\right.}{}$$

Mathematica [A]
time = 0.01, size = 131, normalized size = 0.68

$$\frac{\sqrt{\cot(e+fx)}\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+fx)\right)\right)}{2\sqrt{2}\,f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e
+ f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2]*f
*Sqrt[d*Cot[e + f*x]])
```

Maple [A]
time = 0.11, size = 138, normalized size = 0.72

method	result
derivativedivides	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4fd}$

default	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}\right.\right.}{4fd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/d*(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(
1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^
(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*
arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1))
```

Maxima [A]
time = 0.55, size = 171, normalized size = 0.89

$$-\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d
/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqrt(d)
*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d*cot(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(d*cot(f*x + e)), x)
```

Mupad [B]
time = 2.65, size = 57, normalized size = 0.30

$$\frac{(-1)^{1/4}\operatorname{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)1\mathrm{i}}{\sqrt{d}\;f}+\frac{(-1)^{1/4}\operatorname{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)1\mathrm{i}}{\sqrt{d}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cot(e + f*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*
f) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^
(1/2)*f)
```

3.211 $\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=192

$$\frac{\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,\sqrt{d}\,f}-\frac{\operatorname{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,\sqrt{d}\,f}-\frac{\log\left(\sqrt{d}+\sqrt{d}\,\cot(e+fx)-\right.}{2\sqrt{2}\,\sqrt{d}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)+1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2
)
```

Rubi [A]
time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {16, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,\sqrt{d}\,f}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,\sqrt{d}\,f}-\frac{\log\left(\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,\sqrt{d}\,f}+\frac{\log\left(\sqrt{d}\,\cot(e+fx)+\sqrt{2}\,\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*S
qrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}\,x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} - \frac{\text{Subst}\left(\int \frac{1}{d+\sqrt{2}\sqrt{d}\,x+x^2}\,dx\right.}{2f}$$

$$= -\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}\,f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+\right.}{2\surd}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} - \frac{\log\left(\surd\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 40, normalized size = 0.21

$$-\frac{2(d\cot(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)}{3d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] (-2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2])
/(3*d^2*f)
```

Maple [A]
time = 0.10, size = 135, normalized size = 0.70

method	result

derivativedivides	$-\frac{\sqrt{2}\,\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1 \right.\right.}{4f(d^2)^{\frac{1}{4}}}$
default	$-\frac{\sqrt{2}\,\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1 \right.\right.}{4f(d^2)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/
2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1
/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*ar
ctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1))
```

Maxima [A]
time = 0.52, size = 170, normalized size = 0.89

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e)
))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sq
rt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d/t
an(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/f
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)/sqrt(d*cot(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/sqrt(d*cot(f*x + e)), x)
```

Mupad [B]
time = 2.51, size = 58, normalized size = 0.30

$$\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(d*cot(e + f*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
 - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*
f)
```

3.212 $\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=212

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}-\frac{2\sqrt{d\cot(e+fx)}}{df}-\frac{\log\left(\sqrt{\right.}}{}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)/d^(1/2)-1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)+1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/f*2^(1/2)/d^(1/
2)-2*(d*cot(f*x+e))^(1/2)/d/f
```

Rubi [A]

time = 0.11, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\,f}-\frac{2\sqrt{d\cot(e+fx)}}{df}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}\,f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - (
2*Sqrt[d*Cot[e + f*x]])/(d*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2
]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e
 + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx &= \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d^2}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{df} - \int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{(2d)\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\right.}{}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{df} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}\,x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{\text{Subs}}{}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{df} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}\,f} + \frac{\mathrm{l}}{}\\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} - \frac{2\sqrt{}}{}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 159, normalized size = 0.75

$$-\frac{\sqrt{\cot(e+fx)}\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+8\sqrt{\cot(e+fx)}+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+fx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+fx)\right)\right)}{4f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -1/4*(Sqrt[Cot[e + f*x]]*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]
- 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 8*Sqrt[Cot[e + f*x]] +
```

```
Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]] - Sqrt[2]*Log[1
 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(f*Sqrt[d*Cot[e + f*x]])
```

Maple [A]

time = 0.11, size = 151, normalized size = 0.71

method	result
derivativedivides	$-\frac{2\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\sqrt{\right.\right.}{8}}{fd}$
default	$-\frac{2\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\sqrt{\right.\right.}{8}}{fd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d*((d*cot(f*x+e))^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^2
)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)
*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d
*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)
))
```

Maxima [A]

time = 0.51, size = 188, normalized size = 0.89

$$\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-8\sqrt{\frac{d}{\tan(fx+e)}}}{4\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x + e)))/
(d*f)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(d*cot(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(d*cot(f*x + e)), x)
```

Mupad [B]
time = 2.66, size = 77, normalized size = 0.36

$$-\frac{2\sqrt{d\cot(e+fx)}}{d\,f}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)1\mathrm{i}}{\sqrt{d}\,f}-\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)1\mathrm{i}}{\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(d*cot(e + f*x))^(1/2),x)
```

```
[Out] - (2*(d*cot(e + f*x))^(1/2))/(d*f) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e
+ f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*
cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(1/2)*f)
```

3.213 $\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$

Optimal. Leaf size=214

$$-\frac{\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}+\frac{\operatorname{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}-\frac{2(d\cot(e+fx))^{3/2}}{3d^2f}+\frac{\log\left(\sqrt{\cdots\right.}{\cdots}$$

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/d^2/f-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))/f*2^(1/2)/d^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2)
)/f*2^(1/2)/d^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e)
)^(1/2))/f*2^(1/2)/d^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot
(f*x+e))^(1/2))/f*2^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}f}+\frac{\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}f}-\frac{2(d\cot(e+fx))^{3/2}}{3d^2f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*f) - (
2*(d*Cot[e + f*x])^(3/2))/(3*d^2*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] -
Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f) - Log[Sqrt[d] + Sqrt[d]
*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*Sqrt[d]*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx = \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d^3}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\, \right.}{}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\text{Subst}\left(\int \frac{1}{d-\sqrt{2}\sqrt{d}\,x+x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2f} + \frac{\text{Subst}}{}$$

$$= -\frac{2(d\cot(e+fx))^{3/2}}{3d^2f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\sqrt{d}\,f} - \frac{\mathrm{l}}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\,f} - \frac{2(d}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 47, normalized size = 0.22

$$\frac{2\cot^2(e+fx)\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)\right)}{3f\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[d*Cot[e + f*x]],x]
```

[Out] (2*Cot[e + f*x]^2*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2]))/(
3*f*Sqrt[d*Cot[e + f*x]])

Maple [A]
time = 0.11, size = 156, normalized size = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f d^2}$
default	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e)}}{(d}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f d^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/f/d^2*(1/3*(d*cot(f*x+e))^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f
x+e)-(d^2)^(1/4)(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(
d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2
)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e)
)^(1/2)+1)))

Maxima [A]
time = 0.50, size = 194, normalized size = 0.91

$$\frac{3\,d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right)-8\left(\frac{d}{\tan(fx+e)}\right)^{\frac{3}{2}}}{12\,d^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="maxima")

[Out] 1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f*x +
e))^(3/2))/(d^2*f)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\sqrt{d\cot(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(d*cot(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(d*cot(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/sqrt(d*cot(f*x + e)), x)
```

Mupad [B]
time = 2.67, size = 76, normalized size = 0.36

$$\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}-\frac{2\,(d\cot(e+fx))^{3/2}}{3\,d^2\,f}-\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{d}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(d*cot(e + f*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
- (2*(d*cot(e + f*x))^(3/2))/(3*d^2*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*c
ot(e + f*x))^(1/2))/d^(1/2)))/(d^(1/2)*f)
```

3.214 $\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=232

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{2d}{5f(d\cot(e+fx))^{5/2}}-\frac{}{df\sqrt{d}}$$

```
[Out] 2/5*d/f/(d*cot(f*x+e))^(5/2)+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1
/2))/d^(3/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d
^(3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(
1/2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*
x+e))^(1/2))/d^(3/2)/f*2^(1/2)-2/d/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}+\frac{2d}{5f(d\cot(e+fx))^{5/2}}-\frac{2}{df\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + (2*d
)/(5*f*(d*Cot[e + f*x])^(5/2)) - 2/(d*f*Sqrt[d*Cot[e + f*x]]) - Log[Sqrt[d]
 + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*
f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*
Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = d^2\int \frac{1}{(d\cot(e+fx))^{7/2}}\,dx$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2}$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx\right.}{df}$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+}\right.}{df}$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+}\right.}{df}$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \right.}{2\sqrt{2}\,d^{3/2}f}$$
$$= \frac{2d}{5f(d\cot(e+fx))^{5/2}} - \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\right.}{2\sqrt{2}\,d^{3/2}f}$$
$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f} + \frac{}{5f(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.18, size = 38, normalized size = 0.16

$$\frac{2d\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\cot^2(e+fx)\right)}{5f(d\cot(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (2*d*Hypergeometric2F1[-5/4, 1, -1/4, -Cot[e + f*x]^2])/(5*f*(d*Cot[e + f*x
])^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.56, size = 728, normalized size = 3.14

method	result
default	$\frac{(\cos(fx+e)-1)\left(5i(\cos^2(fx+e))\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\,\sin(fx+e)\,\mathrm{E}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10/f*(cos(f*x+e)-1)*(5*I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(f*x+e)*((cos(f*x+e)-1)/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*cos(f*x+e)^2-5*I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(f*x+e)*((cos(f*x+e)-1)/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2+10*cos(f*x+e)^2*((cos(f*x+e)-1
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-5*cos(f*x+e)^2*((cos(f*x+e)-1)/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*cos(f*x+e)^2*((cos(f
*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-12*cos(f*x+e)
^3*2^(1/2)+12*cos(f*x+e)^2*2^(1/2)+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*(cos(f*x
+e)+1)^2/(d*cos(f*x+e)/sin(f*x+e))^(3/2)/sin(f*x+e)^5/cos(f*x+e)*2^(1/2)
```

Maxima [A]

time = 0.52, size = 215, normalized size = 0.93

$$-\frac{d^3\left(\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right)}{d^4}-\frac{8\left(d^2-\frac{5\,d^2}{\tan(fx+e)^2}\right)}{d^4\left(\frac{d}{\tan(fx+e)}\right)^{\frac{5}{2}}}\right)}{20\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/20*d^3*(5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*
sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*s
qrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^4 - 8*(d^2 - 5
*d^2/tan(f*x + e)^2)/(d^4*(d/tan(f*x + e))^(5/2)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(187) = 374$.
time = 0.39, size = 644, normalized size = 2.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/20*(20*sqrt(2)*d^2*f*(1/(d^6*f^4))^(1/4)*arctan(-sqrt(2)*d*f*sqrt(d*cos(f
*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(1/4) + sqrt(2)*d*f*sqrt((sqrt(2)*d^5*f
^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(3/4)*sin(f*x + e) + d^4
*f^2*sqrt(1/(d^6*f^4))*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*(1/(d^6
*f^4))^(1/4) - 1)*cos(f*x + e)^3 + 20*sqrt(2)*d^2*f*(1/(d^6*f^4))^(1/4)*arc
tan(-sqrt(2)*d*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(1/4) + sq
rt(2)*d*f*sqrt(-(sqrt(2)*d^5*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*
f^4))^(3/4)*sin(f*x + e) - d^4*f^2*sqrt(1/(d^6*f^4))*sin(f*x + e) - d*cos(f
*x + e))/sin(f*x + e))*(1/(d^6*f^4))^(1/4) + 1)*cos(f*x + e)^3 + 5*sqrt(2)*
d^2*f*(1/(d^6*f^4))^(1/4)*cos(f*x + e)^3*log((sqrt(2)*d^5*f^3*sqrt(d*cos(f*
x + e)/sin(f*x + e))*(1/(d^6*f^4))^(3/4)*sin(f*x + e) + d^4*f^2*sqrt(1/(d^6
*f^4))*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) - 5*sqrt(2)*d^2*f*(1/(d
^6*f^4))^(1/4)*cos(f*x + e)^3*log(-(sqrt(2)*d^5*f^3*sqrt(d*cos(f*x + e)/sin
```

```
(f*x + e))*(1/(d^6*f^4))^(3/4)*sin(f*x + e) - d^4*f^2*sqrt(1/(d^6*f^4))*sin
(f*x + e) - d*cos(f*x + e))/sin(f*x + e)) - 8*(6*cos(f*x + e)^2 - 1)*sqrt(d
*cos(f*x + e)/sin(f*x + e))*sin(f*x + e))/(d^2*f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(d*cot(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B]
time = 2.58, size = 93, normalized size = 0.40

$$\frac{\frac{2d}{5}-\frac{2d}{\tan(e+fx)^2}}{f\left(\frac{d}{\tan(e+fx)}\right)^{5/2}}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{d^{3/2}\,f}+\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{\frac{d}{\tan(e+fx)}}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((2*d)/5 - (2*d)/tan(e + f*x)^2)/(f*(d/tan(e + f*x))^(5/2)) - ((-1)^(1/4)*a
tan(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f) + ((-1)^(1/4)
*atanh(((-1)^(1/4)*(d/tan(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

3.215 $\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{2}{3f(d\cot(e+fx))^{3/2}}-\frac{\log\left(\right.}{}$$

```
[Out] 2/3/f/(d*cot(f*x+e))^(3/2)-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2
))/d^(3/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(
3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/
2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+
e))^(1/2))/d^(3/2)/f*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {16, 3555, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}+\frac{2}{3f(d\cot(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + 2
/(3*f*(d*Cot[e + f*x])^(3/2)) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2
]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e
 + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = d\int \frac{1}{(d\cot(e+fx))^{5/2}}\,dx$$
$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d}$$
$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$
$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$
$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x\right.}$$
$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,d^{3/2}f} - \text{Su}$$
$$= \frac{2}{3f(d\cot(e+fx))^{3/2}} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,d^{3/2}f} +$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f} + \frac{}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 37, normalized size = 0.18

$$\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\cot^2(e+fx)\right)}{3f(d\cot(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

[Out] (2*Hypergeometric2F1[-3/4, 1, 1/4, -Cot[e + f*x]^2])/(3*f*(d*Cot[e + f*x])^(3/2))

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.32, size = 540, normalized size = 2.56

method	result
default	$\frac{(\cos(fx+e)-1)\left(3i\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(cos(f*x+e)-1)*(3*I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*
2^(1/2))*cos(f*x+e)-3*I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^
(1/2))*cos(f*x+e)-3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*cos(f*x+e)-3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*c
os(f*x+e)+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*(cos(f*x+e)+1)^2/sin(f*x+e)^4/(d*
cos(f*x+e)/sin(f*x+e))^(3/2)*2^(1/2)
```

Maxima [A]
time = 0.50, size = 197, normalized size = 0.93

$$\frac{d^2\left(\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}\right)}{d^2}+\frac{8}{d^2\left(\frac{d}{\tan(fx+e)}\right)^{\frac{3}{2}}}\right)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*d^2*(3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/d^2 + 8/(d^2*(d/t
an(f*x + e))^(3/2)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(167) = 334$.
time = 0.39, size = 639, normalized size = 3.03

$$-\frac{12\sqrt{2}\,d^{2}f\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,d^{4}f^{3}\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{3}{4}}+\sqrt{2}\,d^{4}f^{3}\sqrt{\frac{d^{4}f^{2}\sqrt{\frac{1}{d^{6}f^{4}}}\sin(fx+e)+\sqrt{2}\,d^{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{3}{4}}-1\right)\cos(fx+e)^{2}+12\sqrt{2}\,d^{2}f\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}\,d^{4}f^{3}\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{3}{4}}+\sqrt{2}\,d^{4}f^{3}\sqrt{\frac{d^{4}f^{2}\sqrt{\frac{1}{d^{6}f^{4}}}\sin(fx+e)-\sqrt{2}\,d^{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{3}{4}}+1\right)\cos(fx+e)^{2}-3\sqrt{2}\,d^{2}f\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{1}{4}}\cos(fx+e)^{2}\log\left(\frac{d^{4}f^{2}\sqrt{\frac{1}{d^{6}f^{4}}}\sin(fx+e)+\sqrt{2}\,d^{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}\right)+3\sqrt{2}\,d^{2}f\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{1}{4}}\cos(fx+e)^{2}\log\left(\frac{d^{4}f^{2}\sqrt{\frac{1}{d^{6}f^{4}}}\sin(fx+e)-\sqrt{2}\,d^{2}f\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}\left(\frac{1}{d^{6}f^{4}}\right)^{\frac{1}{4}}\sin(fx+e)+d\cos(fx+e)}{\sin(fx+e)}\right)+8\left(\cos(fx+e)^{2}-1\right)\sqrt{\frac{d\cos(fx+e)}{\sin(fx+e)}}}{12\,d^{2}f\cos(fx+e)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(2)*d^2*f*(1/(d^6*f^4))^(1/4)*arctan(-sqrt(2)*d^4*f^3*sqrt(d*
cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(3/4) + sqrt(2)*d^4*f^3*sqrt((d^4*
f^2*sqrt(1/(d^6*f^4))*sin(f*x + e) + sqrt(2)*d^2*f*sqrt(d*cos(f*x + e)/sin(
f*x + e))*(1/(d^6*f^4))^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e))*
(1/(d^6*f^4))^(3/4) - 1)*cos(f*x + e)^2 + 12*sqrt(2)*d^2*f*(1/(d^6*f^4))^(1
/4)*arctan(-sqrt(2)*d^4*f^3*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))
^(3/4) + sqrt(2)*d^4*f^3*sqrt((d^4*f^2*sqrt(1/(d^6*f^4))*sin(f*x + e) - sqr
t(2)*d^2*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(1/4)*sin(f*x +
e) + d*cos(f*x + e))/sin(f*x + e))*(1/(d^6*f^4))^(3/4) + 1)*cos(f*x + e)^2
- 3*sqrt(2)*d^2*f*(1/(d^6*f^4))^(1/4)*cos(f*x + e)^2*log((d^4*f^2*sqrt(1/(d
^6*f^4))*sin(f*x + e) + sqrt(2)*d^2*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/
(d^6*f^4))^(1/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) + 3*sqrt(2)*d
^2*f*(1/(d^6*f^4))^(1/4)*cos(f*x + e)^2*log((d^4*f^2*sqrt(1/(d^6*f^4))*sin(
f*x + e) - sqrt(2)*d^2*f*sqrt(d*cos(f*x + e)/sin(f*x + e))*(1/(d^6*f^4))^(1
/4)*sin(f*x + e) + d*cos(f*x + e))/sin(f*x + e)) + 8*(cos(f*x + e)^2 - 1)*s
qrt(d*cos(f*x + e)/sin(f*x + e)))/(d^2*f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)/(d*cot(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B]
time = 0.20, size = 80, normalized size = 0.38

$$\frac{2}{3\,f\,\left(\frac{d}{\tan(e+f\,x)}\right)^{3/2}}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}-\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{\frac{d}{\tan\left(e+f\,x\right)}}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] 2/(3*f*(d/tan(e + f*x))^(3/2)) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d/tan(e + f*
x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d/tan(
e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f)
```

3.216 $\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{2}{df\sqrt{d\cot(e+fx)}}+\frac{\log\left(\sqrt{\right.}}{}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)-1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/
2)+2/d/f/(d*cot(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}+\frac{2}{df\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cot[e + f*x])^(-3/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + 2
/(d*f*Sqrt[d*Cot[e + f*x]]) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*
Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e +
 f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d\cot(e+fx))^{3/2}}\,dx &= \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2}\\
&= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{df}\\
&= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}\\
&= \frac{2}{df\sqrt{d\cot(e+fx)}} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\right.}{}\\
&= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Sub}}{}\\
&= \frac{2}{df\sqrt{d\cot(e+fx)}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{}{d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 38, normalized size = 0.18

$$\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\cot^2(e+fx)\right)}{df\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cot[e + f*x])^(-3/2),x]
```

```
[Out] (2*Hypergeometric2F1[-1/4, 1, 3/4, -Cot[e + f*x]^2])/(d*f*Sqrt[d*Cot[e + f*
x]])
```

Maple [A]
time = 0.06, size = 157, normalized size = 0.74

method	result

derivativedivides	$-\frac{2d\left(-\frac{1}{d^2\sqrt{d\cot(fx+e)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{}}{}\right.\right.}{8d^2(d^2)}\right.}{f}$
default	$-\frac{2d\left(-\frac{1}{d^2\sqrt{d\cot(fx+e)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{}}{}\right.\right.}{8d^2(d^2)}\right.}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*d*(-1/d^2/(d*cot(f*x+e))^(1/2)-1/8/d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(
f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+
(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^
2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e
))^(1/2)+1)))
```

Maxima [A]
time = 0.50, size = 194, normalized size = 0.92

$$\frac{d\left(\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}}{d^2}+\frac{8}{d^2\sqrt{\frac{d}{\tan(fx+e)}}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*d*((2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x +
e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*
sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d
/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(d)
*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/d^2 + 8/(d^2*sqrt(d/ta
n(f*x + e))))/f
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*cot(e + f*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cot(f*x + e))^(-3/2), x)
```

Mupad [B]
time = 2.61, size = 76, normalized size = 0.36

$$\frac{2}{d\,f\,\sqrt{d\cot(e+f\,x)}}+\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f}-\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] 2/(d*f*(d*cot(e + f*x))^(1/2)) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*
x))^(1/2))/d^(1/2)))/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e +
 f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

3.217 $\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=192

$$\frac{\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\right.}{2\sqrt{2}\,d^{3/2}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)-1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2
)
```

Rubi [A]

time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {16, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d
^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}$$

$$= -\frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right.}{df}$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-x^2}\,d\right.}{2\sqrt{2}d^{3}}$$

$$= \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+\right.}{2\sqrt{}}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\log\left(\right.}{}$$

Mathematica [A]
time = 0.03, size = 134, normalized size = 0.70

$$\frac{\sqrt{\cot(e+fx)}\left(2\text{ArcTan}\left(1-\sqrt{2}\sqrt{\cot(e+fx)}\right)-2\text{ArcTan}\left(1+\sqrt{2}\sqrt{\cot(e+fx)}\right)+\log\left(1-\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+fx)\right)-\log\left(1+\sqrt{2}\sqrt{\cot(e+fx)}+\cot(e+fx)\right)\right)}{2\sqrt{2}df\sqrt{d\cot(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[Cot[e + f*x]]*(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 2*ArcTan[1
+ Sqrt[2]*Sqrt[Cot[e + f*x]]] + Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e
+ f*x]] - Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2*Sqrt[2]*d
*f*Sqrt[d*Cot[e + f*x]])
```

Maple [A]
time = 0.08, size = 138, normalized size = 0.72

method	result

derivativedivides	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e}{(d^2)^{\frac{1}{4}}}\right.\right.}{4f\,d^2}$
default	$-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot(fx+e}{(d^2)^{\frac{1}{4}}}\right.\right.}{4f\,d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f*(d^2)^(1/4)/d^2*2^(1/2)*(ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))
^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*
2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-
2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1))
```

Maxima [A]
time = 0.51, size = 170, normalized size = 0.89

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{d^{\frac{3}{2}}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e)
))/sqrt(d))/d^(3/2) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sq
rt(d/tan(f*x + e)))/sqrt(d))/d^(3/2) + sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d/t
an(f*x + e)) + d + d/tan(f*x + e))/d^(3/2) - sqrt(2)*log(-sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/d^(3/2))/f
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)/(d*cot(e + f*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)/(d*cot(f*x + e))^(3/2), x)
```

Mupad [**B**]
time = 2.58, size = 57, normalized size = 0.30

$$\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot\left(e+f\,x\right)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}+\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot\left(e+f\,x\right)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*
f) + ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^
(3/2)*f)
```

3.218 $\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=192

$$\frac{\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\operatorname{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)\right.}{2\sqrt{2}\,d^{3/}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^(1/
2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)+1/4*l
n(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2
)
```

Rubi [A]

time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {16, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\operatorname{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Log[
Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d
^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x
]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx = \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{df}$$

$$= -\frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}$$

$$= \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} - \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right.}{df}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}-2x}{-d+\sqrt{2}\sqrt{d}x-}\right.}{2\sqrt{}}$$

$$= -\frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(\right.}{}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{\log}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 40, normalized size = 0.21

$$-\frac{2(d\cot(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)}{3d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (-2*(d*Cot[e + f*x])^(3/2)*Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2])
/(3*d^3*f)
```

Maple [A]
time = 0.13, size = 138, normalized size = 0.72

method	result

derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)}{4fd(d^2)^{\frac{1}{4}}}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\,\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{d\cot(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)}{4fd(d^2)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/f/d/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(
1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^
(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*
arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1))
```

Maxima [A]
time = 0.50, size = 173, normalized size = 0.90

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}}{4\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f*x + e)
))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sq
rt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*sqrt(d/t
an(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d))/(d*f)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(d*cot(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B]
time = 2.48, size = 58, normalized size = 0.30

$$\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+f\,x)}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
 - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*
f)
```

3.219 $\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{2\sqrt{d\cot(e+fx)}}{d^2f}-\frac{\log\left(\sqrt{d}\right.}{}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^(1
/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)+1/4*
ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot(f*x+e))^(1/2))/d^(3/2)/f*2^(1/
2)-2*(d*cot(f*x+e))^(1/2)/d^2/f
```

Rubi [A]
time = 0.12, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}-\frac{2\sqrt{d\cot(e+fx)}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - (
2*Sqrt[d*Cot[e + f*x]])/(d^2*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt
[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) + Log[Sqrt[d] + Sqrt[d]*Cot
[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx &= \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d^3}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} - \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+fx)\right)}{f}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} + \frac{2\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{f}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} + \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\, \cdots\right.}{\cdots}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,d^{3/2} f} - \frac{\text{Sub}\cdots}{\cdots}\\
&= -\frac{2\sqrt{d\cot(e+fx)}}{d^2 f} - \frac{\log\left(\sqrt{d}+\sqrt{d}\,\cot(e+fx)-\sqrt{2}\,\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}\,d^{3/2} f} + \frac{\text{l}\cdots}{\cdots}\\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2} f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2} f} - \frac{2\sqrt{\cdots}}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 159, normalized size = 0.75

$$
-\frac{\cot^{\frac{3}{2}}(e+fx)\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}\right)+8\sqrt{\cot(e+fx)}+\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)-\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)\right)}{4f(d\cot(e+fx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -1/4*(Cot[e + f*x]^(3/2)*(2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]]
- 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 8*Sqrt[Cot[e + f*x]] +
```

```
Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]] - Sqrt[2]*Log[1
 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(f*(d*Cot[e + f*x])^(3/2))
```

Maple [A]
time = 0.10, size = 151, normalized size = 0.71

method	result
derivativedivides	$-\frac{2\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\right.\right.}{8}\right.}{f\,d^2}$
default	$-\frac{2\left(\sqrt{d\cot(fx+e)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\right.\right.}{8}\right.}{f\,d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d^2*((d*cot(f*x+e))^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d
^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/
4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*
(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+
1)))
```

Maxima [A]
time = 0.51, size = 188, normalized size = 0.89

$$\frac{2\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\sqrt{d}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-8\sqrt{\frac{d}{\tan(fx+e)}}}{4d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d)) + 2*sqrt(2)*sqrt(d)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + sqrt(2)*sqrt(d)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - sqrt(2)*sqrt(d)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 8*sqrt(d/tan(f*x + e)))/
(d^2*f)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e + fx)}{(d\cot(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(d*cot(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^3/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B]
time = 2.62, size = 77, normalized size = 0.36

$$-\frac{2\sqrt{d\cot(e+fx)}}{d^2 f}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}-\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] - (2*(d*cot(e + f*x))^(1/2))/(d^2*f) - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(
e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(
d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)*f)
```

3.220 $\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{2(d\cot(e+fx))^{3/2}}{3d^3f}+\frac{\log\left(\sqrt{\right.}}{}$$

```
[Out] -2/3*(d*cot(f*x+e))^(3/2)/d^3/f-1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))/d^(3/2)/f*2^(1/2)+1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2)
)/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e)
)^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot
(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}-\frac{2(d\cot(e+fx))^{3/2}}{3d^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f)) +
 ArcTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - (
2*(d*Cot[e + f*x])^(3/2))/(3*d^3*f) + Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] -
Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f) - Log[Sqrt[d] + Sqrt[d]
*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx &= \frac{\int (d\cot(e+fx))^{5/2}\,dx}{d^4}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} - \frac{\int \sqrt{d\cot(e+fx)}\,dx}{d^2}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\cot(e+fx)\right)}{df}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{2\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{df} + \frac{\text{Subst}\left(\int \frac{d+x}{d^2+x}\right.}{}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} + \frac{\text{S}}{}\\
&= -\frac{2(d\cot(e+fx))^{3/2}}{3d^3f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}\right)}{2\sqrt{2}d^{3/2}f} -\\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}d^{3/2}f} - \frac{2}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.09, size = 47, normalized size = 0.22

$$\frac{2\cot^3(e+fx)\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\cot^2(e+fx)\right)\right)}{3f(d\cot(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(d*Cot[e + f*x])^(3/2),x]
[Out] (2*Cot[e + f*x]^3*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Cot[e + f*x]^2]))/(
3*f*(d*Cot[e + f*x])^(3/2))
```

Maple [A]
time = 0.09, size = 156, normalized size = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^{2}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^{2}}}{d\cot(fx+e)+(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot}}{(d^{2})}\right.\right.}{8(d^{2})^{\frac{1}{4}}}\right.}{f d^{3}}$
default	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{3}{2}}}{3}-\frac{d^{2}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)-(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^{2}}}{d\cot(fx+e)+(d^{2})^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}+\sqrt{d^{2}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\cot}}{(d^{2})}\right.\right.}{8(d^{2})^{\frac{1}{4}}}\right.}{f d^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d^3*(1/3*(d*cot(f*x+e))^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*cot(f
*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*cot(f*x+e)+(
d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2
)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e)
)^(1/2)+1)))
```

Maxima [A]
time = 0.52, size = 194, normalized size = 0.91

$$\frac{3d^{2}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)}{\sqrt{d}}\right)-8\left(\frac{d}{\tan(fx+e)}\right)^{\frac{3}{2}}}{12d^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/tan(f
*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d/tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(sqrt(2)*sqrt(d)*s
qrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d) + sqrt(2)*log(-sqrt(2)*sq
rt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e))/sqrt(d)) - 8*(d/tan(f*x +
e))^(3/2))/(d^3*f)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{4}\left(e + fx\right)}{\left(d\cot\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(d*cot(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B]
time = 2.65, size = 76, normalized size = 0.36

$$\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot\left(e+f\,x\right)}}{\sqrt{d}}\right)}{d^{3/2}\,f}-\frac{2\,{\left(d\cot(e+f\,x)\right)}^{3/2}}{3\,d^3\,f}-\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot\left(e+f\,x\right)}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
- (2*(d*cot(e + f*x))^(3/2))/(3*d^3*f) - ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*c
ot(e + f*x))^(1/2))/d^(1/2)))/(d^(3/2)*f)
```

3.221 $\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=234

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}+\frac{2\sqrt{d\cot(e+fx)}}{d^2f}-\frac{2(d\cot(e+}{5d^4}$$

```
[Out] -2/5*(d*cot(f*x+e))^(5/2)/d^4/f+1/2*arctan(1-2^(1/2)*(d*cot(f*x+e))^(1/2)/d
^(1/2))/d^(3/2)/f*2^(1/2)-1/2*arctan(1+2^(1/2)*(d*cot(f*x+e))^(1/2)/d^(1/2)
)/d^(3/2)/f*2^(1/2)+1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)-2^(1/2)*(d*cot(f*x+e)
)^(1/2))/d^(3/2)/f*2^(1/2)-1/4*ln(d^(1/2)+cot(f*x+e)*d^(1/2)+2^(1/2)*(d*cot
(f*x+e))^(1/2))/d^(3/2)/f*2^(1/2)+2*(d*cot(f*x+e))^(1/2)/d^2/f
```

Rubi [A]
time = 0.14, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {16, 3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,d^{3/2}f}+\frac{\log\left(\sqrt{d}\cot(e+fx)-\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}-\frac{\log\left(\sqrt{d}\cot(e+fx)+\sqrt{2}\sqrt{d\cot(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,d^{3/2}f}-\frac{2(d\cot(e+fx))^{5/2}}{5d^4f}+\frac{2\sqrt{d\cot(e+fx)}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Cot[e + f*x]])/Sqrt[d]]/(Sqrt[2]*d^(3/2)*f) + (2*S
qrt[d*Cot[e + f*x]])/(d^2*f) - (2*(d*Cot[e + f*x])^(5/2))/(5*d^4*f) + Log[S
qrt[d] + Sqrt[d]*Cot[e + f*x] - Sqrt[2]*Sqrt[d*Cot[e + f*x]]]/(2*Sqrt[2]*d^
(3/2)*f) - Log[Sqrt[d] + Sqrt[d]*Cot[e + f*x] + Sqrt[2]*Sqrt[d*Cot[e + f*x]
]]/(2*Sqrt[2]*d^(3/2)*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{3/2}}\,dx &= \frac{\int (d\cot(e+fx))^{7/2}\,dx}{d^5} \\
&= -\frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{\int (d\cot(e+fx))^{3/2}\,dx}{d^3} \\
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} + \frac{\int \frac{1}{\sqrt{d\cot(e+fx)}}\,dx}{d} \\
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\cot(e+f\right.}{f} \\
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{2\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right.}{f} \\
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} - \frac{\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\cot(e+fx)}\right.}{df} \\
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\,c}\right.}{2\sqrt{2}\,d^{3/2}f} \\
&= \frac{2\sqrt{d\cot(e+fx)}}{d^2f} - \frac{2(d\cot(e+fx))^{5/2}}{5d^4f} + \frac{\log\left(\sqrt{d}+\sqrt{d}\cot(e+fx)-\sqrt{2}\,\sqrt{\right.}}{2\sqrt{2}\,d^{3/2}f} \\
&= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,d^{3/2}f} + 2\sqrt{d}
\end{aligned}$$

Mathematica [A]
time = 0.35, size = 172, normalized size = 0.74

$$\frac{\cot^{\frac{3}{2}}(e+fx)\left(10\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}\right)-10\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}\right)+40\sqrt{\cot(e+fx)}-8\cot^{\frac{5}{2}}(e+fx)+5\sqrt{2}\,\log\left(1-\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)-5\sqrt{2}\,\log\left(1+\sqrt{2}\,\sqrt{\cot(e+fx)}+\cot(e+fx)\right)\right)}{20f(d\cot(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(d*Cot[e + f*x])^(3/2),x]
```

```
[Out] (Cot[e + f*x]^(3/2)*(10*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Cot[e + f*x]]] - 10
*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Cot[e + f*x]]] + 40*Sqrt[Cot[e + f*x]] - 8
*Cot[e + f*x]^(5/2) + 5*Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e
+ f*x]] - 5*Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Cot[e + f*x]] + Cot[e + f*x]]))/(2
0*f*(d*Cot[e + f*x])^(3/2))
```

Maple [A]
time = 0.09, size = 171, normalized size = 0.73

method	result
derivativedivides	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^2\sqrt{d\cot(fx+e)}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}}\right.\right.}{}\right)}{f\,d^4}$
default	$-\frac{2\left(\frac{(d\cot(fx+e))^{\frac{5}{2}}}{5}-d^2\sqrt{d\cot(fx+e)}+\frac{d^2(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\cot(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}}{d\cot(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\cot(fx+e)}\sqrt{2}}\right.\right.}{}\right)}{f\,d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f/d^4*(1/5*(d*cot(f*x+e))^(5/2)-d^2*(d*cot(f*x+e))^(1/2)+1/8*d^2*(d^2)^(
1/4)*2^(1/2)*(ln((d*cot(f*x+e)+(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^
2)^(1/2))/(d*cot(f*x+e)-(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2
)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/
(d^2)^(1/4)*(d*cot(f*x+e))^(1/2)+1)))
```

Maxima [A]
time = 0.51, size = 207, normalized size = 0.88

$$-\frac{10\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+10\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{\frac{d}{\tan(fx+e)}}\right)}{2\sqrt{d}}\right)+5\sqrt{2}\,d^{\frac{5}{2}}\log\left(\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-5\sqrt{2}\,d^{\frac{5}{2}}\log\left(-\sqrt{2}\sqrt{d}\sqrt{\frac{d}{\tan(fx+e)}}+d+\frac{d}{\tan(fx+e)}\right)-40\,d^2\sqrt{\frac{d}{\tan(fx+e)}}+8\left(\frac{d}{\tan(fx+e)}\right)^{\frac{5}{2}}}{20\,d^4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/20*(10*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d/ta
n(f*x + e)))/sqrt(d)) + 10*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqr
t(d) - 2*sqrt(d/tan(f*x + e)))/sqrt(d)) + 5*sqrt(2)*d^(5/2)*log(sqrt(2)*sqr
t(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 5*sqrt(2)*d^(5/2)*log(-sq
rt(2)*sqrt(d)*sqrt(d/tan(f*x + e)) + d + d/tan(f*x + e)) - 40*d^2*sqrt(d/ta
n(f*x + e)) + 8*(d/tan(f*x + e))^(5/2))/(d^4*f)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{(d\cot(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(d*cot(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(d*cot(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(d*cot(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^5/(d*cot(f*x + e))^(3/2), x)
```

Mupad [B]
time = 2.98, size = 93, normalized size = 0.40

$$\frac{2\sqrt{d\cot(e+fx)}}{d^2 f}-\frac{2\,(d\cot(e+fx))^{5/2}}{5\,d^4 f}+\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+fx)}}{\sqrt{d}}\right)\,1\mathrm{i}}{d^{3/2}\,f}+\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\cot(e+fx)}\,1\mathrm{i}}{\sqrt{d}}\right)}{d^{3/2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(d*cot(e + f*x))^(3/2),x)
```

```
[Out] (2*(d*cot(e + f*x))^(1/2))/(d^2*f) - (2*(d*cot(e + f*x))^(5/2))/(5*d^4*f) +
 ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2))/d^(1/2))*1i)/(d^(3/2)
*f) + ((-1)^(1/4)*atan(((-1)^(1/4)*(d*cot(e + f*x))^(1/2)*1i)/d^(1/2)))/(d^
(3/2)*f)
```

3.222 $\int \cot^m(e+fx)\tan^n(e+fx)\,dx$

Optimal. Leaf size=62

$$\frac{\cot^m(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

```
[Out] cot(f*x+e)^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+e)^2
)*tan(f*x+e)^(1+n)/f/(1-m+n)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2684, 3557, 371}

$$\frac{\cot^m(e+fx)\tan^{n+1}(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^m*Tan[e + f*x]^n,x]
```

```
[Out] (Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e +
 f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2684

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \cot^m(e+fx)\tan^n(e+fx)\,dx &= (\cot^m(e+fx)\tan^m(e+fx))\int \tan^{-m+n}(e+fx)\,dx\\
&= \frac{(\cot^m(e+fx)\tan^m(e+fx))\,\text{Subst}\Big(\int \frac{x^{-m+n}}{1+x^2}\,dx, x, \tan(e+fx)\Big)}{f}\\
&= \frac{\cot^m(e+fx)\,{}_2F_1\big(1, \frac{1}{2}(1-m+n); \frac{1}{2}(3-m+n); -\tan^2(e+fx)\big)\tan^{1+n}(e+fx)}{f(1-m+n)}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 62, normalized size = 1.00

$$
\frac{\cot^m(e+fx)\,{}_2F_1\big(1, \frac{1}{2}(1-m+n); \frac{1}{2}(3-m+n); -\tan^2(e+fx)\big)\tan^{1+n}(e+fx)}{f(1-m+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^m*Tan[e + f*x]^n,x]
[Out] (Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e +
 f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$
\int \left(\cot^m\left(fx+e\right)\right)\left(\tan^n\left(fx+e\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^m*tan(f*x+e)^n,x)
[Out] int(cot(f*x+e)^m*tan(f*x+e)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="maxima")
[Out] integrate(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="fricas")
```

```
[Out] integral(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^{n}\left(e+fx\right)\cot^{m}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**m*tan(f*x+e)**n,x)
```

```
[Out] Integral(tan(e + f*x)**n*cot(e + f*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*tan(f*x+e)^n,x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^m*tan(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\mathrm{cot}\left(e+f\,x\right)}^m\,{\mathrm{tan}\left(e+f\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^m*tan(e + f*x)^n,x)
```

```
[Out] int(cot(e + f*x)^m*tan(e + f*x)^n, x)
```

3.223 $\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=67

$$\frac{\cot^m(e+fx)\,{}_2F_1\left(1,\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);-\tan^2(e+fx)\right)(b\tan(e+fx))^{1+n}}{bf(1-m+n)}$$

```
[Out] cot(f*x+e)^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+e)^2
)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2684, 3557, 371}

$$\frac{\cot^m(e+fx)(b\tan(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{bf(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Cot[e + f*x]^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan[e +
 f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2684

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \cot^m(e+fx)(b\tan(e+fx))^n\,dx = (\cot^m(e+fx)(b\tan(e+fx))^m)\int (b\tan(e+fx))^{-m+n}\,dx$$

$$= \frac{(b\cot^m(e+fx)(b\tan(e+fx))^m)\,\text{Subst}\Big(\int \frac{x^{-m+n}}{b^2+x^2}\,dx, x, b\tan(e+fx)}{f}$$

$$= \frac{\cot^m(e+fx)\,{}_2F_1\big(1, \frac{1}{2}(1-m+n); \frac{1}{2}(3-m+n); -\tan^2(e+fx)\big)\,(b}{bf(1-m+n)}$$

Mathematica [A]
time = 0.08, size = 64, normalized size = 0.96

$$\frac{\cot^{-1+m}(e+fx)\,{}_2F_1\big(1, \frac{1}{2}(1-m+n); \frac{1}{2}(3-m+n); -\tan^2(e+fx)\big)\,(b\tan(e+fx))^n}{f(1-m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (Cot[e + f*x]^(-1 + m)*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -
Tan[e + f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - m + n))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (\cot^m(fx+e))\,(b\tan(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int(cot(f*x+e)^m*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\tan{(e+fx)})^n \cot^m{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((b*tan(e + f*x))**n*cot(e + f*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^n*cot(f*x + e)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(e+f\,x)^m\,(b\tan(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^m*(b*tan(e + f*x))^n,x)
```

```
[Out] int(cot(e + f*x)^m*(b*tan(e + f*x))^n, x)
```

3.224 $\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx$

Optimal. Leaf size=64

$$\frac{(a\cot(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

```
[Out] (a*cot(f*x+e))^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+
e)^2)*tan(f*x+e)^(1+n)/f/(1-m+n)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2684, 3557, 371}

$$\frac{\tan^{n+1}(e+fx)(a\cot(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{f(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[e + f*x])^m*Tan[e + f*x]^n,x]
```

```
[Out] ((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan
[e + f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2684

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\int (a\cot(e+fx))^m \tan^n(e+fx)\,dx = ((a\cot(e+fx))^m \tan^m(e+fx))\int \tan^{-m+n}(e+fx)\,dx$$
$$= \frac{((a\cot(e+fx))^m \tan^m(e+fx))\,\text{Subst}\left(\int \frac{x^{-m+n}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{f}$$
$$= \frac{(a\cot(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);-\tan^2(e+fx)\right)}{f(1-m+n)}$$

Mathematica [A]
time = 0.07, size = 64, normalized size = 1.00

$$\frac{(a\cot(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);-\tan^2(e+fx)\right)\tan^{1+n}(e+fx)}{f(1-m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[e + f*x])^m*Tan[e + f*x]^n,x]
[Out] ((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan
[e + f*x]^2]*Tan[e + f*x]^(1 + n))/(f*(1 - m + n))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int (a\cot(fx+e))^m (\tan^n(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(f*x+e))^m*tan(f*x+e)^n,x)
[Out] int((a*cot(f*x+e))^m*tan(f*x+e)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="maxima")
[Out] integrate((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="fricas")
```

```
[Out] integral((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cot{(e+fx)})^m \tan^n{(e+fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))**m*tan(f*x+e)**n,x)
```

```
[Out] Integral((a*cot(e + f*x))**m*tan(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*tan(f*x+e)^n,x, algorithm="giac")
```

```
[Out] integrate((a*cot(f*x + e))^m*tan(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^n\,(a\,\cot(e+f\,x))^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^n*(a*cot(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)^n*(a*cot(e + f*x))^m, x)
```

3.225 $\int (a\cot(e+fx))^m (b\tan(e+fx))^n \, dx$

Optimal. Leaf size=69

$$\frac{(a\cot(e+fx))^m \, {}_2F_1\left(1,\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);-\tan^2(e+fx)\right) (b\tan(e+fx))^{1+n}}{bf(1-m+n)}$$

```
[Out] (a*cot(f*x+e))^m*hypergeom([1, 1/2-1/2*m+1/2*n],[3/2-1/2*m+1/2*n],-tan(f*x+
e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2684, 3557, 371}

$$\frac{(a\cot(e+fx))^m (b\tan(e+fx))^{n+1} \, {}_2F_1\left(1,\frac{1}{2}(-m+n+1);\frac{1}{2}(-m+n+3);-\tan^2(e+fx)\right)}{bf(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((a*Cot[e + f*x])^m*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n)/2, -Tan
[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/(b*f*(1 - m + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2684

```
Int[(cot[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^m, Int[(b*Tan[e +
 f*x])^(n - m), x], x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (a\cot(e+fx))^m(b\tan(e+fx))^n\,dx = ((a\cot(e+fx))^m(b\tan(e+fx))^m)\int (b\tan(e+fx))^{-m+n}\,dx$$
$$= \frac{(b(a\cot(e+fx))^m(b\tan(e+fx))^m)\,\text{Subst}\left(\int \frac{x^{-m+n}}{b^2+x^2}\,dx, x, b\tan(e$$
$$= \frac{(a\cot(e+fx))^m\,{}_2F_1\left(1,\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);-\tan^2(e+$$

Mathematica [A]
time = 0.11, size = 67, normalized size = 0.97

$$\frac{a(a\cot(e+fx))^{-1+m}\,{}_2F_1\left(1,\frac{1}{2}(1-m+n);\frac{1}{2}(3-m+n);-\tan^2(e+fx)\right)(b\tan(e+fx))^n}{f(1-m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cot[e + f*x])^m*(b*Tan[e + f*x])^n,x]
[Out] (a*(a*Cot[e + f*x])^(-1 + m)*Hypergeometric2F1[1, (1 - m + n)/2, (3 - m + n
)/2, -Tan[e + f*x]^2]*(b*Tan[e + f*x])^n)/(f*(1 - m + n))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int (a\cot(fx+e))^m\,(b\tan(fx+e))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x)
[Out] int((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cot(e+fx))^m (b\tan(e+fx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((a*cot(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cot(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cot(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\cot(e+f\,x))^m\,(b\tan(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cot(e + f*x))^m*(b*tan(e + f*x))^n,x)
```

```
[Out] int((a*cot(e + f*x))^m*(b*tan(e + f*x))^n, x)
```

3.226 $\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=67

$$\frac{2(d\tan(e+fx))^{3/2}}{3df}+\frac{4(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{11/2}}{11d^5f}$$

```
[Out] 2/3*(d*tan(f*x+e))^(3/2)/d/f+4/7*(d*tan(f*x+e))^(7/2)/d^3/f+2/11*(d*tan(f*x
+e))^(11/2)/d^5/f
```

Rubi [A]

time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2687, 276}

$$\frac{2(d\tan(e+fx))^{11/2}}{11d^5f}+\frac{4(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f) + (4*(d*Tan[e + f*x])^(7/2))/(7*d^3*f) +
 (2*(d*Tan[e + f*x])^(11/2))/(11*d^5*f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}
\int \sec^6(e+fx)\sqrt{d\tan(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{dx}\,(1+x^2)^2\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\sqrt{dx}+\frac{2(dx)^{5/2}}{d^2}+\frac{(dx)^{9/2}}{d^4}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{2(d\tan(e+fx))^{3/2}}{3df}+\frac{4(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{11/2}}{11d^5f}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 52, normalized size = 0.78

$$\frac{2(45+28\cos(2(e+fx))+4\cos(4(e+fx)))\sec^4(e+fx)(d\tan(e+fx))^{3/2}}{231df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(45 + 28*Cos[2*(e + f*x)] + 4*Cos[4*(e + f*x)])*Sec[e + f*x]^4*(d*Tan[e
+ f*x])^(3/2))/(231*d*f)
```

Maple [A]
time = 3.82, size = 60, normalized size = 0.90

method	result	size
default	$\frac{2\left(32\left(\cos^4(fx+e)\right)+24\left(\cos^2(fx+e)\right)+21\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\,\sin(fx+e)}{231f\cos(fx+e)^5}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/231/f*(32*cos(f*x+e)^4+24*cos(f*x+e)^2+21)*(d*sin(f*x+e)/cos(f*x+e))^(1/2
)*sin(f*x+e)/cos(f*x+e)^5
```

Maxima [A]
time = 0.28, size = 54, normalized size = 0.81

$$\frac{2\left(21\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}+66\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^2+77\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}d^4\right)}{231\,d^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/231*(21*(d*tan(f*x + e))^(11/2) + 66*(d*tan(f*x + e))^(7/2)*d^2 + 77*(d*t
an(f*x + e))^(3/2)*d^4)/(d^5*f)
```

Fricas [A]
time = 0.39, size = 65, normalized size = 0.97

$$\frac{2\left(32\cos\left(fx+e\right)^4+24\cos\left(fx+e\right)^2+21\right)\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{231\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(32*cos(f*x + e)^4 + 24*cos(f*x + e)^2 + 21)*sqrt(d*sin(f*x + e)/cos(
f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\tan{(e+fx)}}\,\sec^6{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**6, x)
```

Giac [A]
time = 0.49, size = 82, normalized size = 1.22

$$\frac{2\left(21\sqrt{d\tan{(fx+e)}}\,d^5\tan{(fx+e)}^5+66\sqrt{d\tan{(fx+e)}}\,d^5\tan{(fx+e)}^3+77\sqrt{d\tan{(fx+e)}}\,d^5\tan{(fx+e)}\right)}{231\,d^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/231*(21*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e)^5 + 66*sqrt(d*tan(f*x + e))
*d^5*tan(f*x + e)^3 + 77*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e))/(d^5*f)
```

Mupad [B]
time = 7.24, size = 334, normalized size = 4.99

$$-\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{231\,f}-\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{231\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}-\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{77\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}+\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,768\mathrm{i}}{77\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3}-\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,160\mathrm{i}}{11\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4}+\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^6,x)
```

```
[Out] ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*768i)/(
77*f*(exp(e*2i + f*x*2i) + 1)^3) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp
(e*2i + f*x*2i) + 1))^(1/2)*64i)/(231*f*(exp(e*2i + f*x*2i) + 1)) - ((-(d*(
exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*32i)/(77*f*(ex
p(e*2i + f*x*2i) + 1)^2) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i +
f*x*2i) + 1))^(1/2)*64i)/(231*f) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp
(e*2i + f*x*2i) + 1))^(1/2)*160i)/(11*f*(exp(e*2i + f*x*2i) + 1)^4) + ((-(d
*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*64i)/(11*f*(
exp(e*2i + f*x*2i) + 1)^5)
```

3.227 $\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{2(d\tan(e+fx))^{3/2}}{3df}+\frac{2(d\tan(e+fx))^{7/2}}{7d^3f}$$

```
[Out] 2/3*(d*tan(f*x+e))^(3/2)/d/f+2/7*(d*tan(f*x+e))^(7/2)/d^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2687, 14}

$$\frac{2(d\tan(e+fx))^{7/2}}{7d^3f}+\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f) + (2*(d*Tan[e + f*x])^(7/2))/(7*d^3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}
\int \sec^4(e+fx)\sqrt{d\tan(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{dx}\,(1+x^2)\,dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(\sqrt{dx}+\frac{(dx)^{5/2}}{d^2}\right)dx, x, \tan(e+fx)\right)}{f}\\
&= \frac{2(d\tan(e+fx))^{3/2}}{3df}+\frac{2(d\tan(e+fx))^{7/2}}{7d^3f}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 34, normalized size = 0.76

$$\frac{2(4+3\sec^2(e+fx))\,(d\tan(e+fx))^{3/2}}{21df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(4 + 3*Sec[e + f*x]^2)*(d*Tan[e + f*x])^(3/2))/(21*d*f)
```

Maple [A]
time = 0.40, size = 50, normalized size = 1.11

method	result	size
default	$\frac{2(4(\cos^2(fx+e))+3)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\,\sin(fx+e)}{21f\cos(fx+e)^3}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/f*(4*cos(f*x+e)^2+3)*(d*sin(f*x+e)/cos(f*x+e))^(1/2)*sin(f*x+e)/cos(f*
x+e)^3
```

Maxima [A]
time = 0.27, size = 38, normalized size = 0.84

$$\frac{2\left(3\,(d\tan(fx+e))^{\frac{7}{2}}+7\,(d\tan(fx+e))^{\frac{3}{2}}\,d^2\right)}{21\,d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(3*(d*tan(f*x + e))^(7/2) + 7*(d*tan(f*x + e))^(3/2)*d^2)/(d^3*f)
```

Fricas [A]
time = 0.38, size = 54, normalized size = 1.20

$$\frac{2\left(4\,\cos(fx+e)^2+3\right)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\,\sin(fx+e)}{21\,f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*cos(f*x + e)^2 + 3)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/
(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\tan\left(e+fx\right)}\,\sec^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**4, x)
```

Giac [A]
time = 0.51, size = 57, normalized size = 1.27

$$\frac{2\left(3\sqrt{d\tan\left(fx+e\right)}\,d^3\tan\left(fx+e\right)^3+7\sqrt{d\tan\left(fx+e\right)}\,d^3\tan\left(fx+e\right)\right)}{21\,d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*(3*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)^3 + 7*sqrt(d*tan(f*x + e))*d^
3*tan(f*x + e))/(d^3*f)
```

Mupad [B]
time = 6.09, size = 218, normalized size = 4.84

$$-\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,8\mathrm{i}}{21\,f}-\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,8\mathrm{i}}{21\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}+\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,24\mathrm{i}}{7\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^4,x)
```

```
[Out] ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*24i)/(7
*f*(exp(e*2i + f*x*2i) + 1)^2) - ((-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e
*2i + f*x*2i) + 1))^(1/2)*8i)/(21*f*(exp(e*2i + f*x*2i) + 1)) - ((-(d*(exp(
e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*8i)/(21*f) - ((-(d
*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*16i)/(7*f*(e
xp(e*2i + f*x*2i) + 1)^3)
```

3.228 $\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=22

$$\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] 2/3*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2687, 32}

$$\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)\sqrt{d\tan(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{dx}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{2(d\tan(e+fx))^{3/2}}{3df}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 22, normalized size = 1.00

$$\frac{2(d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
[Out] (2*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Maple [A]
time = 0.15, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(d\tan(fx+e))^{\frac{3}{2}}}{3df}$	19
default	$\frac{2(d\tan(fx+e))^{\frac{3}{2}}}{3df}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(d*tan(f*x+e))^(3/2)/d/f
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.86

$$\frac{2\left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}}{3\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] 2/3*(d*tan(f*x + e))^(3/2)/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(19) = 38$.
time = 0.37, size = 41, normalized size = 1.86

$$\frac{2\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\tan\left(e+fx\right)}\,\sec^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**2, x)
```

Giac [A]
time = 0.46, size = 23, normalized size = 1.05

$$\frac{2\sqrt{d\tan\left(fx+e\right)}\,\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(d*tan(f*x + e))*tan(f*x + e)/f
```

Mupad [B]
time = 2.57, size = 53, normalized size = 2.41

$$\frac{2\,\sin\left(2\,e+2\,f\,x\right)\,\sqrt{\frac{d\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^2,x)
```

```
[Out] (2*sin(2*e + 2*f*x)*((d*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(3
*f*(cos(2*e + 2*f*x) + 1))
```

3.229 $\int \sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

```
[Out] -1/2*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/2*a
rctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/4*ln(d^(1
/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f*2^(1/2)-1/4*
ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f*2^(1/
2)
```

Rubi [A]
time = 0.08, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3557, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}+\frac{\sqrt{d}\,\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}-\frac{\sqrt{d}\,\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f))
 + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f)
 + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x
]]])/(2*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*
Sqrt[d*Tan[e + f*x]]])/(2*Sqrt[2]*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \sqrt{d\tan(e+fx)}\,dx = \frac{d\text{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(e+fx)\right)}{f}$$

$$= \frac{(2d)\text{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}$$

$$= -\frac{d\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f} + \frac{d\text{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+f\right.}}{f}$$

$$= \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}\,f} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}}{-d+\sqrt{2}\sqrt{}}\right.}{}$$

$$= \frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}\,f} - \frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

$$= -\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f} + \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right.}{\sqrt{2}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 40, normalized size = 0.21

$$\frac{2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]],x]
[Out] (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/
(3*d*f)
```

Maple [A]
time = 0.18, size = 136, normalized size = 0.71

method	result
derivativedivides	$\frac{d\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right)\right.}{4f(d^2)^{\frac{1}{4}}}$

default	$\frac{d\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan(fx+e)}}{(d^2)^{\frac{1}{4}}}+1\right.\right.}{4f(d^2)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*d/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1
/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(
1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1)-2*a
rctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)+1))
```

Maxima [A]
time = 0.50, size = 159, normalized size = 0.83

$$\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x + e
)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*s
qrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) + sqrt(2
)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x + e) -
sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(151) = 302$.
time = 0.37, size = 547, normalized size = 2.85

$$-\sqrt{2}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,df\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}-\sqrt{2}\,f\sqrt{\frac{\sqrt{2}\,df^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\cos(fx+e)+d^2f^2\sqrt{\frac{d^2}{f^4}}\cos(fx+e)+d^3\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}+d^2}{d^2}\right)-\sqrt{2}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,df\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}-\sqrt{2}\,f\sqrt{-\frac{\sqrt{2}\,df^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\cos(fx+e)-d^2f^2\sqrt{\frac{d^2}{f^4}}\cos(fx+e)-d^3\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}-d^2}{d^2}\right)-\frac{1}{4}\sqrt{2}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(\frac{\sqrt{2}\,df^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\cos(fx+e)+d^2f^2\sqrt{\frac{d^2}{f^4}}\cos(fx+e)+d^3\sin(fx+e)}{\cos(fx+e)}\right)+\frac{1}{4}\sqrt{2}\left(\frac{d^2}{f^4}\right)^{\frac{1}{4}}\log\left(-\frac{\sqrt{2}\,df^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d^2}{f^4}\right)^{\frac{3}{4}}\cos(fx+e)-d^2f^2\sqrt{\frac{d^2}{f^4}}\cos(fx+e)-d^3\sin(fx+e)}{\cos(fx+e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*sin(f*x + e)/cos(f*x +
 e))*(d^2/f^4)^(1/4) - sqrt(2)*f*sqrt((sqrt(2)*d*f^3*sqrt(d*sin(f*x + e)/co
s(f*x + e))*(d^2/f^4)^(3/4)*cos(f*x + e) + d^2*f^2*sqrt(d^2/f^4)*cos(f*x +
e) + d^3*sin(f*x + e))/cos(f*x + e))*(d^2/f^4)^(1/4) + d^2)/d^2) - sqrt(2)*
(d^2/f^4)^(1/4)*arctan(-(sqrt(2)*d*f*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2
/f^4)^(1/4) - sqrt(2)*f*sqrt(-(sqrt(2)*d*f^3*sqrt(d*sin(f*x + e)/cos(f*x +
e))*(d^2/f^4)^(3/4)*cos(f*x + e) - d^2*f^2*sqrt(d^2/f^4)*cos(f*x + e) - d^3
```

```
*sin(f*x + e))/cos(f*x + e))*(d^2/f^4)^(1/4) - d^2)/d^2) - 1/4*sqrt(2)*(d^2
/f^4)^(1/4)*log((sqrt(2)*d*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*(d^2/f^4)^
(3/4)*cos(f*x + e) + d^2*f^2*sqrt(d^2/f^4)*cos(f*x + e) + d^3*sin(f*x + e))
/cos(f*x + e)) + 1/4*sqrt(2)*(d^2/f^4)^(1/4)*log(-(sqrt(2)*d*f^3*sqrt(d*sin
(f*x + e)/cos(f*x + e))*(d^2/f^4)^(3/4)*cos(f*x + e) - d^2*f^2*sqrt(d^2/f^4
)*cos(f*x + e) - d^3*sin(f*x + e))/cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \tan(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x)), x)
```

Giac [A]
time = 0.47, size = 182, normalized size = 0.95

$$\frac{\frac{2\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f}+\frac{2\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{f}-\frac{\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{|d|}+|d|\right)}{f}+\frac{\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)-\sqrt{2}\,\sqrt{d\tan(fx+e)}\,\sqrt{|d|}+|d|\right)}{f}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sq
rt(d*tan(f*x + e)))/sqrt(abs(d)))/f + 2*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*sq
rt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f - sqr
t(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(ab
s(d)) + abs(d))/f + sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(
d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f)/d
```

Mupad [B]
time = 2.50, size = 49, normalized size = 0.26

$$\frac{(-1)^{1/4}\sqrt{d}\,\left(\mathrm{atan}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)-\mathrm{atanh}\left(\frac{(-1)^{1/4}\sqrt{d\tan(e+f\,x)}}{\sqrt{d}}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2),x)
```

```
[Out] ((-1)^(1/4)*d^(1/2)*(atan(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2)) - at
anh(((-1)^(1/4)*(d*tan(e + f*x))^(1/2))/d^(1/2))))/f
```

3.230 $\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=227

$$-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,f}+\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,f}+\frac{\sqrt{d}\,\log\left(\sqrt{d}+\sqrt{\right.}$$

```
[Out] -1/8*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/8*a
rctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))*d^(1/2)/f*2^(1/2)+1/16*ln(d^(
1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f*2^(1/2)-1/1
6*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/f*2^(
1/2)+1/2*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]

time = 0.12, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2687, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{d}\,\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,f}+\frac{\sqrt{d}\,\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}\,f}+\frac{\sqrt{d}\,\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{8\sqrt{2}\,f}-\frac{\sqrt{d}\,\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{8\sqrt{2}\,f}+\frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] -1/4*(Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*
f) + (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt[2
]*f) + (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e +
 f*x]]])/(8*Sqrt[2]*f) - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt
[2]*Sqrt[d*Tan[e + f*x]]])/(8*Sqrt[2]*f) + (Cos[e + f*x]^2*(d*Tan[e + f*x])
^(3/2))/(2*d*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{dx}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} + \frac{\text{Subst}\left(\int \frac{\sqrt{dx}}{1+x^2}\,dx, x, \tan(e+fx)\right)}{4f}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} + \frac{\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+f}\right.}{2df}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} - \frac{\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(e+f}\right.}{4df}$$

$$= \frac{\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2df} + \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx,\right.}{8\sqrt{2}f}$$

$$= \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} - \frac{\sqrt{d}\,\text{lo}}{}$$

$$= -\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} + \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{}}{}\right.}{4\sqrt{2}f}$$

Mathematica [A]
time = 0.21, size = 102, normalized size = 0.45

$$-\frac{\left(\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\csc(e+fx)+\csc(e+fx)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)-2\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}\sqrt{d\tan(e+fx)}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] -1/8*((ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x] + Csc[e + f*x]*Log[
Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]] - 2*Sqrt[Sin[2*(e + f
*x)]])*Sqrt[Sin[2*(e + f*x)]]*Sqrt[d*Tan[e + f*x]])/f
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 522, normalized size = 2.30

method	result
default	$\frac{(\cos(fx+e)-1)\left(-i\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(cos(f*x+e)-1)*(-I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2
^(1/2))+I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+((cos(f
*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+((cos(f*x+e)-1)/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(f*x+e)^2*2^(1/2)-2*cos(f*x+e)*2^(1/
2))*(cos(f*x+e)+1)^2*(d*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)^3*2^(1/2)
```

Maxima [A]
time = 0.50, size = 201, normalized size = 0.89

$$\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+\frac{8(d\tan(fx+e))^{\frac{3}{2}}d^2}{d^2\tan(fx+e)^2+d^2}}{16\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f*x
 + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) -
 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) + sq
rt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x + e
) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) + 8*(d*tan(f*x + e))
^(3/2)*d^2/(d^2*tan(f*x + e)^2 + d^2))/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2009 vs. 2(179) = 358.
time = 64.70, size = 2009, normalized size = 8.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")

```
[Out] 1/64*(4*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan((sqrt(4*d^3*f^2*sqrt(d^2/f^4)*cos(
f*x + e)*sin(f*x + e) + d^4 - 2*(sqrt(2)*d^2*f^3*(d^2/f^4)^(3/4)*cos(f*x +
e)*sin(f*x + e) + sqrt(2)*d^3*f*(d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(d*sin(
f*x + e)/cos(f*x + e)))*(2*d^2*cos(f*x + e)*sin(f*x + e) + d*f^2*sqrt(d^2/f
^4) + (sqrt(2)*f^3*(d^2/f^4)^(3/4)*cos(f*x + e)^2 + sqrt(2)*d*f*(d^2/f^4)^(
1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))) + (sqrt(
2)*d^2*f^3*(d^2/f^4)^(3/4)*cos(f*x + e)*sin(f*x + e) + sqrt(2)*d^3*f*(d^2/f
^4)^(1/4)*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*d^4*cos(f*x
 + e)^2 - d^4)) + 4*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-(sqrt(4*d^3*f^2*sqrt(
d^2/f^4)*cos(f*x + e)*sin(f*x + e) + d^4 + 2*(sqrt(2)*d^2*f^3*(d^2/f^4)^(3/
4)*cos(f*x + e)*sin(f*x + e) + sqrt(2)*d^3*f*(d^2/f^4)^(1/4)*cos(f*x + e)^2
)*sqrt(d*sin(f*x + e)/cos(f*x + e)))*(2*d^2*cos(f*x + e)*sin(f*x + e) + d*f
^2*sqrt(d^2/f^4) - (sqrt(2)*f^3*(d^2/f^4)^(3/4)*cos(f*x + e)^2 + sqrt(2)*d*
f*(d^2/f^4)^(1/4)*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x +
e))) - (sqrt(2)*d^2*f^3*(d^2/f^4)^(3/4)*cos(f*x + e)*sin(f*x + e) + sqrt(2)
*d^3*f*(d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(
2*d^4*cos(f*x + e)^2 - d^4)) + 4*sqrt(2)*f*(d^2/f^4)^(1/4)*arctan(-1/2*(2*d
^4*sin(f*x + e) - sqrt(4*d^3*f^2*sqrt(d^2/f^4)*cos(f*x + e)*sin(f*x + e) +
d^4 + 2*(sqrt(2)*d^2*f^3*(d^2/f^4)^(3/4)*cos(f*x + e)*sin(f*x + e) + sqrt(2
)*d^3*f*(d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e)))*
(sqrt(2)*f^3*(d^2/f^4)^(3/4)*cos(f*x + e) + sqrt(2)*d*f*(d^2/f^4)^(1/4)*sin
(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)) + (sqrt(2)*d^2*f^3*(d^2/f^4)^(
3/4)*cos(f*x + e) + sqrt(2)*d^3*f*(d^2/f^4)^(1/4)*sin(f*x + e))*sqrt(d*sin(
f*x + e)/cos(f*x + e)) - 4*(d^3*f^2*cos(f*x + e)^3 - d^3*f^2*cos(f*x + e))*
sqrt(d^2/f^4))/((2*d^4*cos(f*x + e)^2 - d^4)*sin(f*x + e))) + 4*sqrt(2)*f*(
d^2/f^4)^(1/4)*arctan(1/2*(2*d^4*sin(f*x + e) + sqrt(4*d^3*f^2*sqrt(d^2/f^4
)*cos(f*x + e)*sin(f*x + e) + d^4 - 2*(sqrt(2)*d^2*f^3*(d^2/f^4)^(3/4)*cos(
f*x + e)*sin(f*x + e) + sqrt(2)*d^3*f*(d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(
d*sin(f*x + e)/cos(f*x + e)))*(sqrt(2)*f^3*(d^2/f^4)^(3/4)*cos(f*x + e) + s
qrt(2)*d*f*(d^2/f^4)^(1/4)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))
- (sqrt(2)*d^2*f^3*(d^2/f^4)^(3/4)*cos(f*x + e) + sqrt(2)*d^3*f*(d^2/f^4)^(
1/4)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)) - 4*(d^3*f^2*cos(f*x +
 e)^3 - d^3*f^2*cos(f*x + e))*sqrt(d^2/f^4))/((2*d^4*cos(f*x + e)^2 - d^4)*
sin(f*x + e))) - sqrt(2)*f*(d^2/f^4)^(1/4)*log(4*d^3*f^2*sqrt(d^2/f^4)*cos(
f*x + e)*sin(f*x + e) + d^4 + 2*(sqrt(2)*d^2*f^3*(d^2/f^4)^(3/4)*cos(f*x +
e)*sin(f*x + e) + sqrt(2)*d^3*f*(d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(d*sin(
f*x + e)/cos(f*x + e))) + sqrt(2)*f*(d^2/f^4)^(1/4)*log(4*d^3*f^2*sqrt(d^2/
f^4)*cos(f*x + e)*sin(f*x + e) + d^4 - 2*(sqrt(2)*d^2*f^3*(d^2/f^4)^(3/4)*c
os(f*x + e)*sin(f*x + e) + sqrt(2)*d^3*f*(d^2/f^4)^(1/4)*cos(f*x + e)^2)*sq
rt(d*sin(f*x + e)/cos(f*x + e))) - sqrt(2)*f*(d^2/f^4)^(1/4)*log(1/4*d^3*f^
2*sqrt(d^2/f^4)*cos(f*x + e)*sin(f*x + e) + 1/16*d^4 + 1/8*(sqrt(2)*d^2*f^3
```

```
*(d^2/f^4)^(3/4)*cos(f*x + e)*sin(f*x + e) + sqrt(2)*d^3*f*(d^2/f^4)^(1/4)*
cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))) + sqrt(2)*f*(d^2/f^4)^(1
/4)*log(1/4*d^3*f^2*sqrt(d^2/f^4)*cos(f*x + e)*sin(f*x + e) + 1/16*d^4 - 1/
8*(sqrt(2)*d^2*f^3*(d^2/f^4)^(3/4)*cos(f*x + e)*sin(f*x + e) + sqrt(2)*d^3*
f*(d^2/f^4)^(1/4)*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))) + 32*s
qrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\tan\left(e+fx\right)}\,\cos^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*cos(e + f*x)**2, x)
```

Giac [A]
time = 0.51, size = 227, normalized size = 1.00

$$\frac{\frac{8\sqrt{d\tan\left(fx+e\right)}\,d^{3}\tan\left(fx+e\right)}{\left(d^{2}\tan\left(fx+e\right)^{2}+d^{2}\right)f}+\frac{2\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{f}+\frac{2\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{f}-\frac{\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{f}+\frac{\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{f}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(8*sqrt(d*tan(f*x + e))*d^3*tan(f*x + e)/((d^2*tan(f*x + e)^2 + d^2)*f
) + 2*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sqr
t(d*tan(f*x + e)))/sqrt(abs(d)))/f + 2*sqrt(2)*abs(d)^(3/2)*arctan(-1/2*sqr
t(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/f - sqrt
(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs
(d)) + abs(d))/f + sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) - sqrt(2)*sqrt(d
*tan(f*x + e))*sqrt(abs(d)) + abs(d))/f)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(e+f\,x\right)}^2\,\sqrt{d\,\mathrm{tan}\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(d*tan(e + f*x))^(1/2),x)
```

```
[Out] int(cos(e + f*x)^2*(d*tan(e + f*x))^(1/2), x)
```

3.231 $\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=107

$$-\frac{4\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{\sin(2e+2fx)}}+\frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2\sec(e+fx)(d\tan(e$$

```
[Out] 4/5*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(
e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+4/5*cos(
f*x+e)*(d*tan(f*x+e))^(3/2)/d/f+2/5*sec(f*x+e)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2693, 2695, 2652, 2719}

$$\frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df}-\frac{4\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-4*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(5*f*Sq
rt[Sin[2*e + 2*f*x]]) + (4*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(5*d*f) + (
2*Sec[e + f*x]*(d*Tan[e + f*x])^(3/2))/(5*d*f)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{2}{5}\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,d$$
$$= \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df} -$$
$$= \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df} -$$
$$= \frac{4\cos(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{2\sec(e+fx)(d\tan(e+fx))^{3/2}}{5df} -$$
$$= -\frac{4\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{5f\sqrt{\sin(2e+2fx)}} + \frac{4\cos(e+fx)(d}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.48, size = 102, normalized size = 0.95

$$\frac{2\sqrt{d\tan(e+fx)}\left(-4\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(e+fx)\right)\sec(e+fx)\tan(e+fx)+3\sqrt{\sec^2(e+fx)}\,(2\sin(e+fx)+\sec(e+fx)\tan(e+fx))\right)}{15f\sqrt{\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
[Out] (2*Sqrt[d*Tan[e + f*x]]*(-4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^
2]*Sec[e + f*x]*Tan[e + f*x] + 3*Sqrt[Sec[e + f*x]^2]*(2*Sin[e + f*x] + Sec
[e + f*x]*Tan[e + f*x])))/(15*f*Sqrt[Sec[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(118) = 236.
time = 0.37, size = 559, normalized size = 5.22

method	result
default	$-\frac{(\cos(fx+e)-1)^2\left(2\,\mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/f*(cos(f*x+e)-1)^2*(2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*c
os(f*x+e)^3-4*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*
2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^3+2
*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((co
s(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2-4*EllipticE((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((cos(f*x+e)-1)/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*cos(f*x+e)^2+2*cos(f*x+e)^3*2^(1/2)-cos(
f*x+e)^2*2^(1/2)-2^(1/2))*(cos(f*x+e)+1)^2*(d*sin(f*x+e)/cos(f*x+e))^(1/2)/
cos(f*x+e)^2/sin(f*x+e)^5*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e)^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\tan{(e+fx)}}\sec^3{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{{\cos\left(e+f\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^3,x)
```

```
[Out] int((d*tan(e + f*x))^(1/2)/cos(e + f*x)^3, x)
```

3.232 $\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=75

$$-\frac{2\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}+\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}$$

```
[Out] 2*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+
1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+2*cos(f*x+
e)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2693, 2695, 2652, 2719}

$$\frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df}-\frac{2\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt
[Sin[2*e + 2*f*x]]) + (2*Cos[e + f*x]*(d*Tan[e + f*x])^(3/2))/(d*f)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df} - 2\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$$
$$= \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df} - \frac{\left(2\sqrt{\cos(e+fx)}\,\sqrt{d\tan(e+fx)}\right)}{\sqrt{\sin(e}}$$
$$= \frac{2\cos(e+fx)(d\tan(e+fx))^{3/2}}{df} - \frac{\left(2\cos(e+fx)\sqrt{d\tan(e+fx)}\right)\int}{\sqrt{\sin(2e+2fx)}}$$
$$= -\frac{2\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}} + \frac{2\cos(e+fx)(d\,t}{df}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.30, size = 61, normalized size = 0.81

$$-\frac{2\left(-3+2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\right)\sin(e+fx)\sqrt{d\tan(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
[Out] (-2*(-3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e +
f*x]^2])*Sin[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(94) = 188.
time = 0.29, size = 513, normalized size = 6.84

method	result
default	$\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\,(\cos(fx+e)+1)^2(\cos(fx+e)-1)^2\left(2\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{-1+\cos(fx+e)-\sin(}{\sin(fx+e)}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(d*sin(f*x+e)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(2*((
cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)-((cos(f*x+e)-1)/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))-((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)*2^(1
/2)+2^(1/2))/sin(f*x+e)^5*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\tan{(e+fx)}}\,\sec{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))*sec(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(f*x + e))*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/cos(e + f*x),x)
[Out] int((d*tan(e + f*x))^(1/2)/cos(e + f*x), x)
```

3.233 $\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=47

$$\frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

```
[Out] -cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1
/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2695, 2652, 2719}

$$\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(f*Sqrt[Si
n[2*e + 2*f*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\left(\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\right)\int \sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{\sqrt{\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)\sqrt{d\tan(e+fx)}\right)\int \sqrt{\sin(2e+2fx)}\,dx}{\sqrt{\sin(2e+2fx)}}$$

$$= \frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{f\sqrt{\sin(2e+2fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.11, size = 57, normalized size = 1.21

$$\frac{2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\,\sin(e+fx)\sqrt{d\tan(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^2]*S
in[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(69) = 138.
time = 0.33, size = 523, normalized size = 11.13

method	result
default	$-\frac{(\cos(fx+e)-1)^2\left(2\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\,\text{EllipticE}\left(\sqrt{-\frac{-1+c}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)^2*(2*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*c
os(f*x+e)-((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2*((cos
(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
```

```
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-((cos(f*x+e)-1)/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2*2^(1/2))+cos(f*x+e)^2*2^(1/2)-cos(f*x+e)*2^(1/2))*(cos(f*x+e)+1)^2*(d*
sin(f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)^5*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\tan (e+fx)} \cos (e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*tan(e + f*x))*cos(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(e+fx)\ \sqrt{d\tan(e+fx)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(d*tan(e + f*x))^(1/2),x)
```

```
[Out] int(cos(e + f*x)*(d*tan(e + f*x))^(1/2), x)
```

3.234 $\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=81

$$\frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}}+\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] -1/2*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos
(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+1/3*cos
(f*x+e)^3*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2692, 2695, 2652, 2719}

$$\frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df}+\frac{\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(2*f*Sqrt[
Sin[2*e + 2*f*x]]) + (Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df} + \frac{1}{2}\int \cos(e+fx)\sqrt{d\tan(e+fx)}\,dx$$

$$= \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df} + \frac{\left(\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{\sin(e}}$$

$$= \frac{\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{3df} + \frac{\left(\cos(e+fx)\sqrt{d\tan(e+fx)}\right)\int \sqrt{}}{2\sqrt{\sin(2e+2fx)}}$$

$$= \frac{\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{2f\sqrt{\sin(2e+2fx)}} + \frac{\cos^3(e+fx)(d\tan(}{3df}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.48, size = 94, normalized size = 1.16

$$\frac{\sqrt{d\tan(e+fx)}\left(\sqrt{\sec^2(e+fx)}\,(\sin(e+fx)+\sin(3(e+fx)))+4\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(e+fx)\right)\sec(e+fx)\tan(e+fx)\right)}{12f\sqrt{\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Sqrt[d*Tan[e + f*x]]*(Sqrt[Sec[e + f*x]^2]*(Sin[e + f*x] + Sin[3*(e + f*x)
]) + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sec[e + f*x]*Tan[e
 + f*x]))/(12*f*Sqrt[Sec[e + f*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(96) = 192.
time = 0.35, size = 536, normalized size = 6.62

method	result
default	$-\frac{(\cos(fx+e)-1)^2\left(2\sqrt{2}\,(\cos^4(fx+e))+6\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/f*(cos(f*x+e)-1)^2*(2*2^(1/2)*cos(f*x+e)^4+6*((cos(f*x+e)-1)/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2*2^(1/2))*cos(f*x+e)-3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2
^(1/2))*cos(f*x+e)+6*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*El
lipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*((cos
(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+cos(f*x+e)^2*2^(1/2)-3*cos(f*x+e)
*2^(1/2))*(cos(f*x+e)+1)^2*(d*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)^5*2^(
1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeEv
aluation time: 10.28Done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^3\,\sqrt{d\,\tan\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2),x)
```

```
[Out] int(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2), x)
```

3.235 $\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=111

$$\frac{7\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{20f\sqrt{\sin(2e+2fx)}}+\frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df}+\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}$$

```
[Out] -7/20*cos(f*x+e)*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(co
s(e+1/4*Pi+f*x),2^(1/2))*(d*tan(f*x+e))^(1/2)/f/sin(2*f*x+2*e)^(1/2)+7/30*c
os(f*x+e)^3*(d*tan(f*x+e))^(3/2)/d/f+1/5*cos(f*x+e)^5*(d*tan(f*x+e))^(3/2)/
d/f
```

Rubi [A]
time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2692, 2695, 2652, 2719}

$$\frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df}+\frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df}+\frac{7\cos(e+fx)E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(e+fx)}}{20f\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (7*Cos[e + f*x]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Tan[e + f*x]])/(20*f*Sq
rt[Sin[2*e + 2*f*x]]) + (7*Cos[e + f*x]^3*(d*Tan[e + f*x])^(3/2))/(30*d*f)
+ (Cos[e + f*x]^5*(d*Tan[e + f*x])^(3/2))/(5*d*f)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(e+fx)\sqrt{d\tan(e+fx)}\,dx = \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df} + \frac{7}{10}\int \cos^3(e+fx)\sqrt{d\tan(e+fx)}$$
$$= \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df} +$$
$$= \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df} +$$
$$= \frac{7\cos^3(e+fx)(d\tan(e+fx))^{3/2}}{30df} + \frac{\cos^5(e+fx)(d\tan(e+fx))^{3/2}}{5df} +$$
$$= \frac{7\cos(e+fx)E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\tan(e+fx)}}{20f\sqrt{\sin(2e+2fx)}} + \frac{7\cos^3(e+fx)(d\,t}{30d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.83, size = 86, normalized size = 0.77

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}\left(20\sin(2(e+fx))+3\sin(4(e+fx))+28\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\tan(e+fx)\right)}{120f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*Sqrt[d*Tan[e + f*x]]*(20*Sin[2*(e + f*x)] + 3*Sin[4*(e + f*x)
] + 28*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^
2]*Tan[e + f*x]))/(120*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(122) = 244.
time = 0.35, size = 550, normalized size = 4.95

method	result
default	$-\frac{(\cos(fx+e)-1)^2\left(12\sqrt{2}\,(\cos^6(fx+e))+2\sqrt{2}\,(\cos^4(fx+e))-21\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/120/f*(cos(f*x+e)-1)^2*(12*2^(1/2)*cos(f*x+e)^6+2*2^(1/2)*cos(f*x+e)^4-21*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+42*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)-21*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+42*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+7*cos(f*x+e)^2*2^(1/2)-21*cos(f*x+e)*2^(1/2))*(cos(f*x+e)+1)^2*(d*sin(f*x+e)/cos(f*x+e))^(1/2)/sin(f*x+e)^5*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*cos(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^5\,\sqrt{d\,\tan\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2),x)
```

```
[Out] int(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2), x)
```

3.236 $\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=67

$$\frac{2(d\tan(a+bx))^{5/2}}{5bd}+\frac{4(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{13/2}}{13bd^5}$$

```
[Out] 2/5*(d*tan(b*x+a))^(5/2)/b/d+4/9*(d*tan(b*x+a))^(9/2)/b/d^3+2/13*(d*tan(b*x
+a))^(13/2)/b/d^5
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2687, 276}

$$\frac{2(d\tan(a+bx))^{13/2}}{13bd^5}+\frac{4(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d) + (4*(d*Tan[a + b*x])^(9/2))/(9*b*d^3) +
 (2*(d*Tan[a + b*x])^(13/2))/(13*b*d^5)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^6(a+bx)(d\tan(a+bx))^{3/2}\,dx&=\frac{\text{Subst}\left(\int (dx)^{3/2}\left(1+x^2\right)^2\,dx,x,\tan(a+bx)\right)}{b}\\&=\frac{\text{Subst}\left(\int \left((dx)^{3/2}+\frac{2(dx)^{7/2}}{d^2}+\frac{(dx)^{11/2}}{d^4}\right)\,dx,x,\tan(a+bx)\right)}{b}\\&=\frac{2(d\tan(a+bx))^{5/2}}{5bd}+\frac{4(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{13/2}}{13bd^5}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 52, normalized size = 0.78

$$\frac{2d(-32-8\sec^2(a+bx)-5\sec^4(a+bx)+45\sec^6(a+bx))\sqrt{d\tan(a+bx)}}{585b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*(-32 - 8*Sec[a + b*x]^2 - 5*Sec[a + b*x]^4 + 45*Sec[a + b*x]^6)*Sqrt[d
*Tan[a + b*x]])/(585*b)
```

Maple [A]
time = 7.30, size = 60, normalized size = 0.90

method	result	size
default	$\frac{2\left(32\left(\cos^4(bx+a)\right)+40\left(\cos^2(bx+a)\right)+45\right)\sin(bx+a)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}}{585b\cos(bx+a)^5}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/585/b*(32*cos(b*x+a)^4+40*cos(b*x+a)^2+45)*sin(b*x+a)*(d*sin(b*x+a)/cos(b
*x+a))^(3/2)/cos(b*x+a)^5
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.76

$$\frac{2\left(45\,(d\tan(bx+a))^{\frac{13}{2}}+130\,(d\tan(bx+a))^{\frac{9}{2}}\,d^2+117\,(d\tan(bx+a))^{\frac{5}{2}}\,d^4\right)}{585\,bd^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/585*(45*(d*tan(b*x + a))^(13/2) + 130*(d*tan(b*x + a))^(9/2)*d^2 + 117*(d
*tan(b*x + a))^(5/2)*d^4)/(b*d^5)
```

Fricas [A]
time = 0.40, size = 68, normalized size = 1.01

$$-\frac{2\left(32\,d\cos(bx+a)^6+8\,d\cos(bx+a)^4+5\,d\cos(bx+a)^2-45\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{585\,b\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/585*(32*d*cos(b*x + a)^6 + 8*d*cos(b*x + a)^4 + 5*d*cos(b*x + a)^2 - 45*
d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x + a)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan{(a+bx)})^{\frac{3}{2}}\sec^{6}{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**6, x)
```

Giac [A]
time = 0.53, size = 78, normalized size = 1.16

$$\frac{2\left(45\sqrt{d\tan{(bx+a)}}\,d^{6}\tan{(bx+a)}^{6}+130\sqrt{d\tan{(bx+a)}}\,d^{6}\tan{(bx+a)}^{4}+117\sqrt{d\tan{(bx+a)}}\,d^{6}\tan{(bx+a)}^{2}\right)}{585\,bd^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/585*(45*sqrt(d*tan(b*x + a))*d^6*tan(b*x + a)^6 + 130*sqrt(d*tan(b*x + a)
)*d^6*tan(b*x + a)^4 + 117*sqrt(d*tan(b*x + a))*d^6*tan(b*x + a)^2)/(b*d^5)
```

Mupad [B]
time = 9.19, size = 392, normalized size = 5.85

$$-\frac{64\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{585\,b}-\frac{64\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{585\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}-\frac{32\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{195\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2}+\frac{1216\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{117\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3}-\frac{3488\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{117\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^4}+\frac{384\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{13\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^5}-\frac{128\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{13\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^6,x)
```

```
[Out] (1216*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))
/(117*b*(exp(a*2i + b*x*2i) + 1)^3) - (64*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1
i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(585*b*(exp(a*2i + b*x*2i) + 1)) - (32
*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(195
*b*(exp(a*2i + b*x*2i) + 1)^2) - (64*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2))/(585*b) - (3488*d*(-(d*(exp(a*2i + b*x*2i)*
1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(117*b*(exp(a*2i + b*x*2i) + 1)^
4) + (384*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1
/2))/(13*b*(exp(a*2i + b*x*2i) + 1)^5) - (128*d*(-(d*(exp(a*2i + b*x*2i)*1i
 - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(13*b*(exp(a*2i + b*x*2i) + 1)^6)
```

3.237 $\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=45

$$\frac{2(d\tan(a+bx))^{5/2}}{5bd}+\frac{2(d\tan(a+bx))^{9/2}}{9bd^3}$$

```
[Out] 2/5*(d*tan(b*x+a))^(5/2)/b/d+2/9*(d*tan(b*x+a))^(9/2)/b/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2687, 14}

$$\frac{2(d\tan(a+bx))^{9/2}}{9bd^3}+\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d) + (2*(d*Tan[a + b*x])^(9/2))/(9*b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (dx)^{3/2}\left(1+x^2\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left((dx)^{3/2}+\frac{(dx)^{7/2}}{d^2}\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{2(d\tan(a+bx))^{5/2}}{5bd}+\frac{2(d\tan(a+bx))^{9/2}}{9bd^3}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 42, normalized size = 0.93

$$\frac{2d(-4-\sec^2(a+bx)+5\sec^4(a+bx))\sqrt{d\tan(a+bx)}}{45b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*(-4 - Sec[a + b*x]^2 + 5*Sec[a + b*x]^4)*Sqrt[d*Tan[a + b*x]])/(45*b)
```

Maple [A]
time = 0.41, size = 50, normalized size = 1.11

method	result	size
default	$\frac{2\left(4\left(\cos^2(bx+a)\right)+5\right)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}\sin(bx+a)}{45b\cos(bx+a)^3}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/b*(4*cos(b*x+a)^2+5)*(d*sin(b*x+a)/cos(b*x+a))^(3/2)*sin(b*x+a)/cos(b*
x+a)^3
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.80

$$\frac{2\left(5\left(d\tan\left(bx+a\right)\right)^{\frac{9}{2}}+9\left(d\tan\left(bx+a\right)\right)^{\frac{5}{2}}d^2\right)}{45\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45*(5*(d*tan(b*x + a))^(9/2) + 9*(d*tan(b*x + a))^(5/2)*d^2)/(b*d^3)
```

Fricas [A]
time = 0.40, size = 56, normalized size = 1.24

$$-\frac{2\left(4\,d\cos\left(bx+a\right)^4+d\cos\left(bx+a\right)^2-5\,d\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{45\,b\cos\left(bx+a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45*(4*d*cos(b*x + a)^4 + d*cos(b*x + a)^2 - 5*d)*sqrt(d*sin(b*x + a)/cos
(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(a+bx))^{\frac{3}{2}}\sec^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**4, x)
```

Giac [A]
time = 0.46, size = 55, normalized size = 1.22

$$\frac{2\left(5\sqrt{d\tan(bx+a)}\,d^4\tan(bx+a)^4+9\sqrt{d\tan(bx+a)}\,d^4\tan(bx+a)^2\right)}{45\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/45*(5*sqrt(d*tan(b*x + a))*d^4*tan(b*x + a)^4 + 9*sqrt(d*tan(b*x + a))*d^
4*tan(b*x + a)^2)/(b*d^3)
```

Mupad [B]
time = 6.94, size = 276, normalized size = 6.13

$$-\frac{8\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{45\,b}-\frac{8\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{45\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}+\frac{56\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{15\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2}-\frac{64\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{9\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3}+\frac{32\,d\,\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{9\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^4,x)
```

```
[Out] (56*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(
15*b*(exp(a*2i + b*x*2i) + 1)^2) - (8*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/
(exp(a*2i + b*x*2i) + 1))^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)) - (8*d*(-(
d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(45*b) - (
64*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(9
*b*(exp(a*2i + b*x*2i) + 1)^3) + (32*d*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(
exp(a*2i + b*x*2i) + 1))^(1/2))/(9*b*(exp(a*2i + b*x*2i) + 1)^4)
```

3.238 $\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=22

$$\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

```
[Out] 2/5*(d*tan(b*x+a))^(5/2)/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2687, 32}

$$\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)(d\tan(a+bx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (dx)^{3/2}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{2(d\tan(a+bx))^{5/2}}{5bd}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 22, normalized size = 1.00

$$\frac{2(d\tan(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*(d*Tan[a + b*x])^(5/2))/(5*b*d)
```

Maple [A]
time = 0.12, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(d\tan(bx+a))^{\frac{5}{2}}}{5bd}$	19
default	$\frac{2(d\tan(bx+a))^{\frac{5}{2}}}{5bd}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(d*tan(b*x+a))^(5/2)/b/d
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.82

$$\frac{2\,(d\tan\left(bx+a\right))^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(d*tan(b*x + a))^(5/2)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.
time = 0.38, size = 45, normalized size = 2.05

$$-\frac{2\left(d\cos\left(bx+a\right)^{2}-d\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{5\,b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(d*cos(b*x + a)^2 - d)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*cos(b*x +
a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}\sec^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**2, x)
```

Giac [A]
time = 0.48, size = 24, normalized size = 1.09

$$\frac{2\,\sqrt{d\tan\left(bx+a\right)}\,d\tan\left(bx+a\right)^{2}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(d*tan(b*x + a))*d*tan(b*x + a)^2/b
```

Mupad [B]
time = 3.57, size = 100, normalized size = 4.55

$$\frac{2\,d\,\sqrt{\frac{d\,\sin\left(2\,a+2\,b\,x\right)}{\cos\left(2\,a+2\,b\,x\right)+1}}\,\left(\cos\left(2\,a+2\,b\,x\right)-2\,\cos\left(4\,a+4\,b\,x\right)-\cos\left(6\,a+6\,b\,x\right)+2\right)}{5\,b\,\left(15\,\cos\left(2\,a+2\,b\,x\right)+6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^2,x)
```

```
[Out] (2*d*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2)*(cos(2*a + 2*b*x)
- 2*cos(4*a + 4*b*x) - cos(6*a + 6*b*x) + 2))/(5*b*(15*cos(2*a + 2*b*x) + 6
*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) + 10))
```

3.239 $\int (d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=210

$$\frac{d^{3/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\,b}-\frac{d^{3/2}\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\,b}+\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\ \mathrm{ta}\right.}{}$$

```
[Out] 1/2*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-1/2*d^
(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+1/4*d^(3/2)*
ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)-1/4*d
^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2
)+2*d*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3554, 3557, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{d^{3/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\,b}-\frac{d^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,b}+\frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}\,b}-\frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}\,b}+\frac{2d\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(Sqrt[2]*b) -
(d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(Sqrt[2]*b) +
(d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]
)/(2*Sqrt[2]*b) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqr
t[d*Tan[a + b*x]]])/(2*Sqrt[2]*b) + (2*d*Sqrt[d*Tan[a + b*x]])/b
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (d\tan(a+bx))^{3/2}\,dx = \frac{2d\sqrt{d\tan(a+bx)}}{b} - d^2\int \frac{1}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(d^2+x^2)}\,dx, x, d\tan(a+bx)\right)}{b}$$

$$= \frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{b}$$

$$= \frac{2d\sqrt{d\tan(a+bx)}}{b} - \frac{d^2\text{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{b} - \frac{d^2\text{Subst}\left(\int \frac{d+x}{d^2+}\right.}{}$$

$$= \frac{2d\sqrt{d\tan(a+bx)}}{b} + \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}\,b} + \frac{d}{}$$

$$= \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}\,b} - \frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

$$= \frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\,b} - \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\,b}$$

Mathematica [A]
time = 0.29, size = 159, normalized size = 0.76

$$\frac{\left(2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{\tan(a+bx)}\right)-2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{\tan(a+bx)}\right)+\sqrt{2}\log\left(1-\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)-\sqrt{2}\log\left(1+\sqrt{2}\sqrt{\tan(a+bx)}+\tan(a+bx)\right)+8\sqrt{\tan(a+bx)}\right)(d\tan(a+bx))^{3/2}}{4b\tan^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] ((2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*x]]] - 2*Sqrt[2]*ArcTan[1 + S
qrt[2]*Sqrt[Tan[a + b*x]]] + Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[a + b*x]] + T
an[a + b*x]] - Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[Tan[a + b*x]] + Tan[a + b*x]] +
 8*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(4*b*Tan[a + b*x]^(3/2))
```

Maple [A]
time = 0.16, size = 149, normalized size = 0.71

method	result

derivativedivides	$\frac{2d\left(\sqrt{d\tan(bx+a)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\right.\right.}{8}\right.}{b}$
default	$\frac{2d\left(\sqrt{d\tan(bx+a)}-\frac{(d^2)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\right.\right.}{8}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*d*((d*tan(b*x+a))^(1/2)-1/8*(d^2)^(1/4)*2^(1/2)*(ln((d*tan(b*x+a)+(d^2)
^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(b*x+a)-(d^2)^(1/4)*
(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(1/4)*(d*
tan(b*x+a))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)+1))
)
```

Maxima [A]
time = 0.50, size = 170, normalized size = 0.81

$$-\frac{2\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}\,d^{\frac{5}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)-\sqrt{2}\,d^{\frac{5}{2}}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)-8\sqrt{d\tan(bx+a)}\,d^2}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(b*x
+ a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x + a))*d
^2)/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(161) = 322$.
time = 0.47, size = 533, normalized size = 2.54

$$\frac{4\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{1}{4}}b\arctan\left(-\frac{d^6+\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{3}{4}}b^3d\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}-\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{3}{4}}b^3\sqrt{\frac{\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{1}{4}}bd\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)+d^3\sin(bx+a)+\sqrt{\frac{d^6}{b^4}}b^2\cos(bx+a)}{\cos(bx+a)}}}{d^6}\right)+4\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{1}{4}}b\arctan\left(\frac{d^6-\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{3}{4}}b^3d\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}+\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{3}{4}}b^3\sqrt{-\frac{\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{1}{4}}bd\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)-d^3\sin(bx+a)-\sqrt{\frac{d^6}{b^4}}b^2\cos(bx+a)}{\cos(bx+a)}}}{d^6}\right)-\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{1}{4}}b\log\left(\frac{\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{1}{4}}bd\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)+d^3\sin(bx+a)+\sqrt{\frac{d^6}{b^4}}b^2\cos(bx+a)}{\cos(bx+a)}\right)+\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{1}{4}}b\log\left(-\frac{\sqrt{2}\left(\frac{d^6}{b^4}\right)^{\frac{1}{4}}bd\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\cos(bx+a)-d^3\sin(bx+a)-\sqrt{\frac{d^6}{b^4}}b^2\cos(bx+a)}{\cos(bx+a)}\right)+8d\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(-(d^6 + sqrt(2)*(d^6/b^4)^(3/4)*b^3
*d*sqrt(d*sin(b*x + a)/cos(b*x + a)) - sqrt(2)*(d^6/b^4)^(3/4)*b^3*sqrt((sq
rt(2)*(d^6/b^4)^(1/4)*b*d*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a) +
d^3*sin(b*x + a) + sqrt(d^6/b^4)*b^2*cos(b*x + a))/cos(b*x + a)))/d^6) + 4*
sqrt(2)*(d^6/b^4)^(1/4)*b*arctan((d^6 - sqrt(2)*(d^6/b^4)^(3/4)*b^3*d*sqrt(
d*sin(b*x + a)/cos(b*x + a)) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*sqrt(-(sqrt(2)*(
d^6/b^4)^(1/4)*b*d*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a) - d^3*sin
(b*x + a) - sqrt(d^6/b^4)*b^2*cos(b*x + a))/cos(b*x + a)))/d^6) - sqrt(2)*(
d^6/b^4)^(1/4)*b*log((sqrt(2)*(d^6/b^4)^(1/4)*b*d*sqrt(d*sin(b*x + a)/cos(b
*x + a))*cos(b*x + a) + d^3*sin(b*x + a) + sqrt(d^6/b^4)*b^2*cos(b*x + a))/
cos(b*x + a)) + sqrt(2)*(d^6/b^4)^(1/4)*b*log(-(sqrt(2)*(d^6/b^4)^(1/4)*b*d
*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a) - d^3*sin(b*x + a) - sqrt(d
^6/b^4)*b^2*cos(b*x + a))/cos(b*x + a)) + 8*d*sqrt(d*sin(b*x + a)/cos(b*x +
 a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(a+bx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2), x)
```

Mupad [B]
time = 2.65, size = 73, normalized size = 0.35

$$\frac{2\,d\,\sqrt{d\tan(a+b\,x)}}{b}+\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{b}+\frac{(-1)^{1/4}\,d^{3/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan(a+b\,x)}}{\sqrt{d}}\right)\,1\mathrm{i}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2),x)
```

```
[Out] (2*d*(d*tan(a + b*x))^(1/2))/b + ((-1)^(1/4)*d^(3/2)*atan(((-1)^(1/4)*(d*ta
n(a + b*x))^(1/2))/d^(1/2))*1i)/b + ((-1)^(1/4)*d^(3/2)*atanh(((-1)^(1/4)*(
d*tan(a + b*x))^(1/2))/d^(1/2))*1i)/b
```

3.240 $\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=225

$$-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}+\frac{d^{3/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}-\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\right.}{}$$

```
[Out] -1/8*d^(3/2)*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)+1/8*d
^(3/2)*arctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b*2^(1/2)-1/16*d^(3/2
)*ln(d^(1/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(1/2)+1/1
6*d^(3/2)*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b*2^(
1/2)-1/2*d*cos(b*x+a)^2*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.12, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2687, 294, 335, 217, 1179, 642, 1176, 631, 210}

$$-\frac{d^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}\,b}-\frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,b}+\frac{d^{3/2}\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,b}-\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/4*(d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(Sqrt[2]*
b) + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2
]*b) - (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a +
 b*x]]])/(8*Sqrt[2]*b) + (d^(3/2)*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt
[2]*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b) - (d*Cos[a + b*x]^2*Sqrt[d*Tan[a +
 b*x]])/(2*b)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\int \cos^2(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^{3/2}}{(1+x^2)^2}\,dx, x, \tan(a+bx)\right)}{b}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} + \frac{d^2\text{Subst}\left(\int \frac{1}{\sqrt{dx}\,(1+x^2)}\,dx, x, \tan\ldots\right.}{4b}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} + \frac{d\text{Subst}\left(\int \frac{1}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a\ldots}\right.}{2b}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} + \frac{\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a\ldots}\right.}{4b}$$

$$= -\frac{d\cos^2(a+bx)\sqrt{d\tan(a+bx)}}{2b} - \frac{d^{3/2}\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,d\ldots\right.}{8\sqrt{2}\,b}$$

$$= -\frac{d^{3/2}\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}\,b} + \frac{d^{3/2}\ldots}{\ldots}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,b} + \frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\ldots}}{\ldots}\right)}{4\sqrt{2}\,b}$$

Mathematica [A]
time = 0.28, size = 110, normalized size = 0.49

$$-\frac{d\csc(a+bx)\left(\sin(a+bx)+\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}-\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}+\sin(3(a+bx))\right)\sqrt{d\tan(a+bx)}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/8*(d*Csc[a + b*x]*(Sin[a + b*x] + ArcSin[Cos[a + b*x] - Sin[a + b*x]]*Sq
rt[Sin[2*(a + b*x)]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*
x)]]]*Sqrt[Sin[2*(a + b*x)]] + Sin[3*(a + b*x)])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.28, size = 670, normalized size = 2.98

method	result

default	$\frac{(-1+\cos(bx+a))\left(-i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sin(bx+a)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(-1+cos(b*x+a))*(-I*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2
)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/
sin(b*x+a))^(1/2)+I*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2
),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((co
s(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*
x+a))^(1/2)+2*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2
^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*
x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-Ellip
ticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*
sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/si
n(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-EllipticPi((-
(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+
a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a)
)^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b*x+a)^3*2^(1/2
)+2*cos(b*x+a)^2*2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x
+a))^(3/2)/sin(b*x+a)^5*2^(1/2)
```

Maxima [A]
time = 0.51, size = 188, normalized size = 0.84

$$\frac{2\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+2\sqrt{2}\,d^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)+\sqrt{2}\,d^{\frac{5}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)-\sqrt{2}\,d^{\frac{5}{2}}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)-\frac{8\sqrt{d\tan(bx+a)}\,d^4}{d^2\tan(bx+a)^2+d^2}}{16\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*sqrt(2)*d^(5/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
b*x + a)))/sqrt(d)) + 2*sqrt(2)*d^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d*tan(b*x + a)))/sqrt(d)) + sqrt(2)*d^(5/2)*log(d*tan(b*x + a) +
 sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - sqrt(2)*d^(5/2)*log(d*tan(b*x
+ a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d) - 8*sqrt(d*tan(b*x + a))*d
^4/(d^2*tan(b*x + a)^2 + d^2))/(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1558 vs. $2(169) = 338$.
time = 42.02, size = 1558, normalized size = 6.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")

[Out] -1/32*(16*d*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)^2 + 2*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(1/2*(2*d^10*sin(b*x + a) + sqrt(4*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + d^10 + 2*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^7*cos(b*x + a)^3 - b^2*d^7*cos(b*x + a))*sqrt(d^6/b^4) + (sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((2*d^10*cos(b*x + a)^2 - d^10)*sin(b*x + a))) + 2*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(-1/2*(2*d^10*sin(b*x + a) - sqrt(4*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + d^10 - 2*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) - 4*(b^2*d^7*cos(b*x + a)^3 - b^2*d^7*cos(b*x + a))*sqrt(d^6/b^4) - (sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/((2*d^10*cos(b*x + a)^2 - d^10)*sin(b*x + a))) + 2*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(1/2*(sqrt(4*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + d^10 - 2*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(2*d^5*sin(b*x + a) + (sqrt(2)*(d^6/b^4)^(1/4)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + (sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a) - sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(d^10*sin(b*x + a))) + 2*sqrt(2)*(d^6/b^4)^(1/4)*b*arctan(-1/2*(sqrt(4*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + d^10 + 2*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))*(2*d^5*sin(b*x + a) - (sqrt(2)*(d^6/b^4)^(1/4)*b*d^3*cos(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) - (sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a) - sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(d^10*sin(b*x + a))) - sqrt(2)*(d^6/b^4)^(1/4)*b*log(4*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + d^10 + 2*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))) + sqrt(2)*(d^6/b^4)^(1/4)*b*log(4*sqrt(d^6/b^4)*b^2*d^7*cos(b*x + a)*sin(b*x + a) + d^10 - 2*(sqrt(2)*(d^6/b^4)^(1/4)*b*d^8*cos(b*x + a)*sin(b*x + a) + sqrt(2)*(d^6/b^4)^(3/4)*b^3*d^5*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a))))/b

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*(d*tan(b*x+a))**(3/2),x)
[Out] Timed out
```

Giac [A]
time = 0.52, size = 210, normalized size = 0.93

$$\frac{1}{16}\,d\left(\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{2\sqrt{2}\sqrt{|d|}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{|d|}}\right)}{b}+\frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{b}-\frac{\sqrt{2}\sqrt{|d|}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{|d|}+|d|\right)}{b}-\frac{8\sqrt{d\tan(bx+a)}\,d^2}{\left(d^2\tan(bx+a)^2+d^2\right)b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] 1/16*d*(2*sqrt(2)*sqrt(abs(d))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b + 2*sqrt(2)*sqrt(abs(d))*arctan(-1/2
*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/b +
sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan(b*x + a))*sqrt
(abs(d)) + abs(d))/b - sqrt(2)*sqrt(abs(d))*log(d*tan(b*x + a) - sqrt(2)*sq
rt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/b - 8*sqrt(d*tan(b*x + a))*d^2/((
d^2*tan(b*x + a)^2 + d^2)*b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,{\left(d\,\mathrm{tan}\left(a+b\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2),x)
[Out] int(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2), x)
```

3.241 $\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=136

$$-\frac{4d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{77b\sqrt{d\tan(a+bx)}}-\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b}-\frac{2d\sec^3(a+bx)\sqrt{d\,\mathrm{t}}}{77b}$$

```
[Out] 4/77*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*
Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)-4/7
7*d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b-2/77*d*sec(b*x+a)^3*(d*tan(b*x+a))^(1
/2)/b+2/11*d*sec(b*x+a)^5*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2691, 2693, 2694, 2653, 2720}

$$-\frac{4d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{77b\sqrt{d\tan(a+bx)}}+\frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}-\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b}-\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-4*d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
77*b*Sqrt[d*Tan[a + b*x]]) - (4*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(77*b)
 - (2*d*Sec[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(77*b) + (2*d*Sec[a + b*x]^5*S
qrt[d*Tan[a + b*x]])/(11*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
```

```
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b} - \frac{1}{11}d^2\int \frac{\sec^5(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= -\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b} + \frac{2d\sec^5(a+bx)\sqrt{d\tan(a+bx)}}{11b}$$

$$= -\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b} - \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b}$$

$$= -\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b} - \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b}$$

$$= -\frac{4d\sec(a+bx)\sqrt{d\tan(a+bx)}}{77b} - \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{77b}$$

$$= -\frac{4d^2F\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{77b\sqrt{d\tan(a+bx)}} - \frac{4d\sec(a+bx)}{7}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.91, size = 90, normalized size = 0.66

$$-\frac{d\sec^5(a+bx)\left(-23+6\cos(2(a+bx))+\cos(4(a+bx))+16\cos^6(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a+bx)}}{154b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/154*(d*Sec[a + b*x]^5*(-23 + 6*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 16*
Cos[a + b*x]^6*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a
 + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [A]
time = 0.54, size = 251, normalized size = 1.85

method	result
default	$-\frac{(-1+\cos(bx+a))\left(-4\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\left(\cos^5(bx+a)\right)\sin(bx+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/77/b*(-1+cos(b*x+a))*(-4*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)
-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1
/2)*cos(b*x+a)^5*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a
))^(1/2),1/2*2^(1/2))+2*2^(1/2)*cos(b*x+a)^5-2*cos(b*x+a)^4*2^(1/2)+cos(b*x
+a)^3*2^(1/2)-cos(b*x+a)^2*2^(1/2)-7*cos(b*x+a)*2^(1/2)+7*2^(1/2))*(cos(b*x
+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)^5/cos(b*x+a)^4*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^5, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.11, size = 125, normalized size = 0.92

$$\frac{2\left(2\sqrt{i\,d}\,d\cos(bx+a)^5\,\mathrm{ellipticF}(\cos(bx+a)+i\,\sin(bx+a)\,,-1)+2\sqrt{-i\,d}\,d\cos(bx+a)^5\,\mathrm{ellipticF}(\cos(bx+a)-i\,\sin(bx+a)\,,-1)-\left(2\,d\cos(bx+a)^4+d\cos(bx+a)^2-7\,d\right)\sqrt{\frac{d\sin(bx+a)}{\cos(bx+a)}}\right)}{77\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(2*sqrt(I*d)*d*cos(b*x + a)^5*ellipticF(cos(b*x + a) + I*sin(b*x + a),
 -1) + 2*sqrt(-I*d)*d*cos(b*x + a)^5*ellipticF(cos(b*x + a) - I*sin(b*x + a
), -1) - (2*d*cos(b*x + a)^4 + d*cos(b*x + a)^2 - 7*d)*sqrt(d*sin(b*x + a)/
cos(b*x + a)))/(b*cos(b*x + a)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d \tan (a + bx))^{\frac{3}{2}} \sec^5 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*(d*tan(b*x+a))**(3/2),x)
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d \tan(a + b\,x))^{3/2}}{\cos (a + b\,x)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^5,x)
[Out] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^5, x)
```

3.242 $\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=108

$$-\frac{2d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{21b\sqrt{d\tan(a+bx)}}-\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b}+\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}$$

```
[Out] 2/21*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*
Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)-2/2
1*d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b+2/7*d*sec(b*x+a)^3*(d*tan(b*x+a))^(1/
2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2691, 2693, 2694, 2653, 2720}

$$-\frac{2d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{21b\sqrt{d\tan(a+bx)}}+\frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}-\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
21*b*Sqrt[d*Tan[a + b*x]]) - (2*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(21*b)
 + (2*d*Sec[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(7*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
```

```
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b} - \frac{1}{7}d^2\int \frac{\sec^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= -\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}$$

$$= -\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}$$

$$= -\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{21b} + \frac{2d\sec^3(a+bx)\sqrt{d\tan(a+bx)}}{7b}$$

$$= -\frac{2d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{21b\sqrt{d\tan(a+bx)}} - \frac{2d\sec(a+bx)}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.51, size = 80, normalized size = 0.74

$$-\frac{d\sec^3(a+bx)\left(-5+\cos(2(a+bx))+4\cos^4(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/21*(d*Sec[a + b*x]^3*(-5 + Cos[2*(a + b*x)] + 4*Cos[a + b*x]^4*Hypergeom
etric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a
 + b*x]])/b
```

Maple [A]
time = 0.34, size = 225, normalized size = 2.08

method	result
default	$\frac{(-1+\cos(bx+a))\left(2\,\mathrm{EllipticF}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sin(bx+a)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/b*(-1+cos(b*x+a))*(2*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x
+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2)*cos(b*x+a)^3-cos(b*x+a)^3*2^(1/2)+cos(b*x+a)^2*2^(1/2)+3*cos(b*x+a)*
2^(1/2)-3*2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x
+a)^5/cos(b*x+a)^2*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^3, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.10, size = 112, normalized size = 1.04

$$\frac{2\left(\sqrt{i d}\, d\cos\left(bx+a\right)^{3} \mathrm{ellipticF}\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right),-1\right)+\sqrt{-i d}\, d\cos\left(bx+a\right)^{3}\mathrm{ellipticF}\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right),-1\right)-\left(d\cos\left(bx+a\right)^{2}-3\,d\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}\right)}{21\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(sqrt(I*d)*d*cos(b*x + a)^3*ellipticF(cos(b*x + a) + I*sin(b*x + a), -
1) + sqrt(-I*d)*d*cos(b*x + a)^3*ellipticF(cos(b*x + a) - I*sin(b*x + a), -
1) - (d*cos(b*x + a)^2 - 3*d)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x
 + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan (a+bx))^{\frac{3}{2}} \sec ^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,\mathrm{tan}\left(a+b\,x\right)\right)}^{3/2}}{{\cos\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^3,x)
[Out] int((d*tan(a + b*x))^(3/2)/cos(a + b*x)^3, x)
```

3.243 $\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=80

$$-\frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3b\sqrt{d\tan(a+bx)}}+\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

```
[Out] 1/3*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*P
i+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)+2/3*
d*sec(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2691, 2694, 2653, 2720}

$$\frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b}-\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3b\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/3*(d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])
/(b*Sqrt[d*Tan[a + b*x]]) + (2*d*Sec[a + b*x]*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(a+bx)(d\tan(a+bx))^{3/2}\,dx = \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b} - \frac{1}{3}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b} - \frac{\left(d^2\sqrt{\sin(a+bx)}\right)\int \frac{}{\sqrt{\cos(a+bx)}}}{3\sqrt{\cos(a+bx)}\sqrt{d\tan(}}$$

$$= \frac{2d\sec(a+bx)\sqrt{d\tan(a+bx)}}{3b} - \frac{\left(d^2\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int}{3\sqrt{d\tan(a+bx)}}$$

$$= -\frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3b\sqrt{d\tan(a+bx)}} + \frac{2d\sec(a+bx)\sqrt{}}{3b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.32, size = 69, normalized size = 0.86

$$\frac{2d\cos(a+bx)\left(\sec^2(a+bx) - {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*d*Cos[a + b*x]*(Sec[a + b*x]^2 - Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a
 + b*x]^2]*Sqrt[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Maple [A]
time = 0.25, size = 188, normalized size = 2.35

method	result
default	$\frac{(-1+\cos(bx+a))\left(\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\cos(bx+a)\sin(bx+a)\,\mathrm{Elliptic}\right.}{3b\sin(bx+a)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*(-1+cos(b*x+a))*(((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+si
n(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*c
os(b*x+a)*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2
),1/2*2^(1/2))+cos(b*x+a)*2^(1/2)-2^(1/2))*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/c
os(b*x+a))^(3/2)/sin(b*x+a)^5*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.09, size = 95, normalized size = 1.19

$$\frac{\sqrt{i\,d}\,d\cos\left(bx+a\right)\operatorname{ellipticF}(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right),-1)+\sqrt{-i\,d}\,d\cos\left(bx+a\right)\operatorname{ellipticF}(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right),-1)+2\,d\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{3\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 1/3*(sqrt(I*d)*d*cos(b*x + a)*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1)
+ sqrt(-I*d)*d*cos(b*x + a)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1) +
2*d*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(b*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}\sec\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))**(3/2),x)
[Out] Integral((d*tan(a + b*x))**(3/2)*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*sec(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(a+b\,x))^{3/2}}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^(3/2)/cos(a + b*x),x)
```

```
[Out] int((d*tan(a + b*x))^(3/2)/cos(a + b*x), x)
```

3.244 $\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=78

$$\frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

```
[Out] -1/2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*
Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)-d*c
os(b*x+a)*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2690, 2694, 2653, 2720}

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{d\tan(a+bx)}}-\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(2*b
*Sqrt[d*Tan[a + b*x]]) - (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/b
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b} + \frac{1}{2}d^2\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= -\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b} + \frac{\left(d^2\sqrt{\sin(a+bx)}\right)\int \frac{}{\sqrt{\cos(a+bx}}}{2\sqrt{\cos(a+bx)}\sqrt{d\tan(}}$$

$$= -\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{b} + \frac{\left(d^2\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int}{2\sqrt{d\tan(a+bx}}$$

$$= \frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\tan(a+bx)}} - \frac{d\cos(a+bx)\sqrt{dt}}{b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.15, size = 58, normalized size = 0.74

$$\frac{d\cos(a+bx)\left(-1+{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sqrt{d\tan(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*(d*Tan[a + b*x])^(3/2),x]
[Out] (d*Cos[a + b*x]*(-1 + Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[a + b*x]^2]*Sqr
t[Sec[a + b*x]^2])*Sqrt[d*Tan[a + b*x]])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(95) = 190.
time = 0.31, size = 196, normalized size = 2.51

method	result
default	$-\frac{(-1+\cos(bx+a))\left(\mathrm{EllipticF}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sin(bx+a)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a}{\sin(bx+a)}}\right.}{2b\sin(bx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b*(-1+cos(b*x+a))*(EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(
1/2),1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a
)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(
1/2)+cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(
d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)^5*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)*tan(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan{(a+bx)})^{\frac{3}{2}}\cos{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(3/2)*cos(a + b*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument Typeex
t_reduce Error: Bad Argument TypeEvaluation time: 9.92Done
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (a+b x) (d \tan(a+b x))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(cos(a + b*x)*(d*tan(a + b*x))^(3/2), x)
```

3.245 $\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=108

$$\frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\tan(a+bx)}}+\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}$$

```
[Out] -1/12*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4
*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)+1/
6*d*cos(b*x+a)*(d*tan(b*x+a))^(1/2)/b-1/3*d*cos(b*x+a)^3*(d*tan(b*x+a))^(1/
2)/b
```

Rubi [A]

time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2690, 2692, 2694, 2653, 2720}

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{12b\sqrt{d\tan(a+bx)}}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(12*
b*Sqrt[d*Tan[a + b*x]]) + (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/(6*b) - (d*
Cos[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(3*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
```

```
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{1}{6}d^2\int \frac{\cos(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+\frac{1}{12}$$

$$= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+$$

$$= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{6b}-\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{3b}+$$

$$= \frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\tan(a+bx)}}+\frac{d\cos(a+bx)\sqrt{d}}{6b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.20, size = 96, normalized size = 0.89

$$-\frac{\cos(a+bx)\left(\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,\sqrt{\tan(a+bx)}\right)\middle|-1\right)\sqrt{\sec^2(a+bx)}+\cos(2(a+bx))\sqrt{\tan(a+bx)}\right)(d\tan(a+bx))^{3/2}}{6b\tan^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -1/6*(Cos[a + b*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a +
b*x]]], -1]*Sqrt[Sec[a + b*x]^2] + Cos[2*(a + b*x)]*Sqrt[Tan[a + b*x]])*(d*
Tan[a + b*x])^(3/2))/(b*Tan[a + b*x]^(3/2))
```

Maple [A]
time = 0.30, size = 222, normalized size = 2.06

method	result
default	$-\frac{(-1+\cos(bx+a))\left(\mathrm{EllipticF}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sin(bx+a)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/b*(-1+cos(b*x+a))*(EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+
a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^
(1/2)+2*cos(b*x+a)^4*2^(1/2)-2*cos(b*x+a)^3*2^(1/2)-cos(b*x+a)^2*2^(1/2)+co
s(b*x+a)*2^(1/2))*cos(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/
2)/sin(b*x+a)^5*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)^3*tan(b*x + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument Typeex
t_reduce Error: Bad Argument TypeEvaluation time: 10.07Done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)^3\,(d\tan(a+b\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2), x)
```

3.246 $\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{24b\sqrt{d\tan(a+bx)}}+\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a}{30b}$$

```
[Out] -1/24*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4
*Pi+b*x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/(d*tan(b*x+a))^(1/2)+1/
12*d*cos(b*x+a)*(d*tan(b*x+a))^(1/2)/b+1/30*d*cos(b*x+a)^3*(d*tan(b*x+a))^(
1/2)/b-1/5*d*cos(b*x+a)^5*(d*tan(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2690, 2692, 2694, 2653, 2720}

$$\frac{d^2\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{24b\sqrt{d\tan(a+bx)}}-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}+\frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (d^2*EllipticF[a - Pi/4 + b*x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(24*
b*Sqrt[d*Tan[a + b*x]]) + (d*Cos[a + b*x]*Sqrt[d*Tan[a + b*x]])/(12*b) + (d
*Cos[a + b*x]^3*Sqrt[d*Tan[a + b*x]])/(30*b) - (d*Cos[a + b*x]^5*Sqrt[d*Tan
[a + b*x]])/(5*b)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
```

```
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(a+bx)(d\tan(a+bx))^{3/2}\,dx = -\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\frac{1}{10}d^2\int \frac{\cos^3(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx$$

$$= \frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\frac{d\cos^5(a+bx)\sqrt{d\tan(a+bx)}}{5b}+\ldots$$

$$= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\ldots$$

$$= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\ldots$$

$$= \frac{d\cos(a+bx)\sqrt{d\tan(a+bx)}}{12b}+\frac{d\cos^3(a+bx)\sqrt{d\tan(a+bx)}}{30b}-\ldots$$

$$= \frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{24b\sqrt{d\tan(a+bx)}}+\frac{d\cos(a+bx)\sqrt{d\ldots}}{12b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.60, size = 131, normalized size = 0.96

$$\frac{\cos(2(a+bx))\csc(a+bx)\left(10\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\sqrt{\sec^2(a+bx)}+(-3+10\cos(2(a+bx))+3\cos(4(a+bx)))\sqrt{\tan(a+bx)}\right)(d\tan(a+bx))^{3/2}}{120b\sqrt{\tan(a+bx)}\,(-1+\tan^2(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*(d*Tan[a + b*x])^(3/2),x]
```

[Out] (Cos[2*(a + b*x)]*Csc[a + b*x]*(10*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4
)*Sqrt[Tan[a + b*x]]], -1]*Sqrt[Sec[a + b*x]^2] + (-3 + 10*Cos[2*(a + b*x)]
 + 3*Cos[4*(a + b*x)])*Sqrt[Tan[a + b*x]])*(d*Tan[a + b*x])^(3/2))/(120*b*S
qrt[Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))

Maple [A]
time = 0.30, size = 250, normalized size = 1.84

method	result
default	$-\frac{(-1+\cos(bx+a))\left(12(\cos^6(bx+a))\sqrt{2}\,-12\sqrt{2}\,(\cos^5(bx+a))+5\,\mathrm{EllipticF}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sin(bx+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/120/b*(-1+cos(b*x+a))*(12*2^(1/2)*cos(b*x+a)^6-12*2^(1/2)*cos(b*x+a)^5+5
*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b
x+a)((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x
+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-2*cos(b*x+a)^4*2^(
1/2)+2*cos(b*x+a)^3*2^(1/2)-5*cos(b*x+a)^2*2^(1/2)+5*cos(b*x+a)*2^(1/2))*co
s(b*x+a)*(cos(b*x+a)+1)^2*(d*sin(b*x+a)/cos(b*x+a))^(3/2)/sin(b*x+a)^5*2^(1
/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate((d*tan(b*x + a))^(3/2)*cos(b*x + a)^5, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(d*tan(b*x + a))*d*cos(b*x + a)^5*tan(b*x + a), x)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument Typeex
t_reduce Error: Bad Argument TypeEvaluation time: 10.04Done
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^5\,{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2), x)
```

3.247 $\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=67

$$\frac{2(d\tan(e+fx))^{7/2}}{7df}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{15/2}}{15d^5f}$$

```
[Out] 2/7*(d*tan(f*x+e))^(7/2)/d/f+4/11*(d*tan(f*x+e))^(11/2)/d^3/f+2/15*(d*tan(f
*x+e))^(15/2)/d^5/f
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2687, 276}

$$\frac{2(d\tan(e+fx))^{15/2}}{15d^5f}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f) + (4*(d*Tan[e + f*x])^(11/2))/(11*d^3*f)
 + (2*(d*Tan[e + f*x])^(15/2))/(15*d^5*f)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^6(e+fx)(d\tan(e+fx))^{5/2}\,dx&=\frac{\text{Subst}\left(\int (dx)^{5/2}\left(1+x^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int \left((dx)^{5/2}+\frac{2(dx)^{9/2}}{d^2}+\frac{(dx)^{13/2}}{d^4}\right)\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{2(d\tan(e+fx))^{7/2}}{7df}+\frac{4(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{15/2}}{15d^5f}\end{aligned}$$

Mathematica [A]
time = 0.47, size = 52, normalized size = 0.78

$$\frac{2(117+44\cos(2(e+fx))+4\cos(4(e+fx)))\sec^4(e+fx)(d\tan(e+fx))^{7/2}}{1155df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(d*Tan[e + f*x])^(5/2),x]
[Out] (2*(117 + 44*Cos[2*(e + f*x)] + 4*Cos[4*(e + f*x)])*Sec[e + f*x]^4*(d*Tan[e
 + f*x])^(7/2))/(1155*d*f)
```

Maple [A]
time = 3.00, size = 60, normalized size = 0.90

method	result	size
default	$\frac{2\left(32\left(\cos^4(fx+e)\right)+56\left(\cos^2(fx+e)\right)+77\right)\left(\frac{d\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{5}{2}}\sin(fx+e)}{1155f\cos(fx+e)^5}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/1155/f*(32*cos(f*x+e)^4+56*cos(f*x+e)^2+77)*(d*sin(f*x+e)/cos(f*x+e))^(5/
2)*sin(f*x+e)/cos(f*x+e)^5
```

Maxima [A]
time = 0.28, size = 54, normalized size = 0.81

$$\frac{2\left(77\left(d\tan\left(fx+e\right)\right)^{\frac{15}{2}}+210\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}d^2+165\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^4\right)}{1155\,d^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] 2/1155*(77*(d*tan(f*x + e))^(15/2) + 210*(d*tan(f*x + e))^(11/2)*d^2 + 165*
(d*tan(f*x + e))^(7/2)*d^4)/(d^5*f)
```

Fricas [A]
time = 0.44, size = 89, normalized size = 1.33

$$-\frac{2\left(32\,d^2\cos\left(fx+e\right)^6+24\,d^2\cos\left(fx+e\right)^4+21\,d^2\cos\left(fx+e\right)^2-77\,d^2\right)\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{1155\,f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(32*d^2*cos(f*x + e)^6 + 24*d^2*cos(f*x + e)^4 + 21*d^2*cos(f*x + e
)^2 - 77*d^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e
)^7)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [**A**]
time = 0.57, size = 84, normalized size = 1.25

$$\frac{2\left(77\sqrt{d\tan\left(fx+e\right)}\,d^7\tan\left(fx+e\right)^7+210\sqrt{d\tan\left(fx+e\right)}\,d^7\tan\left(fx+e\right)^5+165\sqrt{d\tan\left(fx+e\right)}\,d^7\tan\left(fx+e\right)^3\right)}{1155\,d^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2/1155*(77*sqrt(d*tan(f*x + e))*d^7*tan(f*x + e)^7 + 210*sqrt(d*tan(f*x + e
))*d^7*tan(f*x + e)^5 + 165*sqrt(d*tan(f*x + e))*d^7*tan(f*x + e)^3)/(d^5*f
)
```

Mupad [**B**]
time = 13.29, size = 474, normalized size = 7.07

$$\frac{d^2\sqrt{-\frac{d\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{1155\,f}+\frac{d^2\sqrt{-\frac{d\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,64i}{1155\,f\,\left(e^{e\,2i+f\,x\,2i}+1\right)}+\frac{d^2\sqrt{-\frac{d\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,32i}{385\,f\,{\left(e^{e\,2i+f\,x\,2i}+1\right)}^2}-\frac{d^2\sqrt{-\frac{d\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,2432i}{231\,f\,{\left(e^{e\,2i+f\,x\,2i}+1\right)}^3}+\frac{d^2\sqrt{-\frac{d\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,1504i}{33\,f\,{\left(e^{e\,2i+f\,x\,2i}+1\right)}^4}-\frac{d^2\sqrt{-\frac{d\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,4288i}{55\,f\,{\left(e^{e\,2i+f\,x\,2i}+1\right)}^5}+\frac{d^2\sqrt{-\frac{d\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,896i}{15\,f\,{\left(e^{e\,2i+f\,x\,2i}+1\right)}^6}-\frac{d^2\sqrt{-\frac{d\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)}{e^{e\,2i+f\,x\,2i}+1}}\,256i}{15\,f\,{\left(e^{e\,2i+f\,x\,2i}+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^6,x)
```

```
[Out] (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*64i
)/(1155*f) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) +
1))^(1/2)*64i)/(1155*f*(exp(e*2i + f*x*2i) + 1)) + (d^2*(-(d*(exp(e*2i + f*
x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*32i)/(385*f*(exp(e*2i + f*x
*2i) + 1)^2) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i)
+ 1))^(1/2)*2432i)/(231*f*(exp(e*2i + f*x*2i) + 1)^3) + (d^2*(-(d*(exp(e*2i
 + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*1504i)/(33*f*(exp(e*2i
 + f*x*2i) + 1)^4) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*
x*2i) + 1))^(1/2)*4288i)/(55*f*(exp(e*2i + f*x*2i) + 1)^5) + (d^2*(-(d*(exp
(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*896i)/(15*f*(exp(
e*2i + f*x*2i) + 1)^6) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i
+ f*x*2i) + 1))^(1/2)*256i)/(15*f*(exp(e*2i + f*x*2i) + 1)^7)
```

3.248 $\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=45

$$\frac{2(d\tan(e+fx))^{7/2}}{7df}+\frac{2(d\tan(e+fx))^{11/2}}{11d^3f}$$

```
[Out] 2/7*(d*tan(f*x+e))^(7/2)/d/f+2/11*(d*tan(f*x+e))^(11/2)/d^3/f
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2687, 14}

$$\frac{2(d\tan(e+fx))^{11/2}}{11d^3f}+\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f) + (2*(d*Tan[e + f*x])^(11/2))/(11*d^3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(e+fx)(d\tan(e+fx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (dx)^{5/2}\left(1+x^2\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left((dx)^{5/2}+\frac{(dx)^{9/2}}{d^2}\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{2(d\tan(e+fx))^{7/2}}{7df}+\frac{2(d\tan(e+fx))^{11/2}}{11d^3f}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 42, normalized size = 0.93

$$\frac{2(9+2\cos(2(e+fx)))\sec^2(e+fx)(d\tan(e+fx))^{7/2}}{77df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
[Out] (2*(9 + 2*Cos[2*(e + f*x)])*Sec[e + f*x]^2*(d*Tan[e + f*x])^(7/2))/(77*d*f)
```

Maple [A]
time = 0.32, size = 50, normalized size = 1.11

method	result	size
default	$\frac{2\left(4\left(\cos^2(fx+e)\right)+7\right)\left(\frac{d\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{5}{2}}\sin(fx+e)}{77f\cos(fx+e)^3}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/77/f*(4*cos(f*x+e)^2+7)*(d*sin(f*x+e)/cos(f*x+e))^(5/2)*sin(f*x+e)/cos(f*
x+e)^3
```

Maxima [A]
time = 0.28, size = 38, normalized size = 0.84

$$\frac{2\left(7\left(d\tan\left(fx+e\right)\right)^{\frac{11}{2}}+11\left(d\tan\left(fx+e\right)\right)^{\frac{7}{2}}d^2\right)}{77\,d^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
[Out] 2/77*(7*(d*tan(f*x + e))^(11/2) + 11*(d*tan(f*x + e))^(7/2)*d^2)/(d^3*f)
```

Fricas [A]
time = 0.46, size = 75, normalized size = 1.67

$$-\frac{2\left(4\,d^2\cos\left(fx+e\right)^4+3\,d^2\cos\left(fx+e\right)^2-7\,d^2\right)\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{77\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*d^2*cos(f*x + e)^4 + 3*d^2*cos(f*x + e)^2 - 7*d^2)*sqrt(d*sin(f*x
+ e)/cos(f*x + e))*sin(f*x + e)/(f*cos(f*x + e)^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3878 deep
```

Giac [A]
time = 0.52, size = 59, normalized size = 1.31

$$\frac{2\left(7\sqrt{d\tan\left(fx+e\right)}\,d^5\tan\left(fx+e\right)^5+11\sqrt{d\tan\left(fx+e\right)}\,d^5\tan\left(fx+e\right)^3\right)}{77\,d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2/77*(7*sqrt(d*tan(f*x + e))*d^5*tan(f*x + e)^5 + 11*sqrt(d*tan(f*x + e))*d
^5*tan(f*x + e)^3)/(d^3*f)
```

Mupad [B]
time = 7.22, size = 352, normalized size = 7.82

$$\frac{d^2\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,8\mathrm{i}}{77\,f}+\frac{d^2\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,8\mathrm{i}}{77\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}-\frac{d^2\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,296\mathrm{i}}{77\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}+\frac{d^2\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,944\mathrm{i}}{77\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3}-\frac{d^2\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,160\mathrm{i}}{11\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4}+\frac{d^2\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^4,x)
```

```
[Out] (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*8i)
/(77*f) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))
^(1/2)*8i)/(77*f*(exp(e*2i + f*x*2i) + 1)) - (d^2*(-(d*(exp(e*2i + f*x*2i)*
1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*296i)/(77*f*(exp(e*2i + f*x*2i) +
 1)^2) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^
(1/2)*944i)/(77*f*(exp(e*2i + f*x*2i) + 1)^3) - (d^2*(-(d*(exp(e*2i + f*x*2
i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*160i)/(11*f*(exp(e*2i + f*x*2i
) + 1)^4) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1
))^(1/2)*64i)/(11*f*(exp(e*2i + f*x*2i) + 1)^5)
```

3.249 $\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=22

$$\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

```
[Out] 2/7*(d*tan(f*x+e))^(7/2)/d/f
```

Rubi [A]
time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2687, 32}

$$\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(e+fx)(d\tan(e+fx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (dx)^{5/2}\,dx, x, \tan(e+fx)\right)}{f}\\ &= \frac{2(d\tan(e+fx))^{7/2}}{7df}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 22, normalized size = 1.00

$$\frac{2(d\tan(e+fx))^{7/2}}{7df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*(d*Tan[e + f*x])^(7/2))/(7*d*f)
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7df}$	19
default	$\frac{2(d\tan(fx+e))^{\frac{7}{2}}}{7df}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(d*tan(f*x+e))^(7/2)/d/f
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.86

$$\frac{2\,(d\tan\left(fx+e\right))^{\frac{7}{2}}}{7\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(d*tan(f*x + e))^(7/2)/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(19) = 38.
time = 0.41, size = 60, normalized size = 2.73

$$-\frac{2\left(d^{2}\cos\left(fx+e\right)^{2}-d^{2}\right)\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{7\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/7*(d^2*cos(f*x + e)^2 - d^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x +
 e)/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^{\frac{5}{2}}\sec^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((d*tan(e + f*x))**(5/2)*sec(e + f*x)**2, x)
```

Giac [A]
time = 0.51, size = 28, normalized size = 1.27

$$\frac{2\sqrt{d\tan(fx+e)}\,d^2\tan(fx+e)^3}{7\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2/7*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e)^3/f
```

Mupad [B]
time = 5.58, size = 230, normalized size = 10.45

$$\frac{d^2\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,2\mathrm{i}}{7\,f}-\frac{d^2\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,12\mathrm{i}}{7\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}+\frac{d^2\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,24\mathrm{i}}{7\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}-\frac{d^2\sqrt{-\frac{d\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2)/cos(e + f*x)^2,x)
```

```
[Out] (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*2i)
/(7*f) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^
(1/2)*12i)/(7*f*(exp(e*2i + f*x*2i) + 1)) + (d^2*(-(d*(exp(e*2i + f*x*2i)*1
i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/2)*24i)/(7*f*(exp(e*2i + f*x*2i) + 1)
^2) - (d^2*(-(d*(exp(e*2i + f*x*2i)*1i - 1i))/(exp(e*2i + f*x*2i) + 1))^(1/
2)*16i)/(7*f*(exp(e*2i + f*x*2i) + 1)^3)
```

3.250 $\int (d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=212

$$\frac{d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{d^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\,\text{ta}\right.}{}$$

```
[Out] 1/2*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/2*d^
(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)-1/4*d^(5/2)*
ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/2)+1/4*d
^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/2
)+2/3*d*(d*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.10, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3554, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}\,f}-\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,f}-\frac{d^{5/2}\log\left(\sqrt{d}\,\tan(e+fx)-\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}+\frac{d^{5/2}\log\left(\sqrt{d}\,\tan(e+fx)+\sqrt{2}\,\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{2\sqrt{2}\,f}+\frac{2d(d\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(Sqrt[2]*f) -
(d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan[e + f*x]]]
)/(2*Sqrt[2]*f) + (d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] + Sqrt[2]*Sqr
t[d*Tan[e + f*x]]])/(2*Sqrt[2]*f) + (2*d*(d*Tan[e + f*x])^(3/2))/(3*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (d\tan(e+fx))^{5/2}\,dx = \frac{2d(d\tan(e+fx))^{3/2}}{3f} - d^2\int\sqrt{d\tan(e+fx)}\,dx$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} - \frac{d^3\text{Subst}\left(\int\frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(e+fx)\right)}{f}$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} - \frac{(2d^3)\,\text{Subst}\left(\int\frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f}$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} + \frac{d^3\text{Subst}\left(\int\frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{f} - \frac{d^3\text{Subst}\left(\int \frac{d}{d^2}\right.}{}$$

$$= \frac{2d(d\tan(e+fx))^{3/2}}{3f} - \frac{d^{5/2}\text{Subst}\left(\int\frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x, \sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} -$$

$$= -\frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{2\sqrt{2}f} + \frac{d^{5/2}\log\left(\sqrt{d}+\sqrt{}\right.}{}$$

$$= \frac{d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{\sqrt{2}f} - \frac{d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right.}{\sqrt{2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 40, normalized size = 0.19

$$-\frac{2d\left(-1+{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)\right)(d\tan(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*d*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2])*(d*Tan[e + f*x
])^(3/2))/(3*f)
```

Maple [A]
time = 0.09, size = 154, normalized size = 0.73

method	result

derivativedivides	$\frac{2d\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\ldots}}{(d\ldots}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f}$
default	$\frac{2d\left(\frac{(d\tan(fx+e))^{\frac{3}{2}}}{3}-\frac{d^2\sqrt{2}\left(\ln\left(\frac{d\tan(fx+e)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}{d\tan(fx+e)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(fx+e)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{d\tan\ldots}}{(d\ldots}\right.\right.}{8(d^2)^{\frac{1}{4}}}\right.}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*d*(1/3*(d*tan(f*x+e))^(3/2)-1/8*d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(f*x+
e)-(d^2)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(f*x+e)+(d^2
)^(1/4)*(d*tan(f*x+e))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2)^(
1/4)*(d*tan(f*x+e))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(f*x+e))^(
1/2)+1)))
```

Maxima [A]
time = 0.49, size = 183, normalized size = 0.86

$$-\frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-8\,(d\tan(fx+e))^{\frac{3}{2}}d^2}{12\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) +
 sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x
+ e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(f*x +
e))^(3/2)*d^2)/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(168) = 336$.
time = 0.41, size = 630, normalized size = 2.97

$$\frac{12\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{1}{4}}f\arctan\left(-\frac{d^{10}+\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{3}{4}}d^7f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}-\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{1}{4}}f\sqrt{\frac{d^{15}\sin(fx+e)+\sqrt{\frac{d^{10}}{f^4}}d^{10}f^2\cos(fx+e)+\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{3}{4}}d^7f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{\cos(fx+e)}}}{d^{10}}\right)\cos(fx+e)+12\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{1}{4}}f\arctan\left(\frac{d^{10}-\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{3}{4}}d^7f\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}+\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{1}{4}}f\sqrt{\frac{d^{15}\sin(fx+e)+\sqrt{\frac{d^{10}}{f^4}}d^{10}f^2\cos(fx+e)-\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{3}{4}}d^7f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{\cos(fx+e)}}}{d^{10}}\right)\cos(fx+e)+3\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{1}{4}}f\cos(fx+e)\log\left(\frac{d^{15}\sin(fx+e)+\sqrt{\frac{d^{10}}{f^4}}d^{10}f^2\cos(fx+e)+\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{3}{4}}d^7f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{\cos(fx+e)}\right)-3\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{1}{4}}f\cos(fx+e)\log\left(\frac{d^{15}\sin(fx+e)+\sqrt{\frac{d^{10}}{f^4}}d^{10}f^2\cos(fx+e)-\sqrt{2}\left(\frac{d^{10}}{f^4}\right)^{\frac{3}{4}}d^7f^3\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\cos(fx+e)}{\cos(fx+e)}\right)+8d^2\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\sin(fx+e)}{12f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*sqrt(2)*(d^10/f^4)^(1/4)*f*arctan(-(d^10 + sqrt(2)*(d^10/f^4)^(1/4
)*d^7*f*sqrt(d*sin(f*x + e)/cos(f*x + e)) - sqrt(2)*(d^10/f^4)^(1/4)*f*sqrt
((d^15*sin(f*x + e) + sqrt(d^10/f^4)*d^10*f^2*cos(f*x + e) + sqrt(2)*(d^10/
f^4)^(3/4)*d^7*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/cos(f*x
+ e)))/d^10)*cos(f*x + e) + 12*sqrt(2)*(d^10/f^4)^(1/4)*f*arctan((d^10 - sq
rt(2)*(d^10/f^4)^(1/4)*d^7*f*sqrt(d*sin(f*x + e)/cos(f*x + e)) + sqrt(2)*(d
^10/f^4)^(1/4)*f*sqrt((d^15*sin(f*x + e) + sqrt(d^10/f^4)*d^10*f^2*cos(f*x
+ e) - sqrt(2)*(d^10/f^4)^(3/4)*d^7*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*c
os(f*x + e))/cos(f*x + e)))/d^10)*cos(f*x + e) + 3*sqrt(2)*(d^10/f^4)^(1/4)
*f*cos(f*x + e)*log((d^15*sin(f*x + e) + sqrt(d^10/f^4)*d^10*f^2*cos(f*x +
e) + sqrt(2)*(d^10/f^4)^(3/4)*d^7*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos
(f*x + e))/cos(f*x + e)) - 3*sqrt(2)*(d^10/f^4)^(1/4)*f*cos(f*x + e)*log((d
^15*sin(f*x + e) + sqrt(d^10/f^4)*d^10*f^2*cos(f*x + e) - sqrt(2)*(d^10/f^4
)^(3/4)*d^7*f^3*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e))/cos(f*x + e
)) + 8*d^2*sqrt(d*sin(f*x + e)/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(e+fx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral((d*tan(e + f*x))**(5/2), x)
```

Giac [A]
time = 0.45, size = 218, normalized size = 1.03

$$-\frac{1}{12}d^2\left(\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{6\sqrt{2}|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{|d|}}\right)}{df}-\frac{3\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{df}+\frac{3\sqrt{2}|d|^{\frac{3}{2}}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{|d|}+|d|\right)}{df}-\frac{8\sqrt{d\tan(fx+e)}\tan(fx+e)}{f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*d^2*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
+ 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) + 6*sqrt(2)*abs(d)^(3/2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)
))/(d*f) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f
*x + e))*sqrt(abs(d)) + abs(d))/(d*f) + 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*
x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) - 8*sqrt
(d*tan(f*x + e))*tan(f*x + e)/f)
```

Mupad [B]
time = 2.74, size = 74, normalized size = 0.35

$$\frac{2\,d\,(d\tan(e+f\,x))^{3/2}}{3\,f}-\frac{(-1)^{1/4}\,d^{5/2}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{f}+\frac{(-1)^{1/4}\,d^{5/2}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(e+f\,x\right)}}{\sqrt{d}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(5/2),x)
```

```
[Out] (2*d*(d*tan(e + f*x))^(3/2))/(3*f) - ((-1)^(1/4)*d^(5/2)*atan(((-1)^(1/4)*(
d*tan(e + f*x))^(1/2))/d^(1/2)))/f + ((-1)^(1/4)*d^(5/2)*atanh(((-1)^(1/4)*
(d*tan(e + f*x))^(1/2))/d^(1/2)))/f
```

3.251 $\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=225

$$-\frac{3d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}+\right.}{}$$

```
[Out] -3/8*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/8*d
^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/16*d^(5/2
)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/2)-3/1
6*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(
1/2)-1/2*d*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2687, 294, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{8\sqrt{2}f}-\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{8\sqrt{2}f}-\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-3*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt[2]*
f) + (3*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(4*Sqrt
[2]*f) + (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*Tan
[e + f*x]]])/(8*Sqrt[2]*f) - (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x]
+ Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(8*Sqrt[2]*f) - (d*Cos[e + f*x]^2*(d*Tan[e
 + f*x])^(3/2))/(2*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\int \cos^2(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^{5/2}}{(1+x^2)^2}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{\sqrt{dx}}{1+x^2}\,dx, x, \tan\right.}{4f}$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} + \frac{(3d)\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\,\text{ta}}\right.}{2f}$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} - \frac{(3d)\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\,\text{ta}}\right.}{4f}$$

$$= -\frac{d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{2f} + \frac{(3d^{5/2})\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-}\right.}{8\sqrt{2}}$$

$$= \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{8\sqrt{2}f} - \frac{3d^{5/2}}{}$$

$$= -\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{4\sqrt{2}f} + \frac{3d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{4\sqrt{2}}$$

Mathematica [A]
time = 0.21, size = 107, normalized size = 0.48

$$-\frac{d^2\left(3\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\csc(e+fx)+3\csc(e+fx)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)+2\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}\sqrt{d\tan(e+fx)}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/8*(d^2*(3*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x] + 3*Csc[e + f
*x]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e + f*x)]]] + 2*Sqrt[Sin[
2*(e + f*x)]])*Sqrt[Sin[2*(e + f*x)]]*Sqrt[d*Tan[e + f*x]])/f
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.58, size = 532, normalized size = 2.36

method	result

default	$-\frac{(\cos(fx+e)-1)\left(3i\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\ \sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\ \sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\ \mathrm{EllipticPi}\left(\sqrt{-\frac{-1}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*(cos(f*x+e)-1)*(3*I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)-3*I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*(
(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*((cos(f*x+e)-1)/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(f*x+e)^2*2^(1/2)-2*cos(f*x+
e)*2^(1/2))*cos(f*x+e)^2*(cos(f*x+e)+1)^2*(d*sin(f*x+e)/cos(f*x+e))^(5/2)/s
in(f*x+e)^5*2^(1/2)
```

Maxima [A]
time = 0.50, size = 202, normalized size = 0.90

$$\frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{d}+d\right)}{\sqrt{d}}\right)-\frac{8\left(d\tan(fx+e)\right)^{\frac{3}{2}}d^4}{d^2\tan(fx+e)^2+d^2}}{16\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(f
*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d)
 - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) +
sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x +
 e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) - 8*(d*tan(f*x + e
))^(3/2)*d^4/(d^2*tan(f*x + e)^2 + d^2))/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. $2(177) = 354$.
time = 64.27, size = 2030, normalized size = 9.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/64*(32*d^2*sqrt(d*sin(f*x + e)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) -
 12*sqrt(2)*(d^10/f^4)^(1/4)*f*arctan((sqrt(d^16 + 4*sqrt(d^10/f^4)*d^11*f^
2*cos(f*x + e)*sin(f*x + e) - 2*(sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*cos(f*x +
e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*s
in(f*x + e)/cos(f*x + e)))*(2*d^8*cos(f*x + e)*sin(f*x + e) + sqrt(d^10/f^4
)*d^3*f^2 + (sqrt(2)*(d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)*sin(f*x + e) + sqr
t(2)*(d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))
) + (sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3
/4)*d^8*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(
2*d^16*cos(f*x + e)^2 - d^16)) - 12*sqrt(2)*(d^10/f^4)^(1/4)*f*arctan(-(sqr
t(d^16 + 4*sqrt(d^10/f^4)*d^11*f^2*cos(f*x + e)*sin(f*x + e) + 2*(sqrt(2)*(
d^10/f^4)^(1/4)*d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*co
s(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))*(2*d^8*cos(f*x
+ e)*sin(f*x + e) + sqrt(d^10/f^4)*d^3*f^2 - (sqrt(2)*(d^10/f^4)^(1/4)*d^5*
f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*(d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*
sqrt(d*sin(f*x + e)/cos(f*x + e))) - (sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*cos(f
*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(f*x + e))*sqr
t(d*sin(f*x + e)/cos(f*x + e)))/(2*d^16*cos(f*x + e)^2 - d^16)) - 12*sqrt(2
)*(d^10/f^4)^(1/4)*f*arctan(-1/2*(2*d^16*sin(f*x + e) - sqrt(d^16 + 4*sqrt(
d^10/f^4)*d^11*f^2*cos(f*x + e)*sin(f*x + e) + 2*(sqrt(2)*(d^10/f^4)^(1/4)*
d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(f
*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))*(sqrt(2)*(d^10/f^4)^(1/4)*d^5*f
*sin(f*x + e) + sqrt(2)*(d^10/f^4)^(3/4)*f^3*cos(f*x + e))*sqrt(d*sin(f*x +
 e)/cos(f*x + e)) - 4*(d^11*f^2*cos(f*x + e)^3 - d^11*f^2*cos(f*x + e))*sqr
t(d^10/f^4) + (sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*sin(f*x + e) + sqrt(2)*(d^10
/f^4)^(3/4)*d^8*f^3*cos(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/((2*d^
16*cos(f*x + e)^2 - d^16)*sin(f*x + e))) - 12*sqrt(2)*(d^10/f^4)^(1/4)*f*ar
ctan(1/2*(2*d^16*sin(f*x + e) + sqrt(d^16 + 4*sqrt(d^10/f^4)*d^11*f^2*cos(f
*x + e)*sin(f*x + e) - 2*(sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*cos(f*x + e)^2 +
sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x
+ e)/cos(f*x + e)))*(sqrt(2)*(d^10/f^4)^(1/4)*d^5*f*sin(f*x + e) + sqrt(2)*
(d^10/f^4)^(3/4)*f^3*cos(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)) - 4*(d
^11*f^2*cos(f*x + e)^3 - d^11*f^2*cos(f*x + e))*sqrt(d^10/f^4) - (sqrt(2)*(
d^10/f^4)^(1/4)*d^13*f*sin(f*x + e) + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(
f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/((2*d^16*cos(f*x + e)^2 - d^16
)*sin(f*x + e))) + 3*sqrt(2)*(d^10/f^4)^(1/4)*f*log(729*d^16 + 2916*sqrt(d^
10/f^4)*d^11*f^2*cos(f*x + e)*sin(f*x + e) + 1458*(sqrt(2)*(d^10/f^4)^(1/4)
*d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(
f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))) - 3*sqrt(2)*(d^10/f^4)^(1/4)*f
*log(729*d^16 + 2916*sqrt(d^10/f^4)*d^11*f^2*cos(f*x + e)*sin(f*x + e) - 14
58*(sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/
```

```
4)*d^8*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))) +
3*sqrt(2)*(d^10/f^4)^(1/4)*f*log(729/16*d^16 + 729/4*sqrt(d^10/f^4)*d^11*f^
2*cos(f*x + e)*sin(f*x + e) + 729/8*(sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*cos(f*
x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(f*x + e))*sqrt
(d*sin(f*x + e)/cos(f*x + e))) - 3*sqrt(2)*(d^10/f^4)^(1/4)*f*log(729/16*d^
16 + 729/4*sqrt(d^10/f^4)*d^11*f^2*cos(f*x + e)*sin(f*x + e) - 729/8*(sqrt(
2)*(d^10/f^4)^(1/4)*d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^
3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))))/f
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.51, size = 240, normalized size = 1.07

$$-\frac{1}{16}\left(\frac{8\sqrt{d\tan\left(fx+e\right)}\,d^2\tan\left(fx+e\right)}{\left(d^2\tan\left(fx+e\right)^2+d^2\right)f}-\frac{6\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{df}-\frac{6\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{3\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan\left(fx+e\right)+\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{df}-\frac{3\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan\left(fx+e\right)-\sqrt{2}\sqrt{d\tan\left(fx+e\right)}\sqrt{|d|}+|d|\right)}{df}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/16*(8*sqrt(d*tan(f*x + e))*d^2*tan(f*x + e)/((d^2*tan(f*x + e)^2 + d^2)*
f) - 6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2*sq
rt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) - 6*sqrt(2)*abs(d)^(3/2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*
f) + 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f*x + e
))*sqrt(abs(d)) + abs(d))/(d*f) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e)
 - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f))*d^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^2\,{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(d*tan(e + f*x))^(5/2),x)
```

```
[Out] int(cos(e + f*x)^2*(d*tan(e + f*x))^(5/2), x)
```

3.252 $\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=253

$$-\frac{3d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}+\cdots\right.}{}$$

```
[Out] -3/64*d^(5/2)*arctan(1-2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/64
*d^(5/2)*arctan(1+2^(1/2)*(d*tan(f*x+e))^(1/2)/d^(1/2))/f*2^(1/2)+3/128*d^(
5/2)*ln(d^(1/2)-2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f*2^(1/2)-
3/128*d^(5/2)*ln(d^(1/2)+2^(1/2)*(d*tan(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/f
*2^(1/2)+3/16*d*cos(f*x+e)^2*(d*tan(f*x+e))^(3/2)/f-1/4*d*cos(f*x+e)^4*(d*t
an(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.13, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2687, 294, 296, 335, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3d^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}+1\right)}{32\sqrt{2}f}+\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{64\sqrt{2}f}-\frac{3d^{5/2}\log\left(\sqrt{d}\tan(e+fx)+\sqrt{2}\sqrt{d\tan(e+fx)}+\sqrt{d}\right)}{64\sqrt{2}f}-\frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f}+\frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-3*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(32*Sqrt[2]
*f) + (3*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[e + f*x]])/Sqrt[d]])/(32*Sq
rt[2]*f) + (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*x] - Sqrt[2]*Sqrt[d*T
an[e + f*x]]])/(64*Sqrt[2]*f) - (3*d^(5/2)*Log[Sqrt[d] + Sqrt[d]*Tan[e + f*
x] + Sqrt[2]*Sqrt[d*Tan[e + f*x]]])/(64*Sqrt[2]*f) + (3*d*Cos[e + f*x]^2*(d
*Tan[e + f*x])^(3/2))/(16*f) - (d*Cos[e + f*x]^4*(d*Tan[e + f*x])^(3/2))/(4
*f)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\int \cos^4(e+fx)(d\tan(e+fx))^{5/2}\,dx = \frac{\text{Subst}\left(\int \frac{(dx)^{5/2}}{(1+x^2)^3}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/2}}{4f} + \frac{(3d^2)\,\text{Subst}\left(\int \frac{\sqrt{dx}}{(1+x^2)^2}\,dx, x, \tan\right.}{8f}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/}}{4f}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/}}{4f}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/}}{4f}$$

$$= \frac{3d\cos^2(e+fx)(d\tan(e+fx))^{3/2}}{16f} - \frac{d\cos^4(e+fx)(d\tan(e+fx))^{3/}}{4f}$$

$$= \frac{3d^{5/2}\log\left(\sqrt{d}+\sqrt{d}\tan(e+fx)-\sqrt{2}\sqrt{d\tan(e+fx)}\right)}{64\sqrt{2}f} - \frac{3d^{5/2}}{}$$

$$= -\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(e+fx)}}{\sqrt{d}}\right)}{32\sqrt{2}f} + \frac{3d^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{}}{}\right.}{32\sqrt{2}}$$

Mathematica [A]
time = 0.21, size = 125, normalized size = 0.49

$$-\frac{d^2\left(3\text{ArcSin}(\cos(e+fx)-\sin(e+fx))\csc(e+fx)\sqrt{\sin(2(e+fx))}+3\csc(e+fx)\log\left(\cos(e+fx)+\sin(e+fx)+\sqrt{\sin(2(e+fx))}\right)\sqrt{\sin(2(e+fx))}-2\sin(2(e+fx))+2\sin(4(e+fx))\right)\sqrt{d\tan(e+fx)}}{64f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(d*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/64*(d^2*(3*ArcSin[Cos[e + f*x] - Sin[e + f*x]]*Csc[e + f*x]*Sqrt[Sin[2*(
e + f*x)]] + 3*Csc[e + f*x]*Log[Cos[e + f*x] + Sin[e + f*x] + Sqrt[Sin[2*(e
 + f*x)]]]*Sqrt[Sin[2*(e + f*x)]] - 2*Sin[2*(e + f*x)] + 2*Sin[4*(e + f*x)]
)*Sqrt[d*Tan[e + f*x]])/f
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.30, size = 558, normalized size = 2.21

method	result
default	$-\frac{(\cos(fx+e)-1)\left(3i\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\ \text{EllipticPi}\left(\sqrt{-\frac{-1\cdots}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/f*(cos(f*x+e)-1)*(3*I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)-3*I*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+8*
2^(1/2)*cos(f*x+e)^4-3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(
1/2))-3*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-8*cos(f*x
+e)^3*2^(1/2)-6*cos(f*x+e)^2*2^(1/2)+6*cos(f*x+e)*2^(1/2))*cos(f*x+e)^2*(co
s(f*x+e)+1)^2*(d*sin(f*x+e)/cos(f*x+e))^(5/2)/sin(f*x+e)^5*2^(1/2)
```

Maxima [A]
time = 0.49, size = 235, normalized size = 0.93

$$\frac{3\,d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(fx+e)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(fx+e)+\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(fx+e)-\sqrt{2}\sqrt{d\tan(fx+e)}\sqrt{d}+d\right)}{\sqrt{d}}\right)+\frac{8\left(3\,(d\tan(fx+e))^{\frac{7}{2}}d^4-(d\tan(fx+e))^{\frac{3}{2}}d^6\right)}{d^4\tan(fx+e)^4+2\,d^4\tan(fx+e)^2+d^4}}{128\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/128*(3*d^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(
f*x + e)))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d
```

```
) - 2*sqrt(d*tan(f*x + e)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(f*x + e) +
 sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(f*x
+ e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(d) + d)/sqrt(d)) + 8*(3*(d*tan(f*x
 + e))^(7/2)*d^4 - (d*tan(f*x + e))^(3/2)*d^6)/(d^4*tan(f*x + e)^4 + 2*d^4*
tan(f*x + e)^2 + d^4))/(d*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2047 vs. $2(203) = 406$.
time = 64.32, size = 2047, normalized size = 8.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/512*(12*sqrt(2)*(d^10/f^4)^(1/4)*f*arctan((sqrt(d^16 + 4*sqrt(d^10/f^4)*d
^11*f^2*cos(f*x + e)*sin(f*x + e) - 2*(sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*cos(
f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(f*x + e))*sq
rt(d*sin(f*x + e)/cos(f*x + e)))*(2*d^8*cos(f*x + e)*sin(f*x + e) + sqrt(d^
10/f^4)*d^3*f^2 + (sqrt(2)*(d^10/f^4)^(1/4)*d^5*f*cos(f*x + e)*sin(f*x + e)
 + sqrt(2)*(d^10/f^4)^(3/4)*f^3*cos(f*x + e)^2)*sqrt(d*sin(f*x + e)/cos(f*x
 + e))) + (sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f
^4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x +
e)))/(2*d^16*cos(f*x + e)^2 - d^16)) + 12*sqrt(2)*(d^10/f^4)^(1/4)*f*arctan
(-(sqrt(d^16 + 4*sqrt(d^10/f^4)*d^11*f^2*cos(f*x + e)*sin(f*x + e) + 2*(sqr
t(2)*(d^10/f^4)^(1/4)*d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*
f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))*(2*d^8*co
s(f*x + e)*sin(f*x + e) + sqrt(d^10/f^4)*d^3*f^2 - (sqrt(2)*(d^10/f^4)^(1/4
)*d^5*f*cos(f*x + e)*sin(f*x + e) + sqrt(2)*(d^10/f^4)^(3/4)*f^3*cos(f*x +
e)^2)*sqrt(d*sin(f*x + e)/cos(f*x + e))) - (sqrt(2)*(d^10/f^4)^(1/4)*d^13*f
*cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(f*x + e
))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(2*d^16*cos(f*x + e)^2 - d^16)) + 12*
sqrt(2)*(d^10/f^4)^(1/4)*f*arctan(-1/2*(2*d^16*sin(f*x + e) - sqrt(d^16 + 4
*sqrt(d^10/f^4)*d^11*f^2*cos(f*x + e)*sin(f*x + e) + 2*(sqrt(2)*(d^10/f^4)^
(1/4)*d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e)
*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))*(sqrt(2)*(d^10/f^4)^(1/4)
*d^5*f*sin(f*x + e) + sqrt(2)*(d^10/f^4)^(3/4)*f^3*cos(f*x + e))*sqrt(d*sin
(f*x + e)/cos(f*x + e)) - 4*(d^11*f^2*cos(f*x + e)^3 - d^11*f^2*cos(f*x + e
))*sqrt(d^10/f^4) + (sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*sin(f*x + e) + sqrt(2)
*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/
((2*d^16*cos(f*x + e)^2 - d^16)*sin(f*x + e))) + 12*sqrt(2)*(d^10/f^4)^(1/4
)*f*arctan(1/2*(2*d^16*sin(f*x + e) + sqrt(d^16 + 4*sqrt(d^10/f^4)*d^11*f^2
*cos(f*x + e)*sin(f*x + e) - 2*(sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*cos(f*x + e
)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*si
n(f*x + e)/cos(f*x + e)))*(sqrt(2)*(d^10/f^4)^(1/4)*d^5*f*sin(f*x + e) + sq
```

```
rt(2)*(d^10/f^4)^(3/4)*f^3*cos(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))
- 4*(d^11*f^2*cos(f*x + e)^3 - d^11*f^2*cos(f*x + e))*sqrt(d^10/f^4) - (sqr
t(2)*(d^10/f^4)^(1/4)*d^13*f*sin(f*x + e) + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^
3*cos(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e)))/((2*d^16*cos(f*x + e)^2
- d^16)*sin(f*x + e))) - 3*sqrt(2)*(d^10/f^4)^(1/4)*f*log(729*d^16 + 2916*s
qrt(d^10/f^4)*d^11*f^2*cos(f*x + e)*sin(f*x + e) + 1458*(sqrt(2)*(d^10/f^4)
^(1/4)*d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e
)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))) + 3*sqrt(2)*(d^10/f^4)^(
1/4)*f*log(729*d^16 + 2916*sqrt(d^10/f^4)*d^11*f^2*cos(f*x + e)*sin(f*x + e
) - 1458*(sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f^
4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e
))) - 3*sqrt(2)*(d^10/f^4)^(1/4)*f*log(729/16*d^16 + 729/4*sqrt(d^10/f^4)*d
^11*f^2*cos(f*x + e)*sin(f*x + e) + 729/8*(sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*
cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*d^8*f^3*cos(f*x + e)*sin(f*x + e)
)*sqrt(d*sin(f*x + e)/cos(f*x + e))) + 3*sqrt(2)*(d^10/f^4)^(1/4)*f*log(729
/16*d^16 + 729/4*sqrt(d^10/f^4)*d^11*f^2*cos(f*x + e)*sin(f*x + e) - 729/8*
(sqrt(2)*(d^10/f^4)^(1/4)*d^13*f*cos(f*x + e)^2 + sqrt(2)*(d^10/f^4)^(3/4)*
d^8*f^3*cos(f*x + e)*sin(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e))) - 32*
(4*d^2*cos(f*x + e)^3 - 3*d^2*cos(f*x + e))*sqrt(d*sin(f*x + e)/cos(f*x + e
))*sin(f*x + e))/f
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(d*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 268, normalized size = 1.06

$$\frac{1}{128}d^2\left(\frac{6\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}+2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{df}+\frac{6\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{|d|}-2\sqrt{d\tan\left(fx+e\right)}\right)}{2\sqrt{|d|}}\right)}{df}-\frac{3\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan\left(fx+e\right)+\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{df}+\frac{3\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan\left(fx+e\right)-\sqrt{2}\,\sqrt{d\tan\left(fx+e\right)}\,\sqrt{|d|}+|d|\right)}{df}+\frac{8\left(3\sqrt{d\tan\left(fx+e\right)}\,d^4\tan\left(fx+e\right)^3-\sqrt{d\tan\left(fx+e\right)}\,d^4\tan\left(fx+e\right)\right)}{\left(d^2\tan\left(fx+e\right)^2+d^2\right)^2 f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(d*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/128*d^2*(6*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d))
+ 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)))/(d*f) + 6*sqrt(2)*abs(d)^(3/2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(f*x + e)))/sqrt(abs(d)
))/(d*f) - 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*x + e) + sqrt(2)*sqrt(d*tan(f
*x + e))*sqrt(abs(d)) + abs(d))/(d*f) + 3*sqrt(2)*abs(d)^(3/2)*log(d*tan(f*
x + e) - sqrt(2)*sqrt(d*tan(f*x + e))*sqrt(abs(d)) + abs(d))/(d*f) + 8*(3*s
```

```
qrt(d*tan(f*x + e))*d^4*tan(f*x + e)^3 - sqrt(d*tan(f*x + e))*d^4*tan(f*x +
 e))/((d^2*tan(f*x + e)^2 + d^2)^2*f))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos{(e+f\,x)}^4\,{(d\tan(e+f\,x))}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(d*tan(e + f*x))^(5/2),x)
[Out] int(cos(e + f*x)^4*(d*tan(e + f*x))^(5/2), x)
```

3.253 $\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=109

$$\frac{4F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{7f\sqrt{d\tan(e+fx)}}+\frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}$$

```
[Out] -4/7*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f
*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+4/7*sec
(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f+2/7*sec(f*x+e)^3*(d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2693, 2694, 2653, 2720}

$$\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{4\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{7f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (4*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(7*f*S
qrt[d*Tan[e + f*x]]) + (4*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(7*d*f) + (2*S
ec[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(7*d*f)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx &= \frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{6}{7}\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\\
&= \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{4}{7}\int \frac{\sec(e-}{\sqrt{d\tan(e}}\\
&= \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{\left(4\sqrt{\sin(e+f}\right.}{7\sqrt{}}\\
&= \frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{2\sec^3(e+fx)\sqrt{d\tan(e+fx)}}{7df}+\frac{\left(4\sec(e+fx\right.}{}\\
&= \frac{4F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{7f\sqrt{d\tan(e+fx)}}+\frac{4\sec(e+fx)\sqrt{d\tan(e+fx)}}{7df}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.60, size = 79, normalized size = 0.72

$$\frac{2\left((2+\cos(2(e+fx)))\sec^4(e+fx)+4\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\right)\sin(e+fx)}{7f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*((2 + Cos[2*(e + f*x)])*Sec[e + f*x]^4 + 4*Hypergeometric2F1[1/4, 1/2, 5
/4, -Tan[e + f*x]^2]*Sqrt[Sec[e + f*x]^2])*Sin[e + f*x])/(7*f*Sqrt[d*Tan[e
+ f*x]])
```

Maple [A]
time = 0.38, size = 224, normalized size = 2.06

method	result
default	$-\frac{(\cos(fx+e)-1)\left(4\,\mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\;\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\;\sqrt{}\right.}{7f\sin(fx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/f*(cos(f*x+e)-1)*(4*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*cos
(f*x+e)^3*sin(f*x+e)-2*cos(f*x+e)^3*2^(1/2)+2*cos(f*x+e)^2*2^(1/2)-cos(f*x+
e)*2^(1/2)+2^(1/2))*(cos(f*x+e)+1)^2/sin(f*x+e)^3/cos(f*x+e)^4/(d*sin(f*x+e
)/cos(f*x+e))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^5/sqrt(d*tan(f*x + e)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.11, size = 123, normalized size = 1.13

$$-\frac{2\left(2\sqrt{i\,d}\,\cos\left(fx+e\right)^{3}\mathrm{ellipticF}(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right),-1)+2\sqrt{-i\,d}\,\cos\left(fx+e\right)^{3}\mathrm{ellipticF}(\cos\left(fx+e\right)-i\,\sin\left(fx+e\right),-1)-\left(2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{\frac{d\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\right)}{7\,df\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/7*(2*sqrt(I*d)*cos(f*x + e)^3*ellipticF(cos(f*x + e) + I*sin(f*x + e), -
1) + 2*sqrt(-I*d)*cos(f*x + e)^3*ellipticF(cos(f*x + e) - I*sin(f*x + e), -
1) - (2*cos(f*x + e)^2 + 1)*sqrt(d*sin(f*x + e)/cos(f*x + e)))/(d*f*cos(f*x
 + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{5}\left(e+fx\right)}{\sqrt{d\tan\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**5/sqrt(d*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^5/sqrt(d*tan(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^5\,\sqrt{d\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^5*(d*tan(e + f*x))^(1/2)), x)
```

3.254 $\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{3f\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

```
[Out] -2/3*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f
*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+2/3*sec
(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2693, 2694, 2653, 2720}

$$\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{2\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(3*f*S
qrt[d*Tan[e + f*x]]) + (2*Sec[e + f*x]*Sqrt[d*Tan[e + f*x]])/(3*d*f)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx &= \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{2}{3}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx \\
&= \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{\left(2\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+f}}}{3\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}} \\
&= \frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{\left(2\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2}}}{3\sqrt{d\tan(e+fx)}} \\
&= \frac{2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{3f\sqrt{d\tan(e+fx)}}+\frac{2\sec(e+fx)\sqrt{d\tan(e+fx)}}{3df}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.29, size = 68, normalized size = 0.86

$$\frac{2\left(\sec^2(e+fx)+2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2(e+fx)\right)\sqrt{\sec^2(e+fx)}\right)\sin(e+fx)}{3f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Sec[e + f*x]^2 + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[e + f*x]^2]*Sq
rt[Sec[e + f*x]^2])*Sin[e + f*x])/(3*f*Sqrt[d*Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(94) = 188.
time = 0.60, size = 196, normalized size = 2.48

method	result

default	$-\frac{(\cos(fx+e)-1)\left(2\,\mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{}\right.}{3f\sin(fx+e)^{3}\cos(fx+e)^{2}\sqrt{\frac{d\sin}{\cos}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(cos(f*x+e)-1)*(2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))*((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin
(f*x+e)*cos(f*x+e)-cos(f*x+e)*2^(1/2)+2^(1/2))*(cos(f*x+e)+1)^2/sin(f*x+e)^
3/cos(f*x+e)^2/(d*sin(f*x+e)/cos(f*x+e))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.10, size = 104, normalized size = 1.32

$$-\frac{2\left(\sqrt{i\,d}\,\cos(fx+e)\,\mathrm{ellipticF}(\cos(fx+e)+i\,\sin(fx+e),-1)+\sqrt{-i\,d}\,\cos(fx+e)\,\mathrm{ellipticF}(\cos(fx+e)-i\,\sin(fx+e),-1)-\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}\right)}{3\,df\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(I*d)*cos(f*x + e)*ellipticF(cos(f*x + e) + I*sin(f*x + e), -1) +
 sqrt(-I*d)*cos(f*x + e)*ellipticF(cos(f*x + e) - I*sin(f*x + e), -1) - sqr
t(d*sin(f*x + e)/cos(f*x + e)))/(d*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{3}(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/sqrt(d*tan(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^3\,\sqrt{d\,\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^3*(d*tan(e + f*x))^(1/2)), x)
```

3.255 $\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=47

$$\frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{f\sqrt{d\tan(e+fx)}}$$

```
[Out] -(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),
2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2694, 2653, 2720}

$$\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(f*Sqrt[
d*Tan[e + f*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}dx}{\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}$$
$$= \frac{\left(\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{\sqrt{d\tan(e+fx)}}$$
$$= \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{f\sqrt{d\tan(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 77, normalized size = 1.64

$$-\frac{2\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right)\middle|-1\right)\sec^3(e+fx)\sqrt{\tan(e+fx)}}{f\sqrt{d\tan(e+fx)}\,\left(1+\tan^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[e + f*x]]], -1]*Sec[
e + f*x]^3*Sqrt[Tan[e + f*x]])/(f*Sqrt[d*Tan[e + f*x]]*(1 + Tan[e + f*x]^2)
^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(69) = 138.
time = 0.26, size = 167, normalized size = 3.55

method	result
default	$-\frac{\mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-}{\sin(fx+e}}}{f\sin(fx+e)^2\cos(fx+e)\sqrt{\frac{d\sin(fx+e)}{\cos(fx+e)}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*
((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*(cos(f*x+e)-1)/sin(f*x+
e)^2/cos(f*x+e)*(cos(f*x+e)+1)^2/(d*sin(f*x+e)/cos(f*x+e))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.11, size = 59, normalized size = 1.26

$$-\frac{\sqrt{i\,d}\,\mathrm{ellipticF}(\cos{(fx+e)}+i\,\sin{(fx+e)}\,,-1)+\sqrt{-i\,d}\,\mathrm{ellipticF}(\cos{(fx+e)}-i\,\sin{(fx+e)}\,,-1)}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -(sqrt(I*d)*ellipticF(cos(f*x + e) + I*sin(f*x + e), -1) + sqrt(-I*d)*ellip
ticF(cos(f*x + e) - I*sin(f*x + e), -1))/(d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec{(e+fx)}}{\sqrt{d\tan{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(d*tan(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)/sqrt(d*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sec(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos{(e+f\,x)}\,\sqrt{d\tan{(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(d*tan(e + f*x))^(1/2)),x)
[Out] int(1/(cos(e + f*x)*(d*tan(e + f*x))^(1/2)), x)
```

3.256 $\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=76

$$\frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{2f\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}$$

```
[Out] -1/2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f
*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+cos(f*x
+e)*(d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2692, 2694, 2653, 2720}

$$\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqr
t[d*Tan[e + f*x]]) + (Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(d*f)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx &= \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{1}{2}\int \frac{\sec(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx\\ &= \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\sqrt{\sin(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}\sqrt{\sin(e+fx)}}\,d}{2\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}\\ &= \frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}+\frac{\left(\sec(e+fx)\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2f}}}{2\sqrt{d\tan(e+fx)}}\\ &= \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{2f\sqrt{d\tan(e+fx)}}+\frac{\cos(e+fx)\sqrt{d\tan(e+fx)}}{df}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.60, size = 126, normalized size = 1.66

$$\frac{\cos(2(e+fx))\sec(e+fx)\left(\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right)\middle|-1\right)\sec^2(e+fx)-\sqrt{\sec^2(e+fx)}\sqrt{\tan(e+fx)}\right)\sqrt{\tan(e+fx)}}{f\sqrt{\sec^2(e+fx)}\sqrt{d\tan(e+fx)}\left(-1+\tan^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/Sqrt[d*Tan[e + f*x]],x]
[Out] (Cos[2*(e + f*x)]*Sec[e + f*x]*((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*S
qrt[Tan[e + f*x]]], -1]*Sec[e + f*x]^2 - Sqrt[Sec[e + f*x]^2]*Sqrt[Tan[e +
f*x]])*Sqrt[Tan[e + f*x]])/(f*Sqrt[Sec[e + f*x]^2]*Sqrt[d*Tan[e + f*x]]*(-1
 + Tan[e + f*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(93) = 186.
time = 0.33, size = 198, normalized size = 2.61

method	result

default	$-\frac{(\cos(fx+e)-1)\left(\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\sin(fx+e)\,\mathrm{EllipticF}\left(\sqrt{\right.\right.}{2f\sin(fx+e)^{3}\cos(fx+e)\sqrt{\frac{d\sin(}{\cos(f}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)*(((cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(
1/2))-cos(f*x+e)^2*2^(1/2)+cos(f*x+e)*2^(1/2))*(cos(f*x+e)+1)^2/sin(f*x+e)^
3/cos(f*x+e)/(d*sin(f*x+e)/cos(f*x+e))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)/(d*tan(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)/sqrt(d*tan(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)/sqrt(d*tan(f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)}{\sqrt{d\,\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(d*tan(e + f*x))^(1/2),x)
[Out] int(cos(e + f*x)/(d*tan(e + f*x))^(1/2), x)
```

3.257 $\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=109

$$\frac{5F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{12f\sqrt{d\tan(e+fx)}}+\frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df}+\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}$$

```
[Out] -5/12*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+
f*x),2^(1/2))*sec(f*x+e)*sin(2*f*x+2*e)^(1/2)/f/(d*tan(f*x+e))^(1/2)+5/6*co
s(f*x+e)*(d*tan(f*x+e))^(1/2)/d/f+1/3*cos(f*x+e)^3*(d*tan(f*x+e))^(1/2)/d/f
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2692, 2694, 2653, 2720}

$$\frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df}+\frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df}+\frac{5\sqrt{\sin(2e+2fx)}\sec(e+fx)F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{12f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (5*EllipticF[e - Pi/4 + f*x, 2]*Sec[e + f*x]*Sqrt[Sin[2*e + 2*f*x]])/(12*f*
Sqrt[d*Tan[e + f*x]]) + (5*Cos[e + f*x]*Sqrt[d*Tan[e + f*x]])/(6*d*f) + (Co
s[e + f*x]^3*Sqrt[d*Tan[e + f*x]])/(3*d*f)
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
```

```
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx = \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{5}{6}\int \frac{\cos(e+fx)}{\sqrt{d\tan(e+fx)}}\,dx$$

$$= \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{5}{12}\int \frac{\sec(e}{\sqrt{d\tan}}$$

$$= \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{\left(5\sqrt{\sin(e+}\right.}{12\sqrt{}}$$

$$= \frac{5\cos(e+fx)\sqrt{d\tan(e+fx)}}{6df} + \frac{\cos^3(e+fx)\sqrt{d\tan(e+fx)}}{3df} + \frac{\left(5\sec(e+fx\right.}{}$$

$$= \frac{5F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sec(e+fx)\sqrt{\sin(2e+2fx)}}{12f\sqrt{d\tan(e+fx)}} + \frac{5\cos(e+fx)\sqrt{d\tan(e+fx}}{6df}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.12, size = 94, normalized size = 0.86

$$\frac{11\sin(e+fx)+\sin(3(e+fx))-10\sqrt[4]{-1}\cos(e+fx)F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(e+fx)}\right)\middle|-1\right)\sqrt{\sec^2(e+fx)}\sqrt{\tan(e+fx)}}{12f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (11*Sin[e + f*x] + Sin[3*(e + f*x)] - 10*(-1)^(1/4)*Cos[e + f*x]*EllipticF[
I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[e + f*x]]], -1]*Sqrt[Sec[e + f*x]^2]*Sqrt[Tan
[e + f*x]])/(12*f*Sqrt[d*Tan[e + f*x]])
```

Maple [A]
time = 0.33, size = 226, normalized size = 2.07

method	result

default	$\frac{(\cos(fx+e)-1)\left(2\sqrt{2}\,(\cos^4(fx+e))-5\sqrt{\frac{\cos(fx+e)-1}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\,\sin(\right.}{12f\cos(fx+e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/f*(cos(f*x+e)-1)*(2*2^(1/2)*cos(f*x+e)^4-5*((cos(f*x+e)-1)/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),1/2*2^(1/2))-2*cos(f*x+e)^3*2^(1/2)+5*cos(f*x+e)^2*2^(1/2
)-5*cos(f*x+e)*2^(1/2))*(cos(f*x+e)+1)^2/cos(f*x+e)/(d*sin(f*x+e)/cos(f*x+e
))^(1/2)/sin(f*x+e)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(f*x + e))*cos(f*x + e)^3/(d*tan(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(cos(f*x + e)^3/sqrt(d*tan(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^3}{\sqrt{d\,\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(d*tan(e + f*x))^(1/2),x)
[Out] int(cos(e + f*x)^3/(d*tan(e + f*x))^(1/2), x)
```

3.258 $\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2}{bd\sqrt{d\tan(a+bx)}}+\frac{4(d\tan(a+bx))^{3/2}}{3bd^3}+\frac{2(d\tan(a+bx))^{7/2}}{7bd^5}$$

```
[Out] -2/b/d/(d*tan(b*x+a))^(1/2)+4/3*(d*tan(b*x+a))^(3/2)/b/d^3+2/7*(d*tan(b*x+a
))^(7/2)/b/d^5
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2687, 276}

$$\frac{2(d\tan(a+bx))^{7/2}}{7bd^5}+\frac{4(d\tan(a+bx))^{3/2}}{3bd^3}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]]) + (4*(d*Tan[a + b*x])^(3/2))/(3*b*d^3) + (2*(
d*Tan[a + b*x])^(7/2))/(7*b*d^5)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\int \frac{\sec^6(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(dx)^{3/2}}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(dx)^{3/2}} + \frac{2\sqrt{dx}}{d^2} + \frac{(dx)^{5/2}}{d^4}\right)\,dx, x, \tan(a+bx)\right)}{b}$$
$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} + \frac{4(d\tan(a+bx))^{3/2}}{3bd^3} + \frac{2(d\tan(a+bx))^{7/2}}{7bd^5}$$

Mathematica [A]
time = 0.23, size = 45, normalized size = 0.69

$$\frac{-42+(22+6\sec^2(a+bx))\tan^2(a+bx)}{21bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6/(d*Tan[a + b*x])^(3/2),x]
[Out] (-42 + (22 + 6*Sec[a + b*x]^2)*Tan[a + b*x]^2)/(21*b*d*Sqrt[d*Tan[a + b*x]]
)
```

Maple [A]
time = 1.73, size = 60, normalized size = 0.92

method	result	size
default	$-\frac{2\left(32\left(\cos^4(bx+a)\right)-8\left(\cos^2(bx+a)\right)-3\right)\sin(bx+a)}{21b\cos(bx+a)^5\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/21/b*(32*cos(b*x+a)^4-8*cos(b*x+a)^2-3)*sin(b*x+a)/cos(b*x+a)^5/(d*sin(b
*x+a)/cos(b*x+a))^(3/2)
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.83

$$-\frac{2\left(\frac{21}{\sqrt{d\tan\left(bx+a\right)}} - \frac{3\left(d\tan\left(bx+a\right)\right)^{\frac{7}{2}} + 14\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}d^2}{d^4}\right)}{21\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/21*(21/sqrt(d*tan(b*x + a)) - (3*(d*tan(b*x + a))^(7/2) + 14*(d*tan(b*x
+ a))^(3/2)*d^2)/d^4)/(b*d)
```

Fricas [A]
time = 0.42, size = 64, normalized size = 0.98

$$-\frac{2\left(32\cos\left(bx+a\right)^{4}-8\cos\left(bx+a\right)^{2}-3\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{21\,bd^{2}\cos\left(bx+a\right)^{3}\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/21*(32*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 3)*sqrt(d*sin(b*x + a)/cos(b*
x + a))/(b*d^2*cos(b*x + a)^3*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{6}\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**6/(d*tan(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.62, size = 80, normalized size = 1.23

$$-\frac{2\left(\frac{21}{\sqrt{d\tan\left(bx+a\right)}\,b}-\frac{3\sqrt{d\tan\left(bx+a\right)}\,b^{6}d^{27}\tan(bx+a)^{3}+14\sqrt{d\tan\left(bx+a\right)}\,b^{6}d^{27}\tan(bx+a)}{b^{7}d^{28}}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/21*(21/(sqrt(d*tan(b*x + a))*b) - (3*sqrt(d*tan(b*x + a))*b^6*d^27*tan(b
*x + a)^3 + 14*sqrt(d*tan(b*x + a))*b^6*d^27*tan(b*x + a))/(b^7*d^28))/d
```

Mupad [B]
time = 6.88, size = 268, normalized size = 4.12

$$-\frac{\left(\frac{20\mathrm{i}}{21\,b\,d^2}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,64\mathrm{i}}{21\,b\,d^2}\right)\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1}+\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,20\mathrm{i}}{21\,b\,d^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}+\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,24\mathrm{i}}{7\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{-\frac{d\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,b\,d^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(a + b*x)^6*(d*tan(a + b*x))^(3/2)),x)

[Out] ((-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*20i)/(21*b*d^2*(exp(a*2i + b*x*2i) + 1)) - ((20i/(21*b*d^2) + (exp(a*2i + b*x*2i)*64i)/(21*b*d^2))*(-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2))/(exp(a*2i + b*x*2i) - 1) + ((-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*24i)/(7*b*d^2*(exp(a*2i + b*x*2i) + 1)^2) - ((-(d*(exp(a*2i + b*x*2i)*1i - 1i))/(exp(a*2i + b*x*2i) + 1))^(1/2)*16i)/(7*b*d^2*(exp(a*2i + b*x*2i) + 1)^3)

3.259 $\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{2}{bd\sqrt{d\tan(a+bx)}}+\frac{2(d\tan(a+bx))^{3/2}}{3bd^3}$$

```
[Out] -2/b/d/(d*tan(b*x+a))^(1/2)+2/3*(d*tan(b*x+a))^(3/2)/b/d^3
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {2687, 14}

$$\frac{2(d\tan(a+bx))^{3/2}}{3bd^3}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]]) + (2*(d*Tan[a + b*x])^(3/2))/(3*b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(dx)^{3/2}}\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{(dx)^{3/2}}+\frac{\sqrt{dx}}{d^2}\right)dx,x,\tan(a+bx)\right)}{b}\\ &= -\frac{2}{bd\sqrt{d\tan(a+bx)}}+\frac{2(d\tan(a+bx))^{3/2}}{3bd^3}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 32, normalized size = 0.74

$$\frac{2(-3+\tan^2(a+bx))}{3bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4/(d*Tan[a + b*x])^(3/2),x]
[Out] (2*(-3 + Tan[a + b*x]^2))/(3*b*d*Sqrt[d*Tan[a + b*x]])
```

Maple [A]
time = 0.32, size = 50, normalized size = 1.16

method	result	size
default	$-\frac{2\left(4\left(\cos^{2}(bx+a)\right)-1\right)\sin(bx+a)}{3b\cos(bx+a)^{3}\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^{\frac{3}{2}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3/b*(4*cos(b*x+a)^2-1)*sin(b*x+a)/cos(b*x+a)^3/(d*sin(b*x+a)/cos(b*x+a))
^(3/2)
```

Maxima [A]
time = 0.27, size = 36, normalized size = 0.84

$$-\frac{2\left(\frac{3}{\sqrt{d\tan\left(bx+a\right)}}-\frac{\left(d\tan\left(bx+a\right)\right)^{\frac{3}{2}}}{d^{2}}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2/3*(3/sqrt(d*tan(b*x + a)) - (d*tan(b*x + a))^(3/2)/d^2)/(b*d)
```

Fricas [A]
time = 0.38, size = 54, normalized size = 1.26

$$-\frac{2\left(4\,\cos\left(bx+a\right)^{2}-1\right)\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}}{3\,bd^{2}\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(4*cos(b*x + a)^2 - 1)*sqrt(d*sin(b*x + a)/cos(b*x + a))/(b*d^2*cos(b*
x + a)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**4/(d*tan(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.60, size = 44, normalized size = 1.02

$$\frac{2\left(\frac{\sqrt{d\tan(bx+a)}\tan(bx+a)}{bd}-\frac{3}{\sqrt{d\tan(bx+a)}\,b}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(d*tan(b*x + a))*tan(b*x + a)/(b*d) - 3/(sqrt(d*tan(b*x + a))*b))/
d
```

Mupad [B]
time = 2.97, size = 64, normalized size = 1.49

$$-\frac{4\left(\sin(2\,a+2\,b\,x)+\sin(4\,a+4\,b\,x)\right)\sqrt{\frac{d\sin(2\,a+2\,b\,x)}{\cos(2\,a+2\,b\,x)+1}}}{3\,b\,d^2\sin(2\,a+2\,b\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^4*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] -(4*(sin(2*a + 2*b*x) + sin(4*a + 4*b*x))*((d*sin(2*a + 2*b*x))/(cos(2*a +
2*b*x) + 1))^(1/2))/(3*b*d^2*sin(2*a + 2*b*x)^2)
```

3.260 $\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

```
[Out] -2/b/d/(d*tan(b*x+a))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2687, 32}

$$-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(dx)^{3/2}}\,dx, x, \tan(a+bx)\right)}{b}\\ &= -\frac{2}{bd\sqrt{d\tan(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 20, normalized size = 1.00

$$-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] -2/(b*d*Sqrt[d*Tan[a + b*x]])
```

Maple [A]
time = 0.10, size = 19, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{2}{bd\sqrt{d\tan\left(bx+a\right)}}$	19
default	$-\frac{2}{bd\sqrt{d\tan\left(bx+a\right)}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/d/(d*tan(b*x+a))^(1/2)
```

Maxima [A]
time = 0.29, size = 18, normalized size = 0.90

$$-\frac{2}{\sqrt{d\tan\left(bx+a\right)}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(d*tan(b*x + a))*b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.
time = 0.38, size = 40, normalized size = 2.00

$$-\frac{2\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}\cos\left(bx+a\right)}{bd^{2}\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)/(b*d^2*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Giac [A]
time = 0.55, size = 18, normalized size = 0.90

$$-\frac{2}{\sqrt{d\tan\left(bx+a\right)}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -2/(sqrt(d*tan(b*x + a))*b*d)
```

Mupad [B]
time = 2.57, size = 51, normalized size = 2.55

$$-\frac{\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{d\,\sin\left(2\,a+2\,b\,x\right)}{\cos\left(2\,a+2\,b\,x\right)+1}}}{b\,d^2\,{\sin\left(a+b\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] -(sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(cos(2*a + 2*b*x) + 1))^(1/2))/(b*
d^2*sin(a + b*x)^2)
```

3.261 $\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\,bd^{3/2}}-\frac{\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\,bd^{3/2}}-\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\right.}{2\sqrt{2}\,bd^{3/}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-1/2*ar
ctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-1/4*ln(d^(1/
2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2)+1/4*l
n(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2
)-2/b/d/(d*tan(b*x+a))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3555, 3557, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\,bd^{3/2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{\sqrt{2}\,bd^{3/2}}-\frac{\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}\,bd^{3/2}}+\frac{\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{2\sqrt{2}\,bd^{3/2}}-\frac{2}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[a + b*x])^(-3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*d^(3/2)) - Ar
cTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]]/(Sqrt[2]*b*d^(3/2)) - Log[
Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]]]/(2*Sqrt[2]*b
*d^(3/2)) + Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]*Sqrt[d*Tan[a + b*x
]]]/(2*Sqrt[2]*b*d^(3/2)) - 2/(b*d*Sqrt[d*Tan[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{\int \sqrt{d\tan(a+bx)}\,dx}{d^2}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{x}}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{2\mathrm{Subst}\left(\int \frac{x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} + \frac{\mathrm{Subst}\left(\int \frac{d-x^2}{d^2+x^4}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{bd} - \frac{\mathrm{Subst}\left(\int \frac{d+x^2}{d^2+x^4}\right.}{}$$

$$= -\frac{2}{bd\sqrt{d\tan(a+bx)}} - \frac{\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}\,x-x^2}\,dx, x, \sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}\,bd^{3/2}} - \frac{\mathrm{Su}}{}$$

$$= -\frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{2\sqrt{2}\,bd^{3/2}} + \frac{\log\left(\sqrt{d}+\sqrt{d}\tan(a\right.}{2}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\,bd^{3/2}} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{\sqrt{2}\,bd^{3/2}} - \frac{\log\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 38, normalized size = 0.18

$$-\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};-\tan^2(a+bx)\right)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[a + b*x])^(-3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1, 3/4, -Tan[a + b*x]^2])/(b*d*Sqrt[d*Tan[a + b
*x]])
```

Maple [A]
time = 0.10, size = 157, normalized size = 0.74

method	result

derivativedivides	$\frac{2d\left(-\frac{1}{d^2\sqrt{d\tan(bx+a)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\sqrt{2}\right.\right.}{8d^2(d^2)}\right.}{b}$
default	$\frac{2d\left(-\frac{1}{d^2\sqrt{d\tan(bx+a)}}-\frac{\sqrt{2}\left(\ln\left(\frac{d\tan(bx+a)-(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}{d\tan(bx+a)+(d^2)^{\frac{1}{4}}\sqrt{d\tan(bx+a)}\sqrt{2}+\sqrt{d^2}}\right)+2\arctan\left(\sqrt{2}\right.\right.}{8d^2(d^2)}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*d*(-1/d^2/(d*tan(b*x+a))^(1/2)-1/8/d^2/(d^2)^(1/4)*2^(1/2)*(ln((d*tan(b
*x+a)-(d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2))/(d*tan(b*x+a)+(
d^2)^(1/4)*(d*tan(b*x+a))^(1/2)*2^(1/2)+(d^2)^(1/2)))+2*arctan(2^(1/2)/(d^2
)^(1/4)*(d*tan(b*x+a))^(1/2)+1)-2*arctan(-2^(1/2)/(d^2)^(1/4)*(d*tan(b*x+a)
)^(1/2)+1)))
```

Maxima [A]
time = 0.49, size = 167, normalized size = 0.79

$$-\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan(bx+a)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{\sqrt{2}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan(bx+a)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{8}{\sqrt{d\tan(bx+a)}}}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*x + a)
))/sqrt(d))/sqrt(d) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2*sq
rt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - sqrt(2)*log(d*tan(b*x + a) + sqrt(2)
*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + sqrt(2)*log(d*tan(b*x + a) - s
qrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 8/sqrt(d*tan(b*x + a)))/
(b*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(163) = 326$.
time = 0.39, size = 652, normalized size = 3.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(8*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a)*sin(b*x + a) + 4*(sqr
t(2)*b*d^2*cos(b*x + a)^2 - sqrt(2)*b*d^2)*(1/(b^4*d^6))^(1/4)*arctan(-sqrt
(2)*b*d*sqrt(d*sin(b*x + a)/cos(b*x + a))*(1/(b^4*d^6))^(1/4) + sqrt(2)*b*d
*sqrt((sqrt(2)*b^3*d^5*sqrt(d*sin(b*x + a)/cos(b*x + a))*(1/(b^4*d^6))^(3/4
)*cos(b*x + a) + b^2*d^4*sqrt(1/(b^4*d^6))*cos(b*x + a) + d*sin(b*x + a))/c
os(b*x + a))*(1/(b^4*d^6))^(1/4) - 1) + 4*(sqrt(2)*b*d^2*cos(b*x + a)^2 - s
qrt(2)*b*d^2)*(1/(b^4*d^6))^(1/4)*arctan(-sqrt(2)*b*d*sqrt(d*sin(b*x + a)/c
os(b*x + a))*(1/(b^4*d^6))^(1/4) + sqrt(2)*b*d*sqrt(-(sqrt(2)*b^3*d^5*sqrt(
d*sin(b*x + a)/cos(b*x + a))*(1/(b^4*d^6))^(3/4)*cos(b*x + a) - b^2*d^4*sqr
t(1/(b^4*d^6))*cos(b*x + a) - d*sin(b*x + a))/cos(b*x + a))*(1/(b^4*d^6))^(
1/4) + 1) + (sqrt(2)*b*d^2*cos(b*x + a)^2 - sqrt(2)*b*d^2)*(1/(b^4*d^6))^(1
/4)*log((sqrt(2)*b^3*d^5*sqrt(d*sin(b*x + a)/cos(b*x + a))*(1/(b^4*d^6))^(3
/4)*cos(b*x + a) + b^2*d^4*sqrt(1/(b^4*d^6))*cos(b*x + a) + d*sin(b*x + a))
/cos(b*x + a)) - (sqrt(2)*b*d^2*cos(b*x + a)^2 - sqrt(2)*b*d^2)*(1/(b^4*d^6
))^(1/4)*log(-(sqrt(2)*b^3*d^5*sqrt(d*sin(b*x + a)/cos(b*x + a))*(1/(b^4*d^
6))^(3/4)*cos(b*x + a) - b^2*d^4*sqrt(1/(b^4*d^6))*cos(b*x + a) - d*sin(b*x
 + a))/cos(b*x + a)))/(b*d^2*cos(b*x + a)^2 - b*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d \tan \left(a + bx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral((d*tan(a + b*x))**(-3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 2.69, size = 76, normalized size = 0.36

$$\frac{(-1)^{1/4}\,\mathrm{atanh}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(a+b\,x\right)}}{\sqrt{d}}\right)}{b\,d^{3/2}}-\frac{(-1)^{1/4}\,\mathrm{atan}\left(\frac{(-1)^{1/4}\,\sqrt{d\tan\left(a+b\,x\right)}}{\sqrt{d}}\right)}{b\,d^{3/2}}-\frac{2}{b\,d\,\sqrt{d\tan\left(a+b\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*tan(a + b*x))^(3/2),x)
```

```
[Out] ((-1)^(1/4)*atanh(((-1)^(1/4)*(d*tan(a + b*x))^(1/2))/d^(1/2)))/(b*d^(3/2))
 - ((-1)^(1/4)*atan(((-1)^(1/4)*(d*tan(a + b*x))^(1/2))/d^(1/2)))/(b*d^(3/2
)) - 2/(b*d*(d*tan(a + b*x))^(1/2))
```

3.262 $\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=249

$$\frac{5\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,bd^{3/2}}-\frac{5\mathrm{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,bd^{3/2}}-\frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+b\right.}{8\sqrt{2}\,b}$$

```
[Out] 5/8*arctan(1-2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-5/8*ar
ctan(1+2^(1/2)*(d*tan(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)*2^(1/2)-5/16*ln(d^(1
/2)-2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1/2)+5/16
*ln(d^(1/2)+2^(1/2)*(d*tan(b*x+a))^(1/2)+d^(1/2)*tan(b*x+a))/b/d^(3/2)*2^(1
/2)-5/2/b/d/(d*tan(b*x+a))^(1/2)+1/2*cos(b*x+a)^2/b/d/(d*tan(b*x+a))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$,
Rules used = {2687, 296, 331, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{5\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}\,bd^{3/2}}-\frac{5\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}+1\right)}{4\sqrt{2}\,bd^{3/2}}-\frac{5\log\left(\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,bd^{3/2}}+\frac{5\log\left(\sqrt{d}\tan(a+bx)+\sqrt{2}\sqrt{d\tan(a+bx)}+\sqrt{d}\right)}{8\sqrt{2}\,bd^{3/2}}-\frac{5}{2bd\sqrt{d\tan(a+bx)}}+\frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (5*ArcTan[1 - (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(3/2)
) - (5*ArcTan[1 + (Sqrt[2]*Sqrt[d*Tan[a + b*x]])/Sqrt[d]])/(4*Sqrt[2]*b*d^(
3/2)) - (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] - Sqrt[2]*Sqrt[d*Tan[a + b*x]
]])/(8*Sqrt[2]*b*d^(3/2)) + (5*Log[Sqrt[d] + Sqrt[d]*Tan[a + b*x] + Sqrt[2]
*Sqrt[d*Tan[a + b*x]]])/(8*Sqrt[2]*b*d^(3/2)) - 5/(2*b*d*Sqrt[d*Tan[a + b*x
]]) + Cos[a + b*x]^2/(2*b*d*Sqrt[d*Tan[a + b*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(dx)^{3/2}(1+x^2)^2}\,dx, x, \tan(a+bx)\right)}{b}$$

$$= \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} + \frac{5\text{Subst}\left(\int \frac{1}{(dx)^{3/2}(1+x^2)}\,dx, x, \tan(a+bx)\right)}{4b}$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} - \frac{5\text{Subst}\left(\int \frac{\sqrt{dx}}{1+x^2}\,dx, x, \tan(a+b$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} - \frac{5\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} + \frac{5\text{Subst}\left(\int \frac{d-x^2}{1+\frac{x^4}{d^2}}\,dx, x, \sqrt{d\tan(a$$

$$= -\frac{5}{2bd\sqrt{d\tan(a+bx)}} + \frac{\cos^2(a+bx)}{2bd\sqrt{d\tan(a+bx)}} - \frac{5\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{d}+2x}{-d-\sqrt{2}\sqrt{d}x-x^2}\,dx, x$$

$$= -\frac{5\log\left(\sqrt{d}+\sqrt{d}\tan(a+bx)-\sqrt{2}\sqrt{d\tan(a+bx)}\right)}{8\sqrt{2}bd^{3/2}} + \frac{5\log\left(\sqrt{d}+\sqrt{d}\tan\right.$$

$$= \frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d\tan(a+bx)}}{\sqrt{d}}\right)}{4\sqrt{2}bd^{3/2}} - \frac{5}{}$$

Mathematica [A]
time = 0.31, size = 115, normalized size = 0.46

$$\frac{\csc(a+bx)\left(-17\cos(a+bx)+\cos(3(a+bx))+5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\sqrt{\sin(2(a+bx))}+5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\sqrt{\sin(2(a+bx))}\right)\sqrt{d\tan(a+bx)}}{8bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Csc[a + b*x]*(-17*Cos[a + b*x] + Cos[3*(a + b*x)] + 5*ArcSin[Cos[a + b*x]
- Sin[a + b*x]]*Sqrt[Sin[2*(a + b*x)]] + 5*Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]*Sqrt[Sin[2*(a + b*x)]])*Sqrt[d*Tan[a + b*x]])/(8*
b*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.28, size = 982, normalized size = 3.94

method	result	size
default	Expression too large to display	982

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/b*(5*I*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/
2*I,1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)
-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x
+a)-5*I*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*
x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(co
s(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)+
5*I*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*
2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)-5*I*(-(cos(b*x+
a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(
1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+
a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*EllipticPi((-(cos(b*x+a)-1-s
in(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)-5*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*
x+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/s
in(b*x+a))^(1/2)*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1
/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)-5*EllipticPi((-(cos(b*x+a)-1-sin(b*x+a))/s
in(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)-5*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*((cos(b*x+a
)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipt
icPi((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2
*cos(b*x+a)^3*2^(1/2)+10*cos(b*x+a)*2^(1/2))*sin(b*x+a)/cos(b*x+a)^2/(d*sin
(b*x+a)/cos(b*x+a))^(3/2)*2^(1/2)
```

Maxima [A]

time = 0.49, size = 204, normalized size = 0.82

$$-\frac{\frac{10\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}+2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}+\frac{10\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{d}-2\sqrt{d\tan\left(bx+a\right)}\right)}{2\sqrt{d}}\right)}{\sqrt{d}}-\frac{5\sqrt{2}\log\left(d\tan(bx+a)+\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{5\sqrt{2}\log\left(d\tan(bx+a)-\sqrt{2}\sqrt{d\tan\left(bx+a\right)}\sqrt{d}+d\right)}{\sqrt{d}}+\frac{8\left(5d^2\tan(bx+a)^2+4d^2\right)}{(d\tan(bx+a))^{\frac{5}{2}}+\sqrt{d\tan\left(bx+a\right)}\,d^2}}{16\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(10*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(d) + 2*sqrt(d*tan(b*x +
a)))/sqrt(d))/sqrt(d) + 10*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(d) - 2
*sqrt(d*tan(b*x + a)))/sqrt(d))/sqrt(d) - 5*sqrt(2)*log(d*tan(b*x + a) + sq
rt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 5*sqrt(2)*log(d*tan(b*x +
 a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(d) + d)/sqrt(d) + 8*(5*d^2*tan(b*x
+ a)^2 + 4*d^2)/((d*tan(b*x + a))^(5/2) + sqrt(d*tan(b*x + a))*d^2))/(b*d)
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2037 vs. $2(189) = 378$.
time = 62.54, size = 2037, normalized size = 8.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(32*(cos(b*x + a)^3 - 5*cos(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a
))*sin(b*x + a) - 20*(sqrt(2)*b*d^2*cos(b*x + a)^2 - sqrt(2)*b*d^2)*(1/(b^4
*d^6))^(1/4)*arctan((sqrt(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x
+ a) - 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + s
qrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x
+ a)) + 1)*(b^2*d^3*sqrt(1/(b^4*d^6)) + 2*cos(b*x + a)*sin(b*x + a) + (sqrt
(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*d*(1/(b^4*d^6))^
(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) - (sqrt
(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/
(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(2*cos(
b*x + a)^2 - 1)) - 20*(sqrt(2)*b*d^2*cos(b*x + a)^2 - sqrt(2)*b*d^2)*(1/(b^
4*d^6))^(1/4)*arctan(-(sqrt(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*
x + a) + 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) +
 sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*
x + a)) + 1)*(b^2*d^3*sqrt(1/(b^4*d^6)) + 2*cos(b*x + a)*sin(b*x + a) - (sq
rt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)^2 + sqrt(2)*b*d*(1/(b^4*d^6)
)^(1/4)*cos(b*x + a)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))) + (sq
rt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(
1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)))/(2*co
s(b*x + a)^2 - 1)) + 20*(sqrt(2)*b*d^2*cos(b*x + a)^2 - sqrt(2)*b*d^2)*(1/(
b^4*d^6))^(1/4)*arctan(1/2*((sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x +
```

```
a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(4*b^2*d^3*sqrt(1/(b
^4*d^6))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)
*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2
)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*sin(b*x + a)/cos(b*x + a))
- (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a) + sqrt(2)*b*d*(1/(b^4*d
^6))^(1/4)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 4*(b^2*d^3*cos
(b*x + a)^3 - b^2*d^3*cos(b*x + a))*sqrt(1/(b^4*d^6)) - 2*sin(b*x + a))/((2
*cos(b*x + a)^2 - 1)*sin(b*x + a))) + 20*(sqrt(2)*b*d^2*cos(b*x + a)^2 - sq
rt(2)*b*d^2)*(1/(b^4*d^6))^(1/4)*arctan(1/2*((sqrt(2)*b^3*d^4*(1/(b^4*d^6))
^(3/4)*cos(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(4*
b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^4*(1
/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/
4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) + 1)*sqrt(d*sin(b*x +
a)/cos(b*x + a)) - (sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a) + sqrt
(2)*b*d*(1/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*sin(b*x + a)/cos(b*x + a))
 - 4*(b^2*d^3*cos(b*x + a)^3 - b^2*d^3*cos(b*x + a))*sqrt(1/(b^4*d^6)) + 2*
sin(b*x + a))/((2*cos(b*x + a)^2 - 1)*sin(b*x + a))) - 5*(sqrt(2)*b*d^2*cos
(b*x + a)^2 - sqrt(2)*b*d^2)*(1/(b^4*d^6))^(1/4)*log(4*b^2*d^3*sqrt(1/(b^4*
d^6))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*co
s(b*x + a)*sin(b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*s
qrt(d*sin(b*x + a)/cos(b*x + a)) + 1) + 5*(sqrt(2)*b*d^2*cos(b*x + a)^2 - s
qrt(2)*b*d^2)*(1/(b^4*d^6))^(1/4)*log(4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x +
 a)*sin(b*x + a) - 2*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(
b*x + a) + sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x +
 a)/cos(b*x + a)) + 1) - 5*(sqrt(2)*b*d^2*cos(b*x + a)^2 - sqrt(2)*b*d^2)*(
1/(b^4*d^6))^(1/4)*log(1/4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x +
 a) + 1/8*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) +
sqrt(2)*b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x
 + a)) + 1/16) + 5*(sqrt(2)*b*d^2*cos(b*x + a)^2 - sqrt(2)*b*d^2)*(1/(b^4*d
^6))^(1/4)*log(1/4*b^2*d^3*sqrt(1/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) - 1/
8*(sqrt(2)*b^3*d^4*(1/(b^4*d^6))^(3/4)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*
b*d*(1/(b^4*d^6))^(1/4)*cos(b*x + a)^2)*sqrt(d*sin(b*x + a)/cos(b*x + a)) +
 1/16))/(b*d^2*cos(b*x + a)^2 - b*d^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/(d*tan(a + b*x))**(3/2), x)
```

Giac [A]

time = 0.51, size = 252, normalized size = 1.01

$$-\frac{\frac{10\,\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{|d|}+2\,\sqrt{d\tan\left(bx+a\right)}\right)}{2\,\sqrt{|d|}}\right)}{bd^2}+\frac{10\,\sqrt{2}\,|d|^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\,\left(\sqrt{2}\,\sqrt{|d|}-2\,\sqrt{d\tan\left(bx+a\right)}\right)}{2\,\sqrt{|d|}}\right)}{bd^2}-\frac{5\,\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)+\sqrt{2}\,\sqrt{d\tan\left(bx+a\right)}\,\sqrt{|d|}+|d|\right)}{bd^2}+\frac{5\,\sqrt{2}\,|d|^{\frac{3}{2}}\log\left(d\tan(bx+a)-\sqrt{2}\,\sqrt{d\tan\left(bx+a\right)}\,\sqrt{|d|}+|d|\right)}{bd^2}+\frac{8\,\left(5\,d^2\tan(bx+a)^2+4\,d^2\right)}{\left(\sqrt{d\tan\left(bx+a\right)}\,d^2\tan(bx+a)^2+\sqrt{d\tan\left(bx+a\right)}\,d^2\right)b}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] -1/16*(10*sqrt(2)*abs(d)^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) + 2
*sqrt(d*tan(b*x + a)))/sqrt(abs(d)))/(b*d^2) + 10*sqrt(2)*abs(d)^(3/2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(d)) - 2*sqrt(d*tan(b*x + a)))/sqrt(abs(d)
))/(b*d^2) - 5*sqrt(2)*abs(d)^(3/2)*log(d*tan(b*x + a) + sqrt(2)*sqrt(d*tan
(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) + 5*sqrt(2)*abs(d)^(3/2)*log(d*ta
n(b*x + a) - sqrt(2)*sqrt(d*tan(b*x + a))*sqrt(abs(d)) + abs(d))/(b*d^2) +
8*(5*d^2*tan(b*x + a)^2 + 4*d^2)/((sqrt(d*tan(b*x + a))*d^2*tan(b*x + a)^2
+ sqrt(d*tan(b*x + a))*d^2)*b))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2/(d*tan(a + b*x))^(3/2), x)
```

3.263 $\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=138

$$-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{24\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^2\sqrt{\sin(2a+2bx)}}+\frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}$$

```
[Out] -2*sec(b*x+a)^3/b/d/(d*tan(b*x+a))^(1/2)+24/5*cos(b*x+a)*(sin(a+1/4*Pi+b*x)
^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x
+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)+24/5*cos(b*x+a)*(d*tan(b*x+a))^(3/2)/
b/d^3+12/5*sec(b*x+a)*(d*tan(b*x+a))^(3/2)/b/d^3
```

Rubi [A]
time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2688, 2693, 2695, 2652, 2719}

$$\frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}+\frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}-\frac{24\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sec[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) - (24*Cos[a + b*x]*EllipticE
[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(5*b*d^2*Sqrt[Sin[2*a + 2*b*x]])
+ (24*Cos[a + b*x]*(d*Tan[a + b*x])^(3/2))/(5*b*d^3) + (12*Sec[a + b*x]*(d*
Tan[a + b*x])^(3/2))/(5*b*d^3)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
```

```
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{6\int \sec^3(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}$$

$$= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{12\sec(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\int \sec(a+bx)\sqrt{d\,\mathrm{ta}}}{5d^2}$$

$$= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(}{5bd^3}$$

$$= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(}{5bd^3}$$

$$= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} + \frac{24\cos(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} + \frac{12\sec(a+bx)(d\tan(}{5bd^3}$$

$$= -\frac{2\sec^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{24\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^2\sqrt{\sin(2a+2bx)}} + 24\,\mathrm{co}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.96, size = 104, normalized size = 0.75

$$\frac{2\csc(a+bx)\sqrt{d\tan(a+bx)}\left(-8\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\tan^2(a+bx)+\sqrt{\sec^2(a+bx)}\left(-5+12\sin^2(a+bx)+\tan^2(a+bx)\right)\right)}{5bd^2\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (2*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]]*(-8*Hypergeometric2F1[3/4, 3/2, 7/4, -
Tan[a + b*x]^2]*Tan[a + b*x]^2 + Sqrt[Sec[a + b*x]^2]*(-5 + 12*Sin[a + b*x]
^2 + Tan[a + b*x]^2)))/(5*b*d^2*Sqrt[Sec[a + b*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(147) = 294$.
time = 0.32, size = 535, normalized size = 3.88

method	result
default	$\frac{\left(24\,\mathrm{EllipticE}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\sqrt{-\frac{\cos(bx+a)-1-s}{\sin(bx+a}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b*(24*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/
2))*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a
))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)^3-12*((co
s(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)
*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-si
n(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^3+24*EllipticE((-(cos(b
*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1+sin(b*x+a
))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin
(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)^2-12*((cos(b*x+a)-1+sin(b*x+a))/sin(b
*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))
/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),
1/2*2^(1/2))*cos(b*x+a)^2-12*cos(b*x+a)^3*2^(1/2)+6*cos(b*x+a)^2*2^(1/2)+2^
(1/2))*sin(b*x+a)/cos(b*x+a)^4/(d*sin(b*x+a)/cos(b*x+a))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(a+bx)}{(d\tan(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**5/(d*tan(a + b*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(a+b\,x)^5\,(d\tan(a+b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] int(1/(cos(a + b*x)^5*(d*tan(a + b*x))^(3/2)), x)
```

3.264 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}} \, dx$

Optimal. Leaf size=104

$$-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}+\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}$$

```
[Out] -2*sec(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+4*cos(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(
1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a))^
(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)+4*cos(b*x+a)*(d*tan(b*x+a))^(3/2)/b/d^3
```

Rubi [A]

time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2688, 2693, 2695, 2652, 2719}

$$\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}-\frac{4\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sec[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (4*Cos[a + b*x]*EllipticE[a
- Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]]) + (4*
Cos[a + b*x]*(d*Tan[a + b*x])^(3/2))/(b*d^3)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
```

```
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{2\int \sec(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}\\ &= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}-\frac{4\int \cos(a+bx)\sqrt{d\tan}}{d^2}\\ &= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}-\frac{\left(4\sqrt{\cos(a+bx)}\sqrt{d\,t}\right.}{}\\ &= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}+\frac{4\cos(a+bx)(d\tan(a+bx))^{3/2}}{bd^3}-\frac{\left(4\cos(a+bx)\sqrt{d\tan(}\right.}{d^2\sqrt{}}\\ &= -\frac{2\sec(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}+\frac{4\cos(}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.47, size = 93, normalized size = 0.89

$$-\frac{2\csc(a+bx)\sqrt{d\tan(a+bx)}\left(3\cos(2(a+bx))\sqrt{\sec^2(a+bx)}+4\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\tan^2(a+bx)\right)}{3bd^2\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

[Out] (-2*Csc[a + b*x]*Sqrt[d*Tan[a + b*x]]*(3*Cos[2*(a + b*x)]*Sqrt[Sec[a + b*x]^2] + 4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Tan[a + b*x]^2))/(3*b*d^2*Sqrt[Sec[a + b*x]^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(121) = 242$.
time = 0.34, size = 502, normalized size = 4.83

method	result
default	$-\frac{\left(-4\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticE}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{\cos(bx+a)-1}{\sin(bx}} \right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/b*(-4*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)+2*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)-4*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+2*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+2*cos(b*x+a)*2^(1/2)-2^(1/2))*sin(b*x+a)/cos(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(3/2)*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")

[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3 (a + bx)}{(d \tan (a + bx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)**3/(d*tan(a + b*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos (a + b\,x)^3\,(d \tan (a + b\,x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(3/2)), x)
```

3.265 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{2\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2*cos(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+2*cos(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(
1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a))^
(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2688, 2695, 2652, 2719}

$$-\frac{2\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (2*Cos[a + b*x]*EllipticE[a
- Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{2\int \cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}$$
$$= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{\left(2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\mathrm{si}}$$
$$\overline{d^2\sqrt{\sin(a+bx)}}$$
$$= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{\left(2\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}}$$
$$= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{2\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.41, size = 69, normalized size = 0.88

$$-\frac{2\sin(a+bx)\left(3+2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\,\tan^2(a+bx)\right)}{3b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sin[a + b*x]*(3 + 2*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*S
qrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(3*b*(d*Tan[a + b*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(97) = 194.
time = 0.26, size = 496, normalized size = 6.36

method	result
default	$\left(2\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticE}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{\cos(bx+a)-1-\sin(}{\sin(bx+a)}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(2*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*
x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(
1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)-((cos(b*x+a)
-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(
b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a)
)/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+2*((cos(b*x+a)-1+sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-
1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2)-((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2
),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-cos(b*x+a)*2^(
1/2))*sin(b*x+a)/cos(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (a+bx)}{(d\tan (a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(sec(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

<u>**Giac**</u> [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

<u>**Mupad**</u> [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(a+b\,x\right)\,{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*(d*tan(a + b*x))^(3/2)),x)
```

```
[Out] int(1/(cos(a + b*x)*(d*tan(a + b*x))^(3/2)), x)
```

3.266 $\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{3\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2*cos(b*x+a)/b/d/(d*tan(b*x+a))^(1/2)+3*cos(b*x+a)*(sin(a+1/4*Pi+b*x)^2)^(
1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+a))^
(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2689, 2695, 2652, 2719}

$$-\frac{3\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(b*d*Sqrt[d*Tan[a + b*x]]) - (3*Cos[a + b*x]*EllipticE[a
 - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{3\int \cos(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}$$

$$= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{\left(3\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}\right)\int \sqrt{\cos(a+bx)}\sqrt{\text{si}}}{d^2\sqrt{\sin(a+bx)}}$$

$$= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{\left(3\cos(a+bx)\sqrt{d\tan(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}}$$

$$= -\frac{2\cos(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{3\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.44, size = 66, normalized size = 0.85

$$-\frac{2\sin(a+bx)\left(1+{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Sin[a + b*x]*(1 + Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqr
t[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(b*(d*Tan[a + b*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(97) = 194.
time = 0.26, size = 509, normalized size = 6.53

method	result
default	$\frac{\left(6\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\text{EllipticE}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{\cos(bx+a)-1-\sin(}{\sin(bx+a)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(6*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(
b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2
^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)-3*((cos(b*
x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(-(
cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*
x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+6*((cos(b*x+a)-1+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x
+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a)
)/sin(b*x+a))^(1/2)-3*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos
(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+cos(b*x+
a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2))*sin(b*x+a)/cos(b*x+a)^2/(d*sin(b*x+a)/co
s(b*x+a))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*cos(b*x + a)/(d^2*tan(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(a + bx \right)}}{\left(d \tan{\left(a + bx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)/(d*tan(a + b*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/(d*tan(b*x + a))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)}{{\left(d\tan(a+b\,x)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(cos(a + b*x)/(d*tan(a + b*x))^(3/2), x)
```

3.267 $\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{7\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{2bd^2\sqrt{\sin(2a+2bx)}}-\frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3}$$

```
[Out] -2*cos(b*x+a)^3/b/d/(d*tan(b*x+a))^(1/2)+7/2*cos(b*x+a)*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*x+
a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)-7/3*cos(b*x+a)^3*(d*tan(b*x+a))^(3/2)/
b/d^3
```

Rubi [A]
time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2689, 2692, 2695, 2652, 2719}

$$-\frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3}-\frac{7\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{2bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^3)/(b*d*Sqrt[d*Tan[a + b*x]]) - (7*Cos[a + b*x]*EllipticE[
a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(2*b*d^2*Sqrt[Sin[2*a + 2*b*x]]) -
 (7*Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(3*b*d^3)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
```

```
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx &= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\int \cos^3(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2} \\
&= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3} - \frac{7\int \cos(a+bx)\sqrt{d\tan}}{2d^2} \\
&= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3} - \frac{\left(7\sqrt{\cos(a+bx)}\,\sqrt{d}\right.}{} \\
&= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{3bd^3} - \frac{\left(7\cos(a+bx)\sqrt{d\tan}\right.}{2d^2} \\
&= -\frac{2\cos^3(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{7\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{2bd^2\sqrt{\sin(2a+2bx)}} - \frac{7\cos^3}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.39, size = 77, normalized size = 0.69

$$\frac{\sin(a+bx)\left(-13+\cos(2(a+bx))-14\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\,\tan^2(a+bx)\right)}{6b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Sin[a + b*x]*(-13 + Cos[2*(a + b*x)] - 14*Hypergeometric2F1[3/4, 3/2, 7/4,
 -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*x]^2))/(6*b*(d*Tan[a + b*x
])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(125) = 250$.
time = 0.27, size = 523, normalized size = 4.67

method	result
default	$\frac{\left(42\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticE}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{\cos(bx+a)-1-s}{\sin(bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/b*(42*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2
*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)-21*((cos
(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin
(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)+2*cos(b*x+a)^4*2^(1/2)+4
2*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))
^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))
*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)-21*((cos(b*x+a)-1+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x
+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a)
)/sin(b*x+a))^(1/2)+7*cos(b*x+a)^2*2^(1/2)-21*cos(b*x+a)*2^(1/2))*sin(b*x+a
)/cos(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*cos(b*x + a)^3/(d^2*tan(b*x + a)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3/(d*tan(a + b*x))^(3/2), x)
```

3.268 $\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=142

$$-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}-\frac{77\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{20bd^2\sqrt{\sin(2a+2bx)}}-\frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3}$$

```
[Out] -2*cos(b*x+a)^5/b/d/(d*tan(b*x+a))^(1/2)+77/20*cos(b*x+a)*(sin(a+1/4*Pi+b*x
)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*
x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)-77/30*cos(b*x+a)^3*(d*tan(b*x+a))^(3
/2)/b/d^3-11/5*cos(b*x+a)^5*(d*tan(b*x+a))^(3/2)/b/d^3
```

Rubi [A]

time = 0.13, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2689, 2692, 2695, 2652, 2719}

$$-\frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3}-\frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3}-\frac{77\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{20bd^2\sqrt{\sin(2a+2bx)}}-\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^5)/(b*d*Sqrt[d*Tan[a + b*x]]) - (77*Cos[a + b*x]*EllipticE
[a - Pi/4 + b*x, 2]*Sqrt[d*Tan[a + b*x]])/(20*b*d^2*Sqrt[Sin[2*a + 2*b*x]])
 - (77*Cos[a + b*x]^3*(d*Tan[a + b*x])^(3/2))/(30*b*d^3) - (11*Cos[a + b*x]
^5*(d*Tan[a + b*x])^(3/2))/(5*b*d^3)
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
```

```
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx = -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{11\int \cos^5(a+bx)\sqrt{d\tan(a+bx)}\,dx}{d^2}$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{11\cos^5(a+bx)(d\tan(a+bx))^{3/2}}{5bd^3} - \frac{77\int \cos^3(a+bx)\sqrt{d}}{10d^2}$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan}{5bd^3}$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan}{5bd^3}$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos^3(a+bx)(d\tan(a+bx))^{3/2}}{30bd^3} - \frac{11\cos^5(a+bx)(d\tan}{5bd^3}$$

$$= -\frac{2\cos^5(a+bx)}{bd\sqrt{d\tan(a+bx)}} - \frac{77\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{20bd^2\sqrt{\sin(2a+2bx)}} - 77\cos$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.56, size = 89, normalized size = 0.63

$$\frac{\sin(a+bx)\left(-277+34\cos(2(a+bx))+3\cos(4(a+bx))-308\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sqrt{\sec^2(a+bx)}\tan^2(a+bx)\right)}{120b(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5/(d*Tan[a + b*x])^(3/2),x]
```

```
[Out] (Sin[a + b*x]*(-277 + 34*Cos[2*(a + b*x)] + 3*Cos[4*(a + b*x)] - 308*Hyperg
eometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sqrt[Sec[a + b*x]^2]*Tan[a + b*
x]^2))/(120*b*(d*Tan[a + b*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(151) = 302$.
time = 0.31, size = 536, normalized size = 3.77

method	result
default	$\frac{\left(12\left(\cos^{6}(bx+a)\right)\sqrt{2}+462\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticE}\left(\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120/b*(12*2^(1/2)*cos(b*x+a)^6+462*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2)*cos(b*x+a)-231*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos
(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*El
lipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+
a)+22*cos(b*x+a)^4*2^(1/2)+462*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)
*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/s
in(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)
-231*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+
a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/
2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+77*cos(b*x+a)^2*2^(1/2)-2
31*cos(b*x+a)*2^(1/2))*sin(b*x+a)/cos(b*x+a)^2/(d*sin(b*x+a)/cos(b*x+a))^(3
/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*tan(b*x + a))*cos(b*x + a)^5/(d^2*tan(b*x + a)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/(d*tan(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/(d*tan(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^5/(d*tan(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^5}{{\left(d\,\tan\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5/(d*tan(a + b*x))^(3/2),x)
```

```
[Out] int(cos(a + b*x)^5/(d*tan(a + b*x))^(3/2), x)
```

3.269 $\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}-\frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\tan(a+bx)}}$$

```
[Out] 1/3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*
x),2^(1/2))*sec(b*x+a)*sin(2*b*x+2*a)^(1/2)/b/d^2/(d*tan(b*x+a))^(1/2)-2/3*
sec(b*x+a)/b/d/(d*tan(b*x+a))^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2689, 2694, 2653, 2720}

$$-\frac{\sqrt{\sin(2a+2bx)}\sec(a+bx)F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3bd^2\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
```

```
[Out] (-2*Sec[a + b*x])/(3*b*d*(d*Tan[a + b*x])^(3/2)) - (EllipticF[a - Pi/4 + b*
x, 2]*Sec[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Tan[a + b*x]])
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2694

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol]
:> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[b*Tan[e + f*x]]), Int[1
/(Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]]), x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{5/2}}\,dx &= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}-\frac{\int \frac{\sec(a+bx)}{\sqrt{d\tan(a+bx)}}\,dx}{3d^2}\\ &= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}-\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}\sqrt{\sin(a+bx)}}\,dx}{3d^2\sqrt{\cos(a+bx)}\sqrt{d\tan(a+bx)}}\\ &= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}-\frac{\left(\sec(a+bx)\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2\sqrt{d\tan(a+bx)}}\\ &= -\frac{2\sec(a+bx)}{3bd(d\tan(a+bx))^{3/2}}-\frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sec(a+bx)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\tan(a+bx)}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.48, size = 113, normalized size = 1.38

$$\frac{2\cos(2(a+bx))\csc(a+bx)\sqrt{\sec^2(a+bx)}\left(\sqrt{\sec^2(a+bx)}-\sqrt[4]{-1}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\sqrt{\tan(a+bx)}\right)\middle|-1\right)\tan^{\frac{3}{2}}(a+bx)\right)}{3bd^2\sqrt{d\tan(a+bx)}\,(-1+\tan^2(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]/(d*Tan[a + b*x])^(5/2),x]
[Out] (2*Cos[2*(a + b*x)]*Csc[a + b*x]*Sqrt[Sec[a + b*x]^2]*(Sqrt[Sec[a + b*x]^2]
 - (-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*Sqrt[Tan[a + b*x]]], -1]*Tan[a
 + b*x]^(3/2)))/(3*b*d^2*Sqrt[d*Tan[a + b*x]]*(-1 + Tan[a + b*x]^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(97) = 194.
time = 0.24, size = 306, normalized size = 3.73

method	result
default	$-\frac{(\cos(bx+a)+1)^2(-1+\cos(bx+a))^2\left(\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{\cos(bx+a)-1+\sin(bx+a)}{\sin(bx+a)}}\sqrt{-\frac{\cos(bx+a)-1-\sin(bx+a)}{\sin(bx+a)}}\cos(bx+a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b*(cos(b*x+a)+1)^2*(-1+cos(b*x+a))^2*(((-1+cos(b*x+a))/sin(b*x+a))^(1/
2)*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))
/sin(b*x+a))^(1/2)*cos(b*x+a)*sin(b*x+a)*EllipticF((-(cos(b*x+a)-1-sin(b*x+
a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/si
n(b*x+a))^(1/2)+cos(b*x+a)*2^(1/2))/sin(b*x+a)^3/cos(b*x+a)^3/(d*sin(b*x+a)
/cos(b*x+a))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.10, size = 117, normalized size = 1.43

$$\frac{\left(\cos\left(bx+a\right)^2-1\right)\sqrt{i\,d}\ \mathrm{ellipticF}\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right),-1\right)+\left(\cos\left(bx+a\right)^2-1\right)\sqrt{-i\,d}\ \mathrm{ellipticF}\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right),-1\right)+2\,\sqrt{\frac{d\sin\left(bx+a\right)}{\cos\left(bx+a\right)}}\,\cos\left(bx+a\right)}{3\,\left(bd^3\cos\left(bx+a\right)^2-bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((cos(b*x + a)^2 - 1)*sqrt(I*d)*ellipticF(cos(b*x + a) + I*sin(b*x + a)
, -1) + (cos(b*x + a)^2 - 1)*sqrt(-I*d)*ellipticF(cos(b*x + a) - I*sin(b*x
+ a), -1) + 2*sqrt(d*sin(b*x + a)/cos(b*x + a))*cos(b*x + a))/(b*d^3*cos(b*
x + a)^2 - b*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(a+bx\right)}{\left(d\tan\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))**(5/2),x)
```

```
[Out] Integral(sec(a + b*x)/(d*tan(a + b*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/(d*tan(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)/(d*tan(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(a+b\,x\right)\,{\left(d\,\tan\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*(d*tan(a + b*x))^(5/2)),x)
```

```
[Out] int(1/(cos(a + b*x)*(d*tan(a + b*x))^(5/2)), x)
```

3.270 $\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

```
[Out] -4/5*cos(b*x+a)/b/d^3/(d*tan(b*x+a))^(1/2)+4/5*cos(b*x+a)*(sin(a+1/4*Pi+b*x
)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*tan(b*
x+a))^(1/2)/b/d^4/sin(2*b*x+2*a)^(1/2)-2/5*sec(b*x+a)/b/d/(d*tan(b*x+a))^(5
/2)
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2688, 2695, 2652, 2719}

$$-\frac{4\cos(a+bx)E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{d\tan(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3/(d*Tan[a + b*x])^(7/2),x]
```

```
[Out] (-2*Sec[a + b*x])/(5*b*d*(d*Tan[a + b*x])^(5/2)) - (4*Cos[a + b*x])/(5*b*d^
3*Sqrt[d*Tan[a + b*x]]) - (4*Cos[a + b*x]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt
[d*Tan[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2695

```
Int[Sqrt[(b_.)*tan[(e_.) + (f_.)*(x_)]]/sec[(e_.) + (f_.)*(x_)], x_Symbol]
:> Dist[Sqrt[Cos[e + f*x]]*(Sqrt[b*Tan[e + f*x]]/Sqrt[Sin[e + f*x]]), Int[S
```

```
qrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]], x], x] /; FreeQ[{b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{7/2}}\,dx &= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}+\frac{2\int \frac{\sec(a+bx)}{(d\tan(a+bx))^{3/2}}\,dx}{5d^2}\\
&= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{4\int \cos(a+bx)\sqrt{d\tan(a+bx}}{5d^4}\\
&= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{\left(4\sqrt{\cos(a+bx)}\sqrt{d\tan(a+}\right.}{5d^4}\\
&= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{\left(4\cos(a+bx)\sqrt{d\tan(a+bx)}\right.}{5d^4\sqrt{\sin(2a}}\\
&= -\frac{2\sec(a+bx)}{5bd(d\tan(a+bx))^{5/2}}-\frac{4\cos(a+bx)}{5bd^3\sqrt{d\tan(a+bx)}}-\frac{4\cos(a+bx)E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)}{5bd^4\sqrt{\sin(2a+2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.02, size = 103, normalized size = 0.94

$$-\frac{2\left(4\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\tan^2(a+bx)\right)\sec^2(a+bx)+3\left(-2+\csc^2(a+bx)+\csc^4(a+bx)\right)\sqrt{\sec^2(a+bx)}\right)\sin(a+bx)\sqrt{d\tan(a+bx)}}{15bd^4\sqrt{\sec^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3/(d*Tan[a + b*x])^(7/2),x]
```

```
[Out] (-2*(4*Hypergeometric2F1[3/4, 3/2, 7/4, -Tan[a + b*x]^2]*Sec[a + b*x]^2 + 3
*(-2 + Csc[a + b*x]^2 + Csc[a + b*x]^4)*Sqrt[Sec[a + b*x]^2])*Sin[a + b*x]*
Sqrt[d*Tan[a + b*x]])/(15*b*d^4*Sqrt[Sec[a + b*x]^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 986 vs. 2(121) = 242.
time = 0.34, size = 987, normalized size = 8.97

method	result	size
default	Expression too large to display	987

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/b*(2*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(
b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(co
s(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^3-4*Ellipt
icE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)
-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(
b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)^3+2*((cos(b*x+a)-1+sin(b*
x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-
sin(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))*cos(b*x+a)^2-4*EllipticE((-(cos(b*x+a)-1-sin(b*x+a)
)/sin(b*x+a))^(1/2),1/2*2^(1/2))*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a
))^(1/2)*cos(b*x+a)^2-2*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a))/sin(b*x+a))^(1/2)*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*
EllipticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*cos(b*
x+a)+4*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*
x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(
1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)-2*((cos(b*x+
a)-1+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticF((-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+
a)-1-sin(b*x+a))/sin(b*x+a))^(1/2)+4*((cos(b*x+a)-1+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE((-(cos(b*x+a)-1-sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*(-(cos(b*x+a)-1-sin(b*x+a))/sin(b*x+a))
^(1/2)+2*cos(b*x+a)^3*2^(1/2)-cos(b*x+a)^2*2^(1/2)-2*cos(b*x+a)*2^(1/2))*si
n(b*x+a)/cos(b*x+a)^4/(d*sin(b*x+a)/cos(b*x+a))^(7/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(a+bx)}{(d\tan(a+bx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/(d*tan(b*x+a))**(7/2),x)
```

```
[Out] Integral(sec(a + b*x)**3/(d*tan(a + b*x))**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/(d*tan(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^3/(d*tan(b*x + a))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(a+b\,x)^3\,(d\tan(a+b\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(7/2)),x)
```

```
[Out] int(1/(cos(a + b*x)^3*(d*tan(a + b*x))^(7/2)), x)
```

3.271 $\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{2};-\frac{1}{6};\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/7*hypergeom([-7/6, -1/2],[-1/6],cos(f*x+e)^2)*sec(f*x+e)^(7/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{2};-\frac{1}{6};\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(10/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-7/6, -1/2, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*S
in[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^2(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\,\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{10}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{2};-\frac{1}{6};\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

Mathematica [**A**]
time = 0.15, size = 77, normalized size = 1.45

$$\frac{3\sqrt[3]{\sec(e+fx)}\left(-3\sin(e+fx)+2\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sin(e+fx)+\sec(e+fx)\tan(e+fx)\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(10/3)*Sin[e + f*x]^2,x]
[Out] (3*Sec[e + f*x]^(1/3)*(-3*Sin[e + f*x] + 2*(Cos[e + f*x]^2)^(1/6)*Hypergeom
etric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*Tan[e +
 f*x]))/(7*f)
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}(fx+e)\right)\left(\tan^2(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x)
[Out] int(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^{2}\left(e+fx\right)\sec^{\frac{4}{3}}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(4/3)*tan(f*x+e)**2,x)
```

```
[Out] Integral(tan(e + f*x)**2*sec(e + f*x)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^2\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{4/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(1/cos(e + f*x))^(4/3),x)
```

```
[Out] int(tan(e + f*x)^2*(1/cos(e + f*x))^(4/3), x)
```

3.272 $\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{6};\cos^2(e+fx)\right)\sec^{\frac{5}{3}}(e+fx)\sin(e+fx)}{5f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/5*hypergeom([-5/6, -1/2],[1/6],cos(f*x+e)^2)*sec(f*x+e)^(5/3)*sin(f*x+e)/
f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\frac{3\sin(e+fx)\sec^{\frac{5}{3}}(e+fx)\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{6};\cos^2(e+fx)\right)}{5f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(8/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-5/6, -1/2, 1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(5/3)*Si
n[e + f*x])/(5*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^{\frac{8}{3}}(e+fx)\sin^2(e+fx)\,dx = \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{8}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{6};\cos^2(e+fx)\right)\sec^{\frac{5}{3}}(e+fx)\sin(e+fx)}{5f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 1.06

$$-\frac{3\left(-1+\cos^2(e+fx)^{5/6}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\sin^2(e+fx)\right)\right)\sec^{\frac{5}{3}}(e+fx)\sin(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(8/3)*Sin[e + f*x]^2,x]
```

```
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(5/3)*Sin[e + f*x])/(5*f)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{2}{3}}(fx+e)\right)\left(\tan^2(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x)
```

```
[Out] int(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^2(e+fx)\sec^{\frac{2}{3}}(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(2/3)*tan(f*x+e)**2,x)
```

```
[Out] Integral(tan(e + f*x)**2*sec(e + f*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^2\left(\frac{1}{\cos(e+f\,x)}\right)^{2/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(1/cos(e + f*x))^(2/3),x)
```

```
[Out] int(tan(e + f*x)^2*(1/cos(e + f*x))^(2/3), x)
```

3.273 $\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};\cos^2(e+fx)\right)\sec^{\frac{4}{3}}(e+fx)\sin(e+fx)}{4f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/4*hypergeom([-2/3, -1/2],[1/3],cos(f*x+e)^2)*sec(f*x+e)^(4/3)*sin(f*x+e)/
f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\frac{3\sin(e+fx)\sec^{\frac{4}{3}}(e+fx)\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(7/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-2/3, -1/2, 1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(4/3)*Si
n[e + f*x])/(4*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^{\frac{7}{3}}(e+fx)\sin^2(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\,\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{7}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};\cos^2(e+fx)\right)\sec^{\frac{4}{3}}(e+fx)\sin(e+fx)}{4f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 1.06

$$-\frac{3\left(-1+\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)\right)\sec^{\frac{4}{3}}(e+fx)\sin(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(7/3)*Sin[e + f*x]^2,x]
```

```
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(4/3)*Sin[e + f*x])/(4*f)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{1}{3}}\left(fx+e\right)\right)\left(\tan^2\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x)
```

```
[Out] int(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^2(e+fx)\sqrt[3]{\sec(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(1/3)*tan(f*x+e)**2,x)
```

```
[Out] Integral(tan(e + f*x)**2*sec(e + f*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^2\left(\frac{1}{\cos(e+f\,x)}\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(1/cos(e + f*x))^(1/3),x)
```

```
[Out] int(tan(e + f*x)^2*(1/cos(e + f*x))^(1/3), x)
```

3.274 $\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\cos^2(e+fx)\right)\sec^{\frac{2}{3}}(e+fx)\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/2*hypergeom([-1/2, -1/3],[2/3],cos(f*x+e)^2)*sec(f*x+e)^(2/3)*sin(f*x+e)/
f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\frac{3\sin(e+fx)\sec^{\frac{2}{3}}(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(5/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-1/2, -1/3, 2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(2/3)*Si
n[e + f*x])/(2*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(e+fx)\sin^2(e+fx)\,dx = \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{5}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\cos^2(e+fx)\right)\sec^{\frac{2}{3}}(e+fx)\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 1.06

$$-\frac{3\left(-1+\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)\sec^{\frac{2}{3}}(e+fx)\sin(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(5/3)*Sin[e + f*x]^2,x]
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(2/3)*Sin[e + f*x])/(2*f)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(fx+e)}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/sec(f*x+e)^(1/3),x)
[Out] int(tan(f*x+e)^2/sec(f*x+e)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/sec(f*x+e)**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**2/sec(e + f*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(e+f\,x)^2}{\left(\frac{1}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(1/cos(e + f*x))^(1/3),x)
```

```
[Out] int(tan(e + f*x)^2/(1/cos(e + f*x))^(1/3), x)
```

3.275 $\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx$

Optimal. Leaf size=51

$$\frac{3\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\cos^2(e+fx)\right)\sqrt[3]{\sec(e+fx)}\,\sin(e+fx)}{f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3*hypergeom([-1/2, -1/6],[5/6],cos(f*x+e)^2)*sec(f*x+e)^(1/3)*sin(f*x+e)/f/
(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\frac{3\sin(e+fx)\sqrt[3]{\sec(e+fx)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\cos^2(e+fx)\right)}{f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(4/3)*Sin[e + f*x]^2,x]
```

```
[Out] (3*Hypergeometric2F1[-1/2, -1/6, 5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(1/3)*Si
n[e + f*x])/(f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(e+fx)\sin^2(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\,\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^2(e+fx)}{\cos^{\frac{4}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\cos^2(e+fx)\right)\sqrt[3]{\sec(e+fx)}\,\sin(e+fx)}{f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 1.06

$$-\frac{3\left(-1+\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)\sqrt[3]{\sec(e+fx)}\,\sin(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(4/3)*Sin[e + f*x]^2,x]
[Out] (-3*(-1 + (Cos[e + f*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, Sin[e + f
*x]^2])*Sec[e + f*x]^(1/3)*Sin[e + f*x])/f
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(fx+e)}{\sec(fx+e)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/sec(f*x+e)^(2/3),x)
[Out] int(tan(f*x+e)^2/sec(f*x+e)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{\sec^{\frac{2}{3}}(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/sec(f*x+e)**(2/3),x)
```

```
[Out] Integral(tan(e + f*x)**2/sec(e + f*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/sec(f*x+e)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sec(f*x + e)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(e+f\,x)^2}{\left(\frac{1}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(1/cos(e + f*x))^(2/3),x)
```

```
[Out] int(tan(e + f*x)^2/(1/cos(e + f*x))^(2/3), x)
```

3.276 $\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\,{}_2F_1\left(-\frac{13}{6},-\frac{3}{2};-\frac{7}{6};\cos^2(e+fx)\right)\sec^{\frac{13}{3}}(e+fx)\sin(e+fx)}{13f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/13*hypergeom([-13/6, -3/2],[-7/6],cos(f*x+e)^2)*sec(f*x+e)^(13/3)*sin(f*x
+e)/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\frac{3\sin(e+fx)\sec^{\frac{13}{3}}(e+fx)\,{}_2F_1\left(-\frac{13}{6},-\frac{3}{2};-\frac{7}{6};\cos^2(e+fx)\right)}{13f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(16/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-13/6, -3/2, -7/6, Cos[e + f*x]^2]*Sec[e + f*x]^(13/3)
*Sin[e + f*x])/(13*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^{\frac{16}{3}}(e+fx)\sin^4(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\,\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{16}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{13}{6},-\frac{3}{2};-\frac{7}{6};\cos^2(e+fx)\right)\sec^{\frac{13}{3}}(e+fx)\sin(e+fx)}{13f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.75, size = 89, normalized size = 1.68

$$\frac{3\sqrt[3]{\sec(e+fx)}\left(27\sin(e+fx)-18\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sin(e+fx)+\sec(e+fx)\left(-16+7\sec^2(e+fx)\right)\tan(e+fx)\right)}{91f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(16/3)*Sin[e + f*x]^4,x]
[Out] (3*Sec[e + f*x]^(1/3)*(27*Sin[e + f*x] - 18*(Cos[e + f*x]^2)^(1/6)*Hypergeo
metric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*(-16 +
 7*Sec[e + f*x]^2)*Tan[e + f*x]))/(91*f)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{4}{3}}(fx+e)\right)\left(\tan^4(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x)
[Out] int(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^(4/3)*tan(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^4(e+fx)\sec^{\frac{4}{3}}(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(4/3)*tan(f*x+e)**4,x)
```

```
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(4/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^(4/3)*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^4\left(\frac{1}{\cos(e+f\,x)}\right)^{4/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(1/cos(e + f*x))^(4/3),x)
```

```
[Out] int(tan(e + f*x)^4*(1/cos(e + f*x))^(4/3), x)
```

3.277 $\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\,{}_2F_1\left(-\frac{11}{6},-\frac{3}{2};-\frac{5}{6};\cos^2(e+fx)\right)\sec^{\frac{11}{3}}(e+fx)\sin(e+fx)}{11f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/11*hypergeom([-11/6, -3/2],[-5/6],cos(f*x+e)^2)*sec(f*x+e)^(11/3)*sin(f*x
+e)/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\frac{3\sin(e+fx)\sec^{\frac{11}{3}}(e+fx)\,{}_2F_1\left(-\frac{11}{6},-\frac{3}{2};-\frac{5}{6};\cos^2(e+fx)\right)}{11f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(14/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-11/6, -3/2, -5/6, Cos[e + f*x]^2]*Sec[e + f*x]^(11/3)
*Sin[e + f*x])/(11*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^{\frac{14}{3}}(e+fx)\sin^4(e+fx)\,dx = \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{14}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{11}{6},-\frac{3}{2};-\frac{5}{6};\cos^2(e+fx)\right)\sec^{\frac{11}{3}}(e+fx)\sin(e+fx)}{11f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.59, size = 78, normalized size = 1.47

$$\frac{3\left(\frac{9\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\sin^2(e+fx)\right)}{\sqrt[6]{\cos^2(e+fx)}}-(2+7\cos(2(e+fx)))\sec^4(e+fx)\right)\sin(e+fx)}{55f\sqrt[3]{\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(14/3)*Sin[e + f*x]^4,x]
[Out] (3*((9*Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e + f*x]^2])/(Cos[e + f*x]^2)^(
1/6) - (2 + 7*Cos[2*(e + f*x)])*Sec[e + f*x]^4)*Sin[e + f*x])/(55*f*Sec[e +
 f*x]^(1/3))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{2}{3}}(fx+e)\right)\left(\tan^4(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x)
[Out] int(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="fricas")
[Out] integral(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^4(e+fx)\sec^{\frac{2}{3}}(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(2/3)*tan(f*x+e)**4,x)
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(2/3)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate(sec(f*x + e)^(2/3)*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^4\left(\frac{1}{\cos\left(e+f\,x\right)}\right)^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(1/cos(e + f*x))^(2/3),x)
[Out] int(tan(e + f*x)^4*(1/cos(e + f*x))^(2/3), x)
```

3.278 $\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\,{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};\cos^2(e+fx)\right)\sec^{\frac{10}{3}}(e+fx)\sin(e+fx)}{10f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/10*hypergeom([-5/3, -3/2],[-2/3],cos(f*x+e)^2)*sec(f*x+e)^(10/3)*sin(f*x+
e)/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\frac{3\sin(e+fx)\sec^{\frac{10}{3}}(e+fx)\,{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};\cos^2(e+fx)\right)}{10f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(13/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-5/3, -3/2, -2/3, Cos[e + f*x]^2]*Sec[e + f*x]^(10/3)*
Sin[e + f*x])/(10*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^{\frac{13}{3}}(e+fx)\sin^4(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\,\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{13}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};\cos^2(e+fx)\right)\sec^{\frac{10}{3}}(e+fx)\sin(e+fx)}{10f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.52, size = 77, normalized size = 1.45

$$\frac{3\left(\frac{9\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)}{\sqrt[3]{\cos^2(e+fx)}}+\sec^2(e+fx)\left(-13+4\sec^2(e+fx)\right)\right)\sin(e+fx)}{40f\sec^{\frac{2}{3}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(13/3)*Sin[e + f*x]^4,x]
[Out] (3*((9*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^2])/(Cos[e + f*x]^2)^(
1/3) + Sec[e + f*x]^2*(-13 + 4*Sec[e + f*x]^2))*Sin[e + f*x])/(40*f*Sec[e +
 f*x]^(2/3))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{1}{3}}(fx+e)\right)\left(\tan^4(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x)
[Out] int(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \tan^4(e+fx)\sqrt[3]{\sec(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(1/3)*tan(f*x+e)**4,x)
```

```
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^(1/3)*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^4\,{\left(\frac{1}{\cos(e+f\,x)}\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(1/cos(e + f*x))^(1/3),x)
```

```
[Out] int(tan(e + f*x)^4*(1/cos(e + f*x))^(1/3), x)
```

3.279 $\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};\cos^2(e+fx)\right)\sec^{\frac{8}{3}}(e+fx)\sin(e+fx)}{8f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/8*hypergeom([-3/2, -4/3],[-1/3],cos(f*x+e)^2)*sec(f*x+e)^(8/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\frac{3\sin(e+fx)\sec^{\frac{8}{3}}(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(11/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-3/2, -4/3, -1/3, Cos[e + f*x]^2]*Sec[e + f*x]^(8/3)*S
in[e + f*x])/(8*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^{\frac{11}{3}}(e+fx)\sin^4(e+fx)\,dx = \left(\cos^{\frac{2}{3}}(e+fx)\sec^{\frac{2}{3}}(e+fx)\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{11}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};\cos^2(e+fx)\right)\sec^{\frac{8}{3}}(e+fx)\sin(e+fx)}{8f\sqrt{\sin^2(e+fx)}}$$

Mathematica [**A**]
time = 0.14, size = 78, normalized size = 1.47

$$\frac{3\sec^{\frac{2}{3}}(e+fx)\left(-11\sin(e+fx)+9\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sin(e+fx)+2\sec(e+fx)\tan(e+fx)\right)}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(11/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Sec[e + f*x]^(2/3)*(-11*Sin[e + f*x] + 9*(Cos[e + f*x]^2)^(1/3)*Hypergeo
metric2F1[1/3, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + 2*Sec[e + f*x]*Tan[
e + f*x]))/(16*f)
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(fx+e)}{\sec(fx+e)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/sec(f*x+e)^(1/3),x)
```

```
[Out] int(tan(f*x+e)^4/sec(f*x+e)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/sec(f*x+e)**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**4/sec(e + f*x)**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(e+f\,x)^4}{\left(\frac{1}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(1/cos(e + f*x))^(1/3),x)
```

```
[Out] int(tan(e + f*x)^4/(1/cos(e + f*x))^(1/3), x)
```

3.280 $\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx$

Optimal. Leaf size=53

$$\frac{3\,{}_2F_1\left(-\frac{3}{2},-\frac{7}{6};-\frac{1}{6};\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/7*hypergeom([-3/2, -7/6],[-1/6],cos(f*x+e)^2)*sec(f*x+e)^(7/3)*sin(f*x+e)
/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2712, 2656}

$$\frac{3\sin(e+fx)\sec^{\frac{7}{3}}(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{7}{6};-\frac{1}{6};\cos^2(e+fx)\right)}{7f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(10/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Hypergeometric2F1[-3/2, -7/6, -1/6, Cos[e + f*x]^2]*Sec[e + f*x]^(7/3)*S
in[e + f*x])/(7*f*Sqrt[Sin[e + f*x]^2])
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2712

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1), Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^{\frac{10}{3}}(e+fx)\sin^4(e+fx)\,dx = \left(\sqrt[3]{\cos(e+fx)}\,\sqrt[3]{\sec(e+fx)}\right)\int \frac{\sin^4(e+fx)}{\cos^{\frac{10}{3}}(e+fx)}\,dx$$
$$= \frac{3\,{}_2F_1\left(-\frac{3}{2},-\frac{7}{6};-\frac{1}{6};\cos^2(e+fx)\right)\sec^{\frac{7}{3}}(e+fx)\sin(e+fx)}{7f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 0.18, size = 77, normalized size = 1.45

$$\frac{3\sqrt[3]{\sec(e+fx)}\left(-10\sin(e+fx)+9\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sin(e+fx)+\sec(e+fx)\tan(e+fx)\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(10/3)*Sin[e + f*x]^4,x]
```

```
[Out] (3*Sec[e + f*x]^(1/3)*(-10*Sin[e + f*x] + 9*(Cos[e + f*x]^2)^(1/6)*Hypergeo
metric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*Tan[e
+ f*x]))/(7*f)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(fx+e)}{\sec(fx+e)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/sec(f*x+e)^(2/3),x)
```

```
[Out] int(tan(f*x+e)^4/sec(f*x+e)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{\sec^{\frac{2}{3}}(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/sec(f*x+e)**(2/3),x)
```

```
[Out] Integral(tan(e + f*x)**4/sec(e + f*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/sec(f*x+e)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/sec(f*x + e)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(e+f\,x)^4}{\left(\frac{1}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(1/cos(e + f*x))^(2/3),x)
```

```
[Out] int(tan(e + f*x)^4/(1/cos(e + f*x))^(2/3), x)
```

3.281 $\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{13/6}\,{}_2F_1\left(\frac{3}{2},\frac{13}{6};\frac{5}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^3(e+fx)}{3f}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(13/6)*hypergeom([3/2, 13/6],[5/2],sin(f*x+e)^2)*(d*sec(
f*x+e))^(4/3)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2697}

$$\frac{\cos^2(e+fx)^{13/6}\tan^3(e+fx)(d\sec(e+fx))^{4/3}\,{}_2F_1\left(\frac{3}{2},\frac{13}{6};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[3/2, 13/6, 5/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^3)/(3*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{4/3}\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{13/6}\,{}_2F_1\left(\frac{3}{2},\frac{13}{6};\frac{5}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^3(e+fx)}{3f}$$

Mathematica [A]
time = 0.19, size = 80, normalized size = 1.40

$$\frac{3d\sqrt[3]{d\sec(e+fx)}\left(-3\sin(e+fx)+2\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sin(e+fx)+\sec(e+fx)\tan(e+fx)\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^2,x]
```

```
[Out] (3*d*(d*Sec[e + f*x])^(1/3)*(-3*Sin[e + f*x] + 2*(Cos[e + f*x]^2)^(1/6)*Hyp
ergeometric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*T
an[e + f*x]))/(7*f)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x+e\right)\right)^{\frac{4}{3}} \left(\tan^{2}\left(f x+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x)
```

```
[Out] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*d*sec(f*x + e)*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(e+f x\right)\right)^{\frac{4}{3}} \tan^{2}\left(e+f x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(4/3)*tan(f*x+e)**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**(4/3)*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^2\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(d/cos(e + f*x))^(4/3),x)
[Out] int(tan(e + f*x)^2*(d/cos(e + f*x))^(4/3), x)
```

3.282 $\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{11/6}\,{}_2F_1\left(\frac{3}{2},\frac{11}{6};\frac{5}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^3(e+fx)}{3f}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(11/6)*hypergeom([3/2, 11/6],[5/2],sin(f*x+e)^2)*(d*sec(
f*x+e))^(2/3)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2697}

$$\frac{\cos^2(e+fx)^{11/6}\tan^3(e+fx)(d\sec(e+fx))^{2/3}\,{}_2F_1\left(\frac{3}{2},\frac{11}{6};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^(11/6)*Hypergeometric2F1[3/2, 11/6, 5/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^3)/(3*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{2/3}\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{11/6}\,{}_2F_1\left(\frac{3}{2},\frac{11}{6};\frac{5}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^3(e+fx)}{3f}$$

Mathematica [A]
time = 0.21, size = 80, normalized size = 1.40

$$\frac{3(d\sec(e+fx))^{2/3}\left(-{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\sin^2(e+fx)\right)\sin(2(e+fx))+2\sqrt[6]{\cos^2(e+fx)}\tan(e+fx)\right)}{10f\sqrt[6]{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^2,x]

[Out] (3*(d*Sec[e + f*x])^(2/3)*(-(Hypergeometric2F1[1/2, 5/6, 3/2, Sin[e + f*x]^2]*Sin[2*(e + f*x)]) + 2*(Cos[e + f*x]^2)^(1/6)*Tan[e + f*x]))/(10*f*(Cos[e + f*x]^2)^(1/6))

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x +e \right)\right)^{\frac{2}{3}} \left(\tan^{2}\left(f x +e \right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x)

[Out] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="fricas")

[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec{\left(e + f x \right)}\right)^{\frac{2}{3}} \tan^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))**(2/3)*tan(f*x+e)**2,x)

```
[Out] Integral((d*sec(e + f*x))**(2/3)*tan(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^2\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(d/cos(e + f*x))^(2/3),x)
[Out] int(tan(e + f*x)^2*(d/cos(e + f*x))^(2/3), x)
```

3.283 $\int \sqrt[3]{d\sec(e+fx)}\,\tan^2(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{5/3}\,{}_2F_1\left(\frac{3}{2},\frac{5}{3};\frac{5}{2};\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\,\tan^3(e+fx)}{3f}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(5/3)*hypergeom([3/2, 5/3],[5/2],sin(f*x+e)^2)*(d*sec(f*
x+e))^(1/3)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2697\}$

$$\frac{\cos^2(e+fx)^{5/3}\tan^3(e+fx)\sqrt[3]{d\sec(e+fx)}\,{}_2F_1\left(\frac{3}{2},\frac{5}{3};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^(5/3)*Hypergeometric2F1[3/2, 5/3, 5/2, Sin[e + f*x]^2]*(d
*Sec[e + f*x])^(1/3)*Tan[e + f*x]^3)/(3*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt[3]{d\sec(e+fx)}\,\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{5/3}\,{}_2F_1\left(\frac{3}{2},\frac{5}{3};\frac{5}{2};\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\,\tan^3(e+fx)}{3f}$$

Mathematica [A]
time = 0.20, size = 80, normalized size = 1.40

$$\frac{3\sqrt[3]{d\sec(e+fx)}\left(-{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)\sin(2(e+fx))+2\sqrt[3]{\cos^2(e+fx)}\,\tan(e+fx)\right)}{8f\sqrt[3]{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

```
[Out] (3*(d*Sec[e + f*x])^(1/3)*(-(Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^
2]*Sin[2*(e + f*x)]) + 2*(Cos[e + f*x]^2)^(1/3)*Tan[e + f*x]))/(8*f*(Cos[e
+ f*x]^2)^(1/3))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x +e \right)\right)^{\frac{1}{3}} \left(\tan^{2}\left(f x +e \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{d \sec{\left(e + f x \right)}} \tan^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*tan(f*x+e)**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**(1/3)*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^2\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(d/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^2*(d/cos(e + f*x))^(1/3), x)
```

3.284 $\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{4/3}\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)\tan^3(e+fx)}{3f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(4/3)*hypergeom([4/3, 3/2],[5/2],sin(f*x+e)^2)*tan(f*x+e
)^3/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2697}

$$\frac{\cos^2(e+fx)^{4/3}\tan^3(e+fx)\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((Cos[e + f*x]^2)^(4/3)*Hypergeometric2F1[4/3, 3/2, 5/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{4/3}\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)\tan^3(e+fx)}{3f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A]
time = 0.15, size = 80, normalized size = 1.40

$$\frac{3\left(-{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sin(2(e+fx))+2\cos^2(e+fx)^{2/3}\tan(e+fx)\right)}{4f\cos^2(e+fx)^{2/3}\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (3*(-(Hypergeometric2F1[1/3, 1/2, 3/2, Sin[e + f*x]^2]*Sin[2*(e + f*x)]) +
2*(Cos[e + f*x]^2)^(2/3)*Tan[e + f*x]))/(4*f*(Cos[e + f*x]^2)^(2/3)*(d*Sec[
e + f*x])^(1/3))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^2/(d*sec(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(d*sec(f*x+e))**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**2/(d*sec(e + f*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(e+f\,x)^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(d/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^2/(d/cos(e + f*x))^(1/3), x)
```

3.285 $\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)\tan^3(e+fx)}{3f(d\sec(e+fx))^{2/3}}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(7/6)*hypergeom([7/6, 3/2],[5/2],sin(f*x+e)^2)*tan(f*x+e
)^3/f/(d*sec(f*x+e))^(2/3)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2697}

$$\frac{\cos^2(e+fx)^{7/6}\tan^3(e+fx)\,{}_2F_1\left(\frac{7}{6},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(d*Sec[e + f*x])^(2/3),x]
```

```
[Out] ((Cos[e + f*x]^2)^(7/6)*Hypergeometric2F1[7/6, 3/2, 5/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^3)/(3*f*(d*Sec[e + f*x])^(2/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx=\frac{\cos^2(e+fx)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{3}{2};\frac{5}{2};\sin^2(e+fx)\right)\tan^3(e+fx)}{3f(d\sec(e+fx))^{2/3}}$$

Mathematica [A]
time = 0.14, size = 79, normalized size = 1.39

$$\frac{-\frac{3}{2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sin(2(e+fx))+3\cos^2(e+fx)^{5/6}\tan(e+fx)}{f\cos^2(e+fx)^{5/6}(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(d*Sec[e + f*x])^(2/3),x]
```

```
[Out] ((-3*Hypergeometric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[2*(e + f*x)])/2 +
 3*(Cos[e + f*x]^2)^(5/6)*Tan[e + f*x])/(f*(Cos[e + f*x]^2)^(5/6)*(d*Sec[e
+ f*x])^(2/3))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(fx+e\right)}{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x)
```

```
[Out] int(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^2/(d*sec(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(e+fx\right)}{\left(d\sec\left(e+fx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(d*sec(f*x+e))**(2/3),x)
```

```
[Out] Integral(tan(e + f*x)**2/(d*sec(e + f*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(d*sec(f*x+e))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(d*sec(f*x + e))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(e+f\,x)^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(d/cos(e + f*x))^(2/3),x)
```

```
[Out] int(tan(e + f*x)^2/(d/cos(e + f*x))^(2/3), x)
```

3.286 $\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{19/6}\,{}_2F_1\left(\frac{5}{2},\frac{19}{6};\frac{7}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^5(e+fx)}{5f}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(19/6)*hypergeom([5/2, 19/6],[7/2],sin(f*x+e)^2)*(d*sec(
f*x+e))^(4/3)*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2697}

$$\frac{\cos^2(e+fx)^{19/6}\tan^5(e+fx)(d\sec(e+fx))^{4/3}\,{}_2F_1\left(\frac{5}{2},\frac{19}{6};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^(19/6)*Hypergeometric2F1[5/2, 19/6, 7/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^5)/(5*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{4/3}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{19/6}\,{}_2F_1\left(\frac{5}{2},\frac{19}{6};\frac{7}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{4/3}\tan^5(e+fx)}{5f}$$

Mathematica [A]
time = 0.68, size = 92, normalized size = 1.61

$$\frac{3d\sqrt[3]{d\sec(e+fx)}\left(27\sin(e+fx)-18\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)\sin(e+fx)+\sec(e+fx)\left(-16+7\sec^2(e+fx)\right)\tan(e+fx)\right)}{91f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(4/3)*Tan[e + f*x]^4,x]
```

```
[Out] (3*d*(d*Sec[e + f*x])^(1/3)*(27*Sin[e + f*x] - 18*(Cos[e + f*x]^2)^(1/6)*Hy
pergeometric2F1[1/6, 1/2, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + Sec[e + f*x]*
(-16 + 7*Sec[e + f*x]^2)*Tan[e + f*x]))/(91*f)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x +e \right)\right)^{\frac{4}{3}} \left(\tan^{4}\left(f x +e \right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x)
```

```
[Out] int((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*d*sec(f*x + e)*tan(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec{\left(e + f x \right)}\right)^{\frac{4}{3}} \tan^{4}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(4/3)*tan(f*x+e)**4,x)
```

```
[Out] Integral((d*sec(e + f*x))**(4/3)*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(4/3)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(4/3)*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^4\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(d/cos(e + f*x))^(4/3),x)
[Out] int(tan(e + f*x)^4*(d/cos(e + f*x))^(4/3), x)
```

3.287 $\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{17/6}\,{}_2F_1\left(\frac{5}{2},\frac{17}{6};\frac{7}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^5(e+fx)}{5f}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(17/6)*hypergeom([5/2, 17/6],[7/2],sin(f*x+e)^2)*(d*sec(
f*x+e))^(2/3)*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2697\}$

$$\frac{\cos^2(e+fx)^{17/6}\tan^5(e+fx)(d\sec(e+fx))^{2/3}\,{}_2F_1\left(\frac{5}{2},\frac{17}{6};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^(17/6)*Hypergeometric2F1[5/2, 17/6, 7/2, Sin[e + f*x]^2]*
(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^5)/(5*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{2/3}\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{17/6}\,{}_2F_1\left(\frac{5}{2},\frac{17}{6};\frac{7}{2};\sin^2(e+fx)\right)(d\sec(e+fx))^{2/3}\tan^5(e$$

Mathematica [A]
time = 0.11, size = 69, normalized size = 1.21

$$\frac{3(d\sec(e+fx))^{2/3}\left(-14+9\cos^2(e+fx)^{5/6}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\sin^2(e+fx)\right)+5\sec^2(e+fx)\right)\tan(e+fx)}{55f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*Tan[e + f*x]^4,x]
```

```
[Out] (3*(d*Sec[e + f*x])^(2/3)*(-14 + 9*(Cos[e + f*x]^2)^(5/6)*Hypergeometric2F1
[1/2, 5/6, 3/2, Sin[e + f*x]^2] + 5*Sec[e + f*x]^2)*Tan[e + f*x])/(55*f)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(f x+e\right)\right)^{\frac{2}{3}} \left(\tan^{4}\left(f x+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \sec \left(e+f x\right)\right)^{\frac{2}{3}} \tan^{4}\left(e+f x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)*tan(f*x+e)**4,x)
```

```
[Out] Integral((d*sec(e + f*x))**(2/3)*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^4\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(d/cos(e + f*x))^(2/3),x)
[Out] int(tan(e + f*x)^4*(d/cos(e + f*x))^(2/3), x)
```

3.288 $\int \sqrt[3]{d\sec(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{8/3}\,{}_2F_1\left(\frac{5}{2},\frac{8}{3};\frac{7}{2};\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\,\tan^5(e+fx)}{5f}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(8/3)*hypergeom([5/2, 8/3],[7/2],sin(f*x+e)^2)*(d*sec(f*
x+e))^(1/3)*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2697}

$$\frac{\cos^2(e+fx)^{8/3}\tan^5(e+fx)\sqrt[3]{d\sec(e+fx)}\,{}_2F_1\left(\frac{5}{2},\frac{8}{3};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^(8/3)*Hypergeometric2F1[5/2, 8/3, 7/2, Sin[e + f*x]^2]*(d
*Sec[e + f*x])^(1/3)*Tan[e + f*x]^5)/(5*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt[3]{d\sec(e+fx)}\,\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{8/3}\,{}_2F_1\left(\frac{5}{2},\frac{8}{3};\frac{7}{2};\sin^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\,\tan^5(e+fx)}{5f}$$

Mathematica [A]
time = 0.11, size = 69, normalized size = 1.21

$$\frac{3\sqrt[3]{d\sec(e+fx)}\left(-13+9\cos^2(e+fx)^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};\sin^2(e+fx)\right)+4\sec^2(e+fx)\right)\tan(e+fx)}{40f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] (3*(d*Sec[e + f*x])^(1/3)*(-13 + 9*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1
[1/2, 2/3, 3/2, Sin[e + f*x]^2] + 4*Sec[e + f*x]^2)*Tan[e + f*x])/(40*f)
```

Maple [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int (d\sec (fx+e))^{\frac{1}{3}} \left(\tan^{4}(fx+e)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{d\sec (e+fx)} \tan^{4}(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*tan(f*x+e)**4,x)
```

```
[Out] Integral((d*sec(e + f*x))**(1/3)*tan(e + f*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^4\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(d/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^4*(d/cos(e + f*x))^(1/3), x)
```

3.289 $\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{7/3}\,{}_2F_1\left(\frac{7}{3},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)\tan^5(e+fx)}{5f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(7/3)*hypergeom([7/3, 5/2],[7/2],sin(f*x+e)^2)*tan(f*x+e
)^5/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2697}

$$\frac{\cos^2(e+fx)^{7/3}\tan^5(e+fx)\,{}_2F_1\left(\frac{7}{3},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)}{5f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((Cos[e + f*x]^2)^(7/3)*Hypergeometric2F1[7/3, 5/2, 7/2, Sin[e + f*x]^2]*Ta
n[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{7/3}\,{}_2F_1\left(\frac{7}{3},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)\tan^5(e+fx)}{5f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A]
time = 0.11, size = 69, normalized size = 1.21

$$\frac{3\left(-11+9\sqrt[3]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)+2\sec^2(e+fx)\right)\tan(e+fx)}{16f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (3*(-11 + 9*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, Sin[e +
 f*x]^2] + 2*Sec[e + f*x]^2)*Tan[e + f*x])/(16*f*(d*Sec[e + f*x])^(1/3))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(2/3)*tan(f*x + e)^4/(d*sec(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(d*sec(f*x+e))**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**4/(d*sec(e + f*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(e+f\,x)^4}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(d/cos(e + f*x))^(1/3),x)
[Out] int(tan(e + f*x)^4/(d/cos(e + f*x))^(1/3), x)
```

3.290 $\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}} dx$

Optimal. Leaf size=57

$$\frac{\cos^2(e+fx)^{13/6}\,{}_2F_1\left(\frac{13}{6},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)\tan^5(e+fx)}{5f(d\sec(e+fx))^{2/3}}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(13/6)*hypergeom([13/6, 5/2],[7/2],sin(f*x+e)^2)*tan(f*x
+e)^5/f/(d*sec(f*x+e))^(2/3)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2697}

$$\frac{\cos^2(e+fx)^{13/6}\tan^5(e+fx)\,{}_2F_1\left(\frac{13}{6},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)}{5f(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(d*Sec[e + f*x])^(2/3),x]
```

```
[Out] ((Cos[e + f*x]^2)^(13/6)*Hypergeometric2F1[13/6, 5/2, 7/2, Sin[e + f*x]^2]*
Tan[e + f*x]^5)/(5*f*(d*Sec[e + f*x])^(2/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(d\sec(e+fx))^{2/3}}\,dx = \frac{\cos^2(e+fx)^{13/6}\,{}_2F_1\left(\frac{13}{6},\frac{5}{2};\frac{7}{2};\sin^2(e+fx)\right)\tan^5(e+fx)}{5f(d\sec(e+fx))^{2/3}}$$

Mathematica [A]
time = 0.10, size = 67, normalized size = 1.18

$$\frac{3\left(-10+9\sqrt[6]{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\sin^2(e+fx)\right)+\sec^2(e+fx)\right)\tan(e+fx)}{7f(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(d*Sec[e + f*x])^(2/3),x]
```

```
[Out] (3*(-10 + 9*(Cos[e + f*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, Sin[e +
 f*x]^2] + Sec[e + f*x]^2)*Tan[e + f*x])/(7*f*(d*Sec[e + f*x])^(2/3))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(fx+e\right)}{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x)
```

```
[Out] int(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*tan(f*x + e)^4/(d*sec(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}\left(e+fx\right)}{\left(d\sec\left(e+fx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(d*sec(f*x+e))**(2/3),x)
```

```
[Out] Integral(tan(e + f*x)**4/(d*sec(e + f*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(d*sec(f*x+e))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(d*sec(f*x + e))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(e+f\,x)^4}{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(d/cos(e + f*x))^(2/3),x)
```

```
[Out] int(tan(e + f*x)^4/(d/cos(e + f*x))^(2/3), x)
```

3.291 $\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal. Leaf size=178

$$-\frac{\sqrt{b}\,d^3\operatorname{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}+\frac{\sqrt{b}\,d^3\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}$$

```
[Out] -1/4*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*b^(1/2)*(b*tan(f*x+e))^(1/2)/
f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+1/4*d^3*arctanh((b*sin(f*x+e))^
(1/2)/b^(1/2))*b^(1/2)*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f
*x+e))^(1/2)+1/2*d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/b/f
```

Rubi [A]
time = 0.11, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2693, 2696, 2644, 335, 304, 209, 212}

$$-\frac{\sqrt{b}\,d^3\sqrt{b\tan(e+fx)}\operatorname{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\,\sqrt{d\sec(e+fx)}}+\frac{\sqrt{b}\,d^3\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\,\sqrt{d\sec(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -1/4*(Sqrt[b]*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]]
)/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) + (Sqrt[b]*d^3*ArcTanh[Sqrt
[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*f*Sqrt[d*Sec[e + f*x]]*S
qrt[b*Sin[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2
*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}\,dx = \frac{d^2\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{2bf} + \frac{1}{4}d^2\int\sqrt{d\sec(e+fx)}$$

$$= \frac{d^2\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{2bf} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\int}{4\sqrt{d\sec(e+}}$$

$$= \frac{d^2\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{2bf} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\mathrm{S}}{4bf\sqrt{d\sec(e}}$$

$$= \frac{d^2\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{2bf} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\mathrm{S}}{2bf\sqrt{d\sec(}}$$

$$= \frac{d^2\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{2bf} + \frac{\left(bd^3\sqrt{b\tan(e+fx)}\right)\mathrm{S}}{4f\sqrt{d\sec(e}}$$

$$= -\frac{\sqrt{b}\,d^3\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}} + \frac{\sqrt{b}\,d^3\,\mathrm{ta}}{4}$$

Mathematica [A]
time = 1.32, size = 174, normalized size = 0.98

$$\frac{b(d\sec(e+fx))^{5/2}\left(-4\sqrt{\sec(e+fx)}+4\sec^{\frac{5}{2}}(e+fx)+2\mathrm{ArcTan}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\tan^2(e+fx)}+\left(-\log\left(1-\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)+\log\left(1+\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\right)\sqrt[4]{\tan^2(e+fx)}\right)}{8f\sec^{\frac{5}{2}}(e+fx)\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (b*(d*Sec[e + f*x])^(5/2)*(-4*Sqrt[Sec[e + f*x]] + 4*Sec[e + f*x]^(5/2) + 2
*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)]*(Tan[e + f*x]^2)^(1/4) +
 (-Log[1 - Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)] + Log[1 + Sqrt[Sec[e
+ f*x]]/(Tan[e + f*x]^2)^(1/4)])*(Tan[e + f*x]^2)^(1/4)))/(8*f*Sec[e + f*x]
^(5/2)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 4.49, size = 600, normalized size = 3.37

method	result
default	$\frac{\left(i\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(f}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(I*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-s
in(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticP
i(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos
(f*x+e)^2-I*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-
I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*Ellipt
icPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*
cos(f*x+e)^2-((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)
-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*Ellip
ticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))
*cos(f*x+e)^2-((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e
)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*Elli
pticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)*cos(f*x+e)^2+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))*cos(f*x+e)*(b*sin(f*x+e)/cos
(f*x+e))^(1/2)*(d/cos(f*x+e))^(5/2)*sin(f*x+e)/(cos(f*x+e)-1)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(154) = 308.
time = 0.57, size = 858, normalized size = 4.82

$$\left[-\frac{2\sqrt{-bd}\,d^2\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{-bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{4\left(bd\cos(fx+e)^2-bd-\left(bd\cos(fx+e)+bd\right)\sin(fx+e)\right)}\right)\cos(fx+e)-\sqrt{-bd}\,d^2\cos(fx+e)\log\left(\frac{bd\cos(fx+e)^4-72bd\cos(fx+e)^2-8\left(7\cos(fx+e)^3-\left(\cos(fx+e)^3-8\cos(fx+e)\right)\sin(fx+e)-8\cos(fx+e)\right)\sqrt{-bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}+72bd+28\left(bd\cos(fx+e)^2-2bd\right)\sin(fx+e)}{\cos(fx+e)^4-8\cos(fx+e)^2-4\left(\cos(fx+e)^2-2\right)\sin(fx+e)+8}\right)-16d^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin(fx+e)}{32f\cos(fx+e)}, -\frac{2\sqrt{bd}\,d^2\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2+\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{4\left(bd\cos(fx+e)^2-bd+\left(bd\cos(fx+e)+bd\right)\sin(fx+e)\right)}\right)\cos(fx+e)-\sqrt{bd}\,d^2\cos(fx+e)\log\left(\frac{bd\cos(fx+e)^4-72bd\cos(fx+e)^2-8\left(7\cos(fx+e)^3+\left(\cos(fx+e)^3-8\cos(fx+e)\right)\sin(fx+e)-8\cos(fx+e)\right)\sqrt{bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}+72bd-28\left(bd\cos(fx+e)^2-2bd\right)\sin(fx+e)}{\cos(fx+e)^4-8\cos(fx+e)^2+4\left(\cos(fx+e)^2-2\right)\sin(fx+e)+8}\right)-16d^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin(fx+e)}{32f\cos(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(2*sqrt(-b*d)*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (c
os(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt
(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x
+ e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) - sqrt(
-b*d)*d^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*
(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(
```

```
f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))
 + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 -
 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*d^2*sqrt
(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
 + e)), -1/32*(2*sqrt(b*d)*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^
2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4
)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos
(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) -
sqrt(b*d)*d^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2
- 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*
cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^
4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*d^2*s
qrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(
f*x + e))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\tan\left(e+f\,x\right)}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2), x)
```

3.292 $\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx$

Optimal. Leaf size=93

$$-\frac{d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}$$

```
[Out] d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(
1/2)/sin(f*x+e)^(1/2)+d^2*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2693, 2696, 2721, 2719}

$$\frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}}-\frac{d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -((d^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec
[e + f*x]]*Sqrt[Sin[e + f*x]])) + (d^2*(b*Tan[e + f*x])^(3/2))/(b*f*Sqrt[d*
Sec[e + f*x]])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\,dx = \frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}} - \frac{1}{2}d^2\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$$

$$= \frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}} - \frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b\sin(e+fx)}}{2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= \frac{d^2(b\tan(e+fx))^{3/2}}{bf\sqrt{d\sec(e+fx)}} - \frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{\sin(e+fx)}\,d}{2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

$$= -\frac{d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} + \frac{d^2(b\tan(e+fx))^3}{bf\sqrt{d\sec(e+fx}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 11.09, size = 71, normalized size = 0.76

$$\frac{d\sqrt{d\sec(e+fx)}\sin(e+fx)\sqrt{b\tan(e+fx)}\left(1-\frac{{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)}{\left(-\tan^2(e+fx)\right)^{3/4}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d*Sqrt[d*Sec[e + f*x]]*Sin[e + f*x]*Sqrt[b*Tan[e + f*x]]*(1 - Hypergeometr
ic2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]/(-Tan[e + f*x]^2)^(3/4)))/f
```

Maple [C] Result contains complex when optimal does not.
time = 0.61, size = 572, normalized size = 6.15

method	result
default	$\frac{\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\left(\frac{d}{\cos(fx+e)}\right)^{\frac{3}{2}}\cos(fx+e)\left(2\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx}{\sin(fx+e)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(b*sin(f*x+e)/cos(f*x+e))^(1/2)*(d/cos(f*x+e))^(3/2)*cos(f*x+e)*(2*(-
I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))
^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*
x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2-(-I*(cos(f*x
+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(
I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin
(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2+2*(-I*(cos(f*x+e)-1)/s
in(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*
x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e))
/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)-(-I*(cos(f*x+e)-1)/sin(f*x+e))^(
1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f
*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))*cos(f*x+e)-cos(f*x+e)*2^(1/2)+2^(1/2))/sin(f*x+e)*2^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 113, normalized size = 1.22

$$\frac{2\,d\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)-i\sqrt{-2i\,bd}\,\mathrm{dweierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+i\sqrt{2i\,bd}\,\mathrm{dweierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e
) - I*sqrt(-2*I*b*d)*d*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(
f*x + e) + I*sin(f*x + e))) + I*sqrt(2*I*b*d)*d*weierstrassZeta(4, 0, weier
strassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\tan\left(e+fx\right)}\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b \tan (e+f x)} \left(\frac{d}{\cos (e+f x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2),x)
[Out] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2), x)
```

3.293 $\int \sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}\,dx$

Optimal. Leaf size=132

$$-\frac{\sqrt{b}\,d\,\mathrm{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}+\frac{\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}$$

```
[Out] -d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*b^(1/2)*(b*tan(f*x+e))^(1/2)/f/(d*s
ec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+d*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2
))*b^(1/2)*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2696, 2644, 335, 304, 209, 212}

$$\frac{\sqrt{b}\,d\sqrt{b\tan(e+fx)}\,\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f\sqrt{b\sin(e+fx)}\,\sqrt{d\sec(e+fx)}}-\frac{\sqrt{b}\,d\sqrt{b\tan(e+fx)}\,\mathrm{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f\sqrt{b\sin(e+fx)}\,\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -((Sqrt[b]*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(f*
Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])) + (Sqrt[b]*d*ArcTanh[Sqrt[b*Sin
[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Si
n[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}\,dx &= \frac{\left(d\sqrt{b\tan(e+fx)}\right)\int \sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}\\
&= \frac{\left(d\sqrt{b\tan(e+fx)}\right)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx, x, b\sin(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}\\
&= \frac{\left(2d\sqrt{b\tan(e+fx)}\right)\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{bf\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}\\
&= \frac{\left(bd\sqrt{b\tan(e+fx)}\right)\mathrm{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{f\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}} - \ldots\\
&= -\frac{\sqrt{b}\,d\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}} + \frac{\sqrt{b}\,d\tanh \ldots}{f \ldots}
\end{aligned}$$

Mathematica [A]
time = 0.79, size = 136, normalized size = 1.03

$$\frac{b\left(2\text{ArcTan}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)-\log\left(1-\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)+\log\left(1+\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\right)\sqrt{d\sec(e+fx)}\sqrt[4]{\tan^2(e+fx)}}{2f\sqrt{\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (b*(2*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)] - Log[1 - Sqrt[Sec[
e + f*x]]/(Tan[e + f*x]^2)^(1/4)] + Log[1 + Sqrt[Sec[e + f*x]]/(Tan[e + f*x
]^2)^(1/4)])*Sqrt[d*Sec[e + f*x]]*(Tan[e + f*x]^2)^(1/4))/(2*f*Sqrt[Sec[e +
 f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 306, normalized size = 2.32

method	result
default	$\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\,\sqrt{2}\,\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\sin(fx+e)\sqrt{\frac{d}{\cos(fx+e)}}\,\sqrt{\frac{b\sin(}{\cos(f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*2^(1/2)*((I*cos(f*x+e)-I+sin(f*x
+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(
f*x+e)*(d/cos(f*x+e))^(1/2)*(b*sin(f*x+e)/cos(f*x+e))^(1/2)*cos(f*x+e)*(I*E
llipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1
/2))-I*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,
1/2*2^(1/2))-EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-
1/2*I,1/2*2^(1/2))-EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2
),1/2+1/2*I,1/2*2^(1/2)))/(cos(f*x+e)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(116) = 232$.
time = 0.52, size = 710, normalized size = 5.38

$$\left[-\frac{2\sqrt{-bd}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{-bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{4\left(bd\cos(fx+e)^2-bd-(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\right)-\sqrt{-bd}\log\left(\frac{bd\cos(fx+e)^4-72bd\cos(fx+e)^2-8\left(7\cos(fx+e)^3-\left(\cos(fx+e)^3-8\cos(fx+e)\right)\sin(fx+e)-8\cos(fx+e)\right)\sqrt{-bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}+72bd+28\left(bd\cos(fx+e)^2-2bd\right)\sin(fx+e)}{\cos(fx+e)^4-8\cos(fx+e)^2-4\left(\cos(fx+e)^2-2\right)\sin(fx+e)+8}\right)}{8f}, -\frac{2\sqrt{bd}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2+\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{4\left(bd\cos(fx+e)^2-bd+(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\right)-\sqrt{bd}\log\left(\frac{bd\cos(fx+e)^4-72bd\cos(fx+e)^2-8\left(7\cos(fx+e)^3+\left(\cos(fx+e)^3-8\cos(fx+e)\right)\sin(fx+e)-8\cos(fx+e)\right)\sqrt{bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}+72bd-28\left(bd\cos(fx+e)^2-2bd\right)\sin(fx+e)}{\cos(fx+e)^4-8\cos(fx+e)^2+4\left(\cos(fx+e)^2-2\right)\sin(fx+e)+8}\right)}{8f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*sqrt(-b*d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(f*
x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(-b*d
)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^
2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e))) - sqrt(-b*d)*log((b*d*cos
(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3
 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x
 + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2
- 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)
^2 - 2)*sin(f*x + e) + 8)))/f, -1/8*(2*sqrt(b*d)*arctan(1/4*(cos(f*x + e)^3
 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) -
2*cos(f*x + e) + 4)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(
f*x + e))/(b*d*cos(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)
)) - sqrt(b*d)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*(7*cos(f
*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e)
)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d
 - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*
x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\tan(e+fx)}\,\sqrt{d\sec(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*tan(e + f*x))*sqrt(d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{b\tan\left(e+f\,x\right)}\,\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2), x)
```

3.294 $\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=55

$$\frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1
/2)/sin(f*x+e)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2696, 2721, 2719}

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Sec[e +
 f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\sqrt{b\tan(e+fx)}\int\sqrt{b\sin(e+fx)}\,dx}{\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$
$$= \frac{\sqrt{b\tan(e+fx)}\int\sqrt{\sin(e+fx)}\,dx}{\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$
$$= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.60, size = 62, normalized size = 1.13

$$-\frac{2b\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-2*b*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(
1/4))/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.50, size = 551, normalized size = 10.02

method	result
risch	$-\frac{i\sqrt{2}\sqrt{-\frac{ib\left(e^{2i(fx+e)}-1\right)}{e^{2i(fx+e)}+1}}}{f\sqrt{\frac{d\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}}+\frac{i\left(\frac{2i\left(-ibd\,e^{2i(fx+e)}+idb\right)}{bd\sqrt{e^{i(fx+e)}\left(-ibd\,e^{2i(fx+e)}+idb\right)}}-\frac{\sqrt{e^{i(fx+e)}+1}\sqrt{-2\,e^{i(fx+e)}+2}}{}\right.}{}$
default	$-\frac{\left(2\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(f}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(2*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/s
in(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(
```

```
((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)-(-I*
(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(
1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+
e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2*(-I*(cos(f*x+e
)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*
cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f
*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*
((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2
),1/2*2^(1/2))+cos(f*x+e)*2^(1/2)-2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(1/2)/
(d/cos(f*x+e))^(1/2)/sin(f*x+e)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(d*sec(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 70, normalized size = 1.27

$$\frac{i\sqrt{-2ibd}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e)))-i\sqrt{2ibd}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)-i\sin(fx+e)))}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] (I*sqrt(-2*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x +
 e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*weierstrassZeta(4, 0, weierstrassP
Inverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*tan(e + f*x))/sqrt(d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(f*x + e))/sqrt(d*sec(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(1/2), x)
```

3.295 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

```
[Out] 2/3*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2685}

$$\frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(3/2))/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

Mathematica [A]
time = 0.14, size = 34, normalized size = 1.00

$$\frac{2(b\tan(e+fx))^{3/2}}{3bf(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(3/2))/(3*b*f*(d*Sec[e + f*x])^(3/2))
```

Maple [A]
time = 0.36, size = 50, normalized size = 1.47

method	result	size
default	$\frac{2\sin(fx+e)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}{3f\cos(fx+e)\left(\frac{d}{\cos(fx+e)}\right)^{\frac{3}{2}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/f*sin(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(1/2)/cos(f*x+e)/(d/cos(f*x+e))^
(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(3/2), x)
```

Fricas [A]
time = 0.40, size = 55, normalized size = 1.62

$$\frac{2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{3d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)*sin
(f*x + e)/(d^2*f)
```

Sympy [A]
time = 12.04, size = 53, normalized size = 1.56

$$\begin{cases} \frac{2\sqrt{b\tan(e+fx)}\tan(e+fx)}{3f(d\sec(e+fx))^{\frac{3}{2}}} & \text{for } f \neq 0 \\ \frac{x\sqrt{b\tan(e)}}{(d\sec(e))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Piecewise((2*sqrt(b*tan(e + f*x))*tan(e + f*x)/(3*f*(d*sec(e + f*x))**(3/2)
), Ne(f, 0)), (x*sqrt(b*tan(e))/(d*sec(e))**(3/2), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(3/2), x)
```

Mupad [**B**]
time = 3.48, size = 55, normalized size = 1.62

$$\frac{\sin\left(2\,e+2\,f\,x\right)\,\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{3\,d^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(3/2),x)
```

```
[Out] (sin(2*e + 2*f*x)*(d/cos(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2
*f*x) + 1))^(1/2))/(3*d^2*f)
```

3.296 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

```
[Out] -4/5*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^2/f/(d*sec(f*x+
e))^(1/2)/sin(f*x+e)^(1/2)+2/5*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(5/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2692, 2696, 2721, 2719}

$$\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (4*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt[d*S
ec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(5*b*f*(d*Sec
[e + f*x])^(5/2))
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx &= \frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}+\frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{5d^2}\\
&= \frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}+\frac{\left(2\sqrt{b\tan(e+fx)}\right)\int \sqrt{b\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= \frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}+\frac{\left(2\sqrt{b\tan(e+fx)}\right)\int \sqrt{\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\
&= \frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{5bf(d\sec(e+fx))^{5/2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.77, size = 79, normalized size = 0.83

$$-\frac{b\left(-1+\cos(2(e+fx))+4\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/5*(b*(-1 + Cos[2*(e + f*x)] + 4*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e
+ f*x]^2]*(-Tan[e + f*x]^2)^(1/4)))/(d^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[
e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.47, size = 571, normalized size = 6.01

method	result
default	$-\frac{\left(-2\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\,\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\ \text{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/f*(-2*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e)
)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+4
*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+
e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos
(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2^(1/2)*cos
(f*x+e)^3-2*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e
))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+4*(-I*(cos(
f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*
(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+
sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+cos(f*x+e)*2^(1/2)-2*2^(1/2))*(b
*sin(f*x+e)/cos(f*x+e))^(1/2)/(d/cos(f*x+e))^(5/2)/cos(f*x+e)^2/sin(f*x+e)*
2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 121, normalized size = 1.27

$$\frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2\sin(fx+e)+i\sqrt{-2i\,bd}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e)))-i\sqrt{2i\,bd}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(fx+e)-i\sin(fx+e)))\right)}{5\,d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/5*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2*
sin(f*x + e) + I*sqrt(-2*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse(4
```

```
, 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*weierstrassZeta(4, 0
, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \tan{\left(e + f x \right)}}}{\left(d \sec{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(5/2),x)
[Out] Integral(sqrt(b*tan(e + f*x))/(d*sec(e + f*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\mathrm{tan}\left(e+f\,x\right)}}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(5/2), x)
```

3.297 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}+\frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}$$

```
[Out] 2/7*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(7/2)+8/21*(b*tan(f*x+e))^(3/2)
/b/d^2/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2692, 2685}

$$\frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}+\frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(3/2))/(7*b*f*(d*Sec[e + f*x])^(7/2)) + (8*(b*Tan[e + f
*x])^(3/2))/(21*b*d^2*f*(d*Sec[e + f*x])^(3/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{7/2}}\,dx &= \frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}+\frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}\,dx}{7d^2}\\ &= \frac{2(b\tan(e+fx))^{3/2}}{7bf(d\sec(e+fx))^{7/2}}+\frac{8(b\tan(e+fx))^{3/2}}{21bd^2f(d\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 53, normalized size = 0.74

$$\frac{(19\sin(e+fx)+3\sin(3(e+fx)))\sqrt{b\tan(e+fx)}}{42d^3f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] ((19*Sin[e + f*x] + 3*Sin[3*(e + f*x)])*Sqrt[b*Tan[e + f*x]])/(42*d^3*f*Sqr
t[d*Sec[e + f*x]])
```

Maple [A]
time = 0.54, size = 62, normalized size = 0.86

method	result	size
default	$\frac{2(3(\cos^2(fx+e))+4)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\,\sin(fx+e)}{21f\left(\frac{d}{\cos(fx+e)}\right)^{\frac{7}{2}}\cos(fx+e)^3}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/f*(3*cos(f*x+e)^2+4)*(b*sin(f*x+e)/cos(f*x+e))^(1/2)*sin(f*x+e)/(d/cos
(f*x+e))^(7/2)/cos(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(7/2), x)
```

Fricas [A]
time = 0.40, size = 69, normalized size = 0.96

$$\frac{2\left(3\cos\left(fx+e\right)^3+4\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{21\,d^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e))*sin(f*x + e)/(d^4*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(7/2), x)
```

Mupad [B]
time = 3.35, size = 69, normalized size = 0.96

$$\frac{\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,\left(22\,\sin\left(2\,e+2\,f\,x\right)+3\,\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{84\,d^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(7/2),x)
```

```
[Out] ((d/cos(e + f*x))^(1/2)*(22*sin(2*e + 2*f*x) + 3*sin(4*e + 4*f*x))*((b*sin(
2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(84*d^4*f)
```

3.298 $\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=132

$$\frac{8E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}+\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}$$

```
[Out] -8/15*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipti
cE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^4/f/(d*sec(f*x
+e))^(1/2)/sin(f*x+e)^(1/2)+2/9*(b*tan(f*x+e))^(3/2)/b/f/(d*sec(f*x+e))^(9/
2)+4/15*(b*tan(f*x+e))^(3/2)/b/d^2/f/(d*sec(f*x+e))^(5/2)
```

Rubi [A]
time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2692, 2696, 2721, 2719}

$$\frac{8E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}}+\frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (8*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(15*d^4*f*Sqrt[d*
Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) + (2*(b*Tan[e + f*x])^(3/2))/(9*b*f*(d*Se
c[e + f*x])^(9/2)) + (4*(b*Tan[e + f*x])^(3/2))/(15*b*d^2*f*(d*Sec[e + f*x]
)^(5/2))
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{9/2}}\,dx = \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}} + \frac{2\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx}{3d^2}$$

$$= \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}} + \frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}} + \frac{4\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{15d^4}$$

$$= \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}} + \frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}} + \frac{\left(4\sqrt{b\tan(e+fx)}\right)\int \sqrt{b\,\mathrm{si}}}{15d^4\sqrt{d\sec(e+fx)}\,\sqrt{b\,\mathrm{s}}}$$

$$= \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}} + \frac{4(b\tan(e+fx))^{3/2}}{15bd^2f(d\sec(e+fx))^{5/2}} + \frac{\left(4\sqrt{b\tan(e+fx)}\right)\int \sqrt{\sin}}{15d^4\sqrt{d\sec(e+fx)}\,\sqrt{\sin}}$$

$$= \frac{8E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}} + \frac{2(b\tan(e+fx))^{3/2}}{9bf(d\sec(e+fx))^{9/2}} + \frac{4(b\tan(e+}{15bd^2f(d\sec(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 11.07, size = 92, normalized size = 0.70

$$\frac{b(17+5\cos(2(e+fx)))\sin^2(e+fx) - 24b\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Tan[e + f*x]]/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (b*(17 + 5*Cos[2*(e + f*x)])*Sin[e + f*x]^2 - 24*b*Hypergeometric2F1[-1/4,
1/4, 3/4, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(1/4))/(45*d^4*f*Sqrt[d*Sec[e +
 f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 586, normalized size = 4.44

method	result
default	$\frac{\left(-5\sqrt{2}\,(\cos^5(fx+e))+12\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\,\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\;\mathrm{EllipticF}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45/f*(-5*2^(1/2)*cos(f*x+e)^5+12*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I
*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/s
in(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1
/2*2^(1/2))*cos(f*x+e)-24*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+
e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2
))*cos(f*x+e)+12*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(
f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*E
llipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-24*(-I
*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^
(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x
+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2^(1/2)*cos(f*x+e)^3-6*cos
(f*x+e)*2^(1/2)+12*2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(1/2)/(d/cos(f*x+e))^
(9/2)/cos(f*x+e)^4/sin(f*x+e)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 135, normalized size = 1.02

$$\frac{2\left((5\cos(fx+e)^4+6\cos(fx+e)^2)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin(fx+e)+6i\sqrt{-2i\,bd}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e)))-6i\sqrt{2i\,bd}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)-i\sin(fx+e)))\right)}{45\,d^5 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] 2/45*((5*cos(f*x + e)^4 + 6*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e
))*sqrt(d/cos(f*x + e))*sin(f*x + e) + 6*I*sqrt(-2*I*b*d)*weierstrassZeta(4
```

```
, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2
*I*b*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*si
n(f*x + e))))/(d^5*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(1/2)/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(1/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tan(f*x + e))/(d*sec(f*x + e))^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\tan\left(e+f\,x\right)}}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(9/2),x)
```

```
[Out] int((b*tan(e + f*x))^(1/2)/(d/cos(e + f*x))^(9/2), x)
```

3.299 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=131

$$-\frac{b^2d^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{6f\sqrt{b\tan(e+fx)}}-\frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f}+\frac{b(d\sec}{}$$

```
[Out] 1/6*b^2*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*E
llipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)
^(1/2)/f/(b*tan(f*x+e))^(1/2)+1/3*b*(d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(1/
2)/f-1/6*b*d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2691, 2693, 2696, 2721, 2720}

$$-\frac{b^2d^2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{6f\sqrt{b\tan(e+fx)}}-\frac{bd^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{6f}+\frac{b\sqrt{b\tan(e+fx)}\,(d\sec(e+fx))^{5/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -1/6*(b^2*d^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Si
n[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]]) - (b*d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*
Tan[e + f*x]])/(6*f) + (b*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]])/(3*f
)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}\,dx = \frac{b(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}{3f} - \frac{1}{6}b^2\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}$$

$$= -\frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b}}{3f}$$

$$= -\frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b}}{3f}$$

$$= -\frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{6f} + \frac{b(d\sec(e+fx))^{5/2}\sqrt{b}}{3f}$$

$$= -\frac{b^2d^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{6f\sqrt{b\tan(e+fx)}} - bd$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.14, size = 95, normalized size = 0.73

$$\frac{bd^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\left({}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\sec^2(e+fx)\right)+\left(-1+2\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{6f\sqrt[4]{-\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (b*d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]*(Hypergeometric2F1[1/4, 3/
4, 5/4, Sec[e + f*x]^2] + (-1 + 2*Sec[e + f*x]^2)*(-Tan[e + f*x]^2)^(1/4)))
/(6*f*(-Tan[e + f*x]^2)^(1/4))
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 239, normalized size = 1.82

method	result
default	$\frac{\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}\left(\frac{d}{\cos(fx+e)}\right)^{\frac{5}{2}}\cos(fx+e)\left(i\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(}{}}\right.\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/f*(b*sin(f*x+e)/cos(f*x+e))^(3/2)*(d/cos(f*x+e))^(5/2)*cos(f*x+e)*(I*(
(I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))
/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*sin(f*x+e)*cos(f*x+e)^3-
2^(1/2)*cos(f*x+e)^3+cos(f*x+e)^2*2^(1/2)+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))/(
cos(f*x+e)-1)/sin(f*x+e)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 151, normalized size = 1.15

$$-\frac{\sqrt{-2i\,bd}\,bd^2\cos\left(fx+e\right)^2\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+\sqrt{2i\,bd}\,bd^2\cos\left(fx+e\right)^2\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right))+2\left(bd^2\cos\left(fx+e\right)^2-2\,bd^2\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{12\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(-2*I*b*d)*b*d^2*cos(f*x + e)^2*weierstrassPInverse(4, 0, cos(f*
x + e) + I*sin(f*x + e)) + sqrt(2*I*b*d)*b*d^2*cos(f*x + e)^2*weierstrassPI
nverse(4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(b*d^2*cos(f*x + e)^2 - 2*b
*d^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x +
e)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\tan(e+f\,x))^{3/2}\,\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2),x)
```

```
[Out] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2), x)
```

3.300 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=169

$$-\frac{b^{3/2}d\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4f\sqrt{b\tan(e+fx)}}-\frac{b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4f\sqrt{b\tan(e+fx)}}$$

```
[Out] -1/4*b^(3/2)*d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b
*sin(f*x+e))^(1/2)/f/(b*tan(f*x+e))^(1/2)-1/4*b^(3/2)*d*arctanh((b*sin(f*x+
e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/f/(b*tan(f*x+e
))^(1/2)+1/2*b*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.12, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2691, 2696, 2644, 335, 218, 212, 209}

$$-\frac{b^{3/2}d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\tan(e+fx)}}-\frac{b^{3/2}d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\tan(e+fx)}}+\frac{b\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{3/2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -1/4*(b^(3/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*S
qrt[b*Sin[e + f*x]])/(f*Sqrt[b*Tan[e + f*x]]) - (b^(3/2)*d*ArcTanh[Sqrt[b*S
in[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(4*f*Sqrt[
b*Tan[e + f*x]]) + (b*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}\,dx = \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f} - \frac{1}{4}b^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}$$

$$= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f} - \frac{\left(b^2d\sqrt{d\sec(e+fx)}\,\sqrt{\right.}{4\sqrt{}}$$

$$= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f} - \frac{\left(bd\sqrt{d\sec(e+fx)}\,\sqrt{b\right.}{}$$

$$= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f} - \frac{\left(bd\sqrt{d\sec(e+fx)}\,\sqrt{b\right.}{}$$

$$= \frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2f} - \frac{\left(b^2d\sqrt{d\sec(e+fx)}\,\sqrt{\right.}{}$$

$$= -\frac{b^{3/2}d\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+}}{4f\sqrt{b\tan(e+fx)}}$$

Mathematica [A]
time = 6.81, size = 129, normalized size = 0.76

$$\frac{b(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}\left(\mathrm{ArcTan}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)-\tanh^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)+2\sec^{\frac{3}{2}}(e+fx)\sqrt[4]{\tan^2(e+fx)}\right)}{4f\sec^{\frac{3}{2}}(e+fx)\sqrt[4]{\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (b*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]*(ArcTan[Sqrt[Sec[e + f*x]]/(
Tan[e + f*x]^2)^(1/4)] - ArcTanh[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)]
 + 2*Sec[e + f*x]^(3/2)*(Tan[e + f*x]^2)^(1/4)))/(4*f*Sec[e + f*x]^(3/2)*(T
an[e + f*x]^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 759, normalized size = 4.49

method	result

default	$\frac{\left(-2i(\cos^2(fx+e))\sin(fx+e)\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\ \text{EllipticF}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*(-2*I*cos(f*x+e)^2*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e)
)^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/
sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))+I*cos(f*x+e)^2*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x
+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-
1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1
/2),1/2-1/2*I,1/2*2^(1/2))+I*cos(f*x+e)^2*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f
*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-
I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/
sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-cos(f*x+e)^2*sin(f*x+e)*((I*cos(f*
x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+
e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+
sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+cos(f*x+e)^2*sin(f*x+e
)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+
e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*c
os(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(f*x+
e)*2^(1/2)-2*2^(1/2))*cos(f*x+e)*(d/cos(f*x+e))^(3/2)*(b*sin(f*x+e)/cos(f*x
+e))^(3/2)/(cos(f*x+e)-1)/sin(f*x+e)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(145) = 290$.
time = 0.54, size = 837, normalized size = 4.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(2*sqrt(-b*d)*b*d*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (co
s(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(
-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x +
 e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) + sqrt(-
b*d)*b*d*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 - 8*(
7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f
*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))
+ 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 -
8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*b*d*sqrt(
b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)), -1/32*
(2*sqrt(b*d)*b*d*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x +
 e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*d)*sq
rt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^2 -
b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) - sqrt(b*d)*b*d*
cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 + 8*(7*cos(f*x
 + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*
sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b*d -
 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(f*x
+ e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*b*d*sqrt(b*sin(f*x
+ e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\tan(e+f\,x))^{3/2}\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2),x)
[Out] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2), x)
```

3.301 $\int \sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=88

$$-\frac{b^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}+\frac{b\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}}{f}$$

```
[Out] b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
/(b*tan(f*x+e))^(1/2)+b*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2691, 2696, 2721, 2720}

$$\frac{b\sqrt{b\tan(e+fx)}\,\sqrt{d\sec(e+fx)}}{f}-\frac{b^2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -((b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(f*Sqrt[b*Tan[e + f*x]])) + (b*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*
x]])/f
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}\,dx = \frac{b\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}}{f} - \frac{1}{2}b^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$$
$$= \frac{b\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}}{f} - \frac{\left(b^2\sqrt{d\sec(e+fx)}\,\sqrt{b\sin$$
$$2\sqrt{b\,t$$
$$= \frac{b\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}}{f} - \frac{\left(b^2\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e$$
$$2\sqrt{b\tan$$
$$= -\frac{b^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}} + \frac{b\sqrt{d$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.83, size = 105, normalized size = 1.19

$$\frac{b\sqrt{d\sec(e+fx)}\left(\sec^{\frac{3}{2}}(e+fx)-\frac{{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{b\tan(e+fx)}}{f\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (b*Sqrt[d*Sec[e + f*x]]*(Sec[e + f*x]^(3/2) - (Hypergeometric2F1[1/4, 1/2,
5/4, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Sqrt[1 + Sec[e + f*x]])/Sqrt[2
])*Sqrt[b*Tan[e + f*x]]))/(f*Sec[e + f*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 211, normalized size = 2.40

method	result
default	$\frac{\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}\sqrt{\frac{d}{\cos(fx+e)}}\,\cos(fx+e)\left(i\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\,\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e}{\sin(fx+e)}}\right.}{2f(\cos(fx+e)-1)\sin(fx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(b*sin(f*x+e)/cos(f*x+e))^(3/2)*(d/cos(f*x+e))^(1/2)*cos(f*x+e)*(I*(-
I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))
^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*
x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*cos(f*x+e)+cos
(f*x+e)*2^(1/2)-2^(1/2))/(cos(f*x+e)-1)/sin(f*x+e)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 98, normalized size = 1.11

$$-\frac{\sqrt{-2i\,bd}\;b\mathrm{weierstrassPInverse}\left(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)+\sqrt{2i\,bd}\;b\mathrm{weierstrassPInverse}\left(4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)-2\,b\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x +
 e)) + sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
 e)) - 2*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\tan\left(e+fx\right)\right)^{\frac{3}{2}}\sqrt{d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x))**(3/2)*sqrt(d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\tan(e+f\,x))^{3/2}\,\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2),x)
```

```
[Out] int((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2), x)
```

3.302 $\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=167

$$-\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}}+\frac{b^{3/2}d\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}+\frac{b^{3/2}d\tanh^{-1}\left(\sqrt{\cdots}\right.}{f(d\sec(e\cdots}$$

```
[Out] -2*d*csc(f*x+e)*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)+b^(3/2)*d*arcta
n((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)
/(b*sin(f*x+e))^(3/2)+b^(3/2)*d*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*ta
n(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)/(b*sin(f*x+e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2696, 2644, 327, 335, 218, 212, 209}

$$\frac{b^{3/2}d(b\tan(e+fx))^{3/2}\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}+\frac{b^{3/2}d(b\tan(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{f(b\sin(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}-\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-2*d*Csc[e + f*x]*(b*Tan[e + f*x])^(3/2))/(f*(d*Sec[e + f*x])^(3/2)) + (b^
(3/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*(b*Tan[e + f*x])^(3/2))/(f*(d*
Sec[e + f*x])^(3/2)*(b*Sin[e + f*x])^(3/2)) + (b^(3/2)*d*ArcTanh[Sqrt[b*Sin
[e + f*x]]/Sqrt[b]]*(b*Tan[e + f*x])^(3/2))/(f*(d*Sec[e + f*x])^(3/2)*(b*Si
n[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{3/2}}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\left(d(b\tan(e+fx))^{3/2}\right)\int \sec(e+fx)(b\sin(e+fx))^{3/2}\,dx}{(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}$$

$$= \frac{\left(d(b\tan(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{b^2}}\,dx, x, b\sin(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}$$

$$= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}} + \frac{\left(bd(b\tan(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{b^2}\right.}\right.}{f(d\sec(e+fx))^{3/2}(b\sin(e+}$$

$$= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}} + \frac{\left(2bd(b\tan(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{b^2}}\,dx,\right.}{f(d\sec(e+fx))^{3/2}(b\sin(e+}$$

$$= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}} + \frac{\left(b^2d(b\tan(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x\right.}{f(d\sec(e+fx))^{3/2}(b\sin(e+}$$

$$= -\frac{2d\csc(e+fx)(b\tan(e+fx))^{3/2}}{f(d\sec(e+fx))^{3/2}} + \frac{b^{3/2}d\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)(b\tan(e+}{f(d\sec(e+fx))^{3/2}(b\sin(e+fx))^{3/}}$$

Mathematica [A]
time = 4.91, size = 128, normalized size = 0.77

$$-\frac{b\sqrt{b\tan(e+fx)}\left(\text{ArcTan}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt{\sec(e+fx)} - \tanh^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt{\sec(e+fx)} + 2\sqrt[4]{\tan^2(e+fx)}\right)}{f\sqrt{d\sec(e+fx)}\sqrt[4]{\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] -((b*Sqrt[b*Tan[e + f*x]]*(ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)
]*Sqrt[Sec[e + f*x]] - ArcTanh[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)]*S
qrt[Sec[e + f*x]] + 2*(Tan[e + f*x]^2)^(1/4)))/(f*Sqrt[d*Sec[e + f*x]]*(Tan
[e + f*x]^2)^(1/4)))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 719, normalized size = 4.31

method	result

default	$\frac{\left(2i\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(fx-}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(2*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e)
)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)-I
*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+
e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*co
s(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(f*x+e)-
I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x
+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*c
os(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(f*x+e)
+(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+
e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*co
s(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(f*x+e)-
(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e
))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos
(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(f*x+e)-2
*cos(f*x+e)*2^(1/2)+2*2^(1/2))*cos(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)/(
cos(f*x+e)-1)/(d/cos(f*x+e))^(1/2)/sin(f*x+e)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(d*sec(f*x + e)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(150) = 300.
time = 0.67, size = 805, normalized size = 4.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*b*d*sqrt(-b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (co
s(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(
b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e))/(b*cos(f*x + e
)^2 - (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - b*d*sqrt(-b/d)*log((b*cos(f
*x + e)^4 - 72*b*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8
*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x +
 e))*sqrt(-b/d)*sqrt(d/cos(f*x + e)) + 28*(b*cos(f*x + e)^2 - 2*b)*sin(f*x
+ e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*si
n(f*x + e) + 8)) + 16*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
e))*cos(f*x + e))/(d*f), 1/8*(2*b*d*sqrt(b/d)*arctan(1/4*(cos(f*x + e)^3 -
5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*c
os(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x
 + e))/(b*cos(f*x + e)^2 + (b*cos(f*x + e) + b)*sin(f*x + e) - b)) + b*d*sq
rt(b/d)*log((b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 +
 (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*si
n(f*x + e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e)) - 28*(b*cos(f*x + e
)^2 - 2*b)*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos
(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*b*sqrt(b*sin(f*x + e)/cos(f*x + e)
)*sqrt(d/cos(f*x + e))*cos(f*x + e))/(d*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\tan{(e+fx)})^{\frac{3}{2}}}{\sqrt{d\sec{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((b*tan(e + f*x))**(3/2)/sqrt(d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/sqrt(d*sec(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^{3/2}}{\sqrt{\frac{d}{\cos{(e+f\,x)}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(1/2), x)
```

3.303 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2b^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

```
[Out] -2/3*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1
/2)/d^2/f/(b*tan(f*x+e))^(1/2)-2/3*b*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^
(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2690, 2696, 2721, 2720}

$$\frac{2b^2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) - (2*b*Sqrt[b*Tan[e + f*x]])/(3*f*(d*S
ec[e + f*x])^(3/2))
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{3/2}}\,dx &= -\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}+\frac{b^2\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3d^2}\\ &= -\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}+\frac{\left(b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int\frac{1}{\sqrt{b\sin(e+fx)}}}{3d^2\sqrt{b\tan(e+fx)}}\\ &= -\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}+\frac{\left(b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,d}{3d^2\sqrt{b\tan(e+fx)}}\\ &= \frac{2b^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.66, size = 98, normalized size = 1.02

$$-\frac{2b\left(-\sqrt{2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sec^{\frac{3}{2}}(e+fx)+\sqrt{1+\sec(e+fx)}\right)\sqrt{b\tan(e+fx)}}{3f(d\sec(e+fx))^{3/2}\sqrt{1+\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b*(-(Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[(e + f*x)/2]^2]*Sec[
e + f*x]^(3/2)) + Sqrt[1 + Sec[e + f*x]])*Sqrt[b*Tan[e + f*x]])/(3*f*(d*Sec
[e + f*x])^(3/2)*Sqrt[1 + Sec[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.41, size = 207, normalized size = 2.16

method	result
default	$-\frac{\left(i\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\,\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(}{\sin(fx+e)}}\right.\right.}{3f(\cos(fx+e)-1)\left(\frac{d}{\cos(fx+e)}\right)^{\frac{3}{2}}\sin(fx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*sin(f*x+e)*((I*cos(f*x+e)-I+
sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/
2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+co
s(f*x+e)^2*2^(1/2)-cos(f*x+e)*2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(3/2)/(cos
(f*x+e)-1)/(d/cos(f*x+e))^(3/2)/sin(f*x+e)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 112, normalized size = 1.17

$$-\frac{2\,b\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}-\sqrt{-2i\,bd}\,b\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))-\sqrt{2i\,bd}\,b\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right))}{3\,d^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x +
e)^2 - sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
+ e)) - sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
+ e)))/(d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\tan\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*tan(e + f*x))**(3/2)/(d*sec(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^{3/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(3/2),x)
```

```
[Out] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(3/2), x)
```

3.304 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=34

$$\frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

```
[Out] 2/5*(b*tan(f*x+e))^(5/2)/b/f/(d*sec(f*x+e))^(5/2)
```

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2685}

$$\frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(5/2))/(5*b*f*(d*Sec[e + f*x])^(5/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{5/2}}\,dx = \frac{2(b\tan(e+fx))^{5/2}}{5bf(d\sec(e+fx))^{5/2}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(34) = 68.
time = 1.68, size = 141, normalized size = 4.15

$$-\frac{b\sec^{\frac{3}{2}}(e+fx)\left(\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{\sec(e+fx)}+\sqrt{\frac{1}{1+\cos(e+fx)}}\cos(3(e+fx))\sec^{\frac{3}{2}}(e+fx)-\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{1+\sec(e+fx)}\right)\sqrt{b\tan(e+fx)}}{10f\sqrt{\frac{1}{1+\cos(e+fx)}}(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/10*(b*Sec[e + f*x]^(3/2)*(Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[Sec[e + f*x
]] + Sqrt[(1 + Cos[e + f*x])^(-1)]*Cos[3*(e + f*x)]*Sec[e + f*x]^(3/2) - Se
c[(e + f*x)/2]^2*Sqrt[1 + Sec[e + f*x]])*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[(1 +
 Cos[e + f*x])^(-1)]*(d*Sec[e + f*x])^(5/2))
```

Maple [A]
time = 0.33, size = 50, normalized size = 1.47

method	result	size
default	$\frac{2 \sin(fx+e) \left(\frac{b \sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}}{5 f \cos(fx+e) \left(\frac{d}{\cos(fx+e)}\right)^{\frac{5}{2}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/f*sin(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)/cos(f*x+e)/(d/cos(f*x+e))^
(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(30) = 60$.
time = 0.41, size = 63, normalized size = 1.85

$$-\frac{2 \left(b \cos \left(fx+e\right)^{3} - b \cos \left(fx+e\right)\right) \sqrt{\frac{b \sin \left(fx+e\right)}{\cos \left(fx+e\right)}} \sqrt{\frac{d}{\cos \left(fx+e\right)}}}{5 \, d^{3} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/5*(b*cos(f*x + e)^3 - b*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e))/(d^3*f)
```

Sympy [A]
time = 62.86, size = 53, normalized size = 1.56

$$\begin{cases} \frac{2(b\tan(e+fx))^{\frac{3}{2}}\tan(e+fx)}{5f(d\sec(e+fx))^{\frac{5}{2}}} & \text{for } f \neq 0 \\ \frac{x(b\tan(e))^{\frac{3}{2}}}{(d\sec(e))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Piecewise((2*(b*tan(e + f*x))**(3/2)*tan(e + f*x)/(5*f*(d*sec(e + f*x))**(5
/2)), Ne(f, 0)), (x*(b*tan(e))**(3/2)/(d*sec(e))**(5/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(5/2), x)
```

Mupad [B]
time = 3.16, size = 65, normalized size = 1.91

$$\frac{b\sqrt{\frac{d}{\cos(e+fx)}}\,(\cos(e+fx)-\cos(3e+3fx))\,\sqrt{\frac{b\sin(2e+2fx)}{\cos(2e+2fx)+1}}}{10\,d^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(5/2),x)
```

```
[Out] (b*(d/cos(e + f*x))^(1/2)*(cos(e + f*x) - cos(3*e + 3*f*x))*((b*sin(2*e + 2
*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(10*d^3*f)
```

3.305 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=131

$$\frac{4b^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{21d^4f\sqrt{b\tan(e+fx)}}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}+\frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}}$$

```
[Out] -4/21*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ell
ipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(
1/2)/d^4/f/(b*tan(f*x+e))^(1/2)-2/7*b*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))
^(7/2)+2/21*b*(b*tan(f*x+e))^(1/2)/d^2/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2690, 2692, 2696, 2721, 2720}

$$\frac{4b^2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f\sqrt{b\tan(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}}-\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (4*b^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f
*x]])/(21*d^4*f*Sqrt[b*Tan[e + f*x]]) - (2*b*Sqrt[b*Tan[e + f*x]])/(7*f*(d*
Sec[e + f*x])^(7/2)) + (2*b*Sqrt[b*Tan[e + f*x]])/(21*d^2*f*(d*Sec[e + f*x]
)^(3/2))
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{b^2\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx}{7d^2}$$

$$= -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}} + \frac{(2b^2)\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{21d^4}$$

$$= -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}} + \frac{\left(2b^2\sqrt{d\sec(e+fx)}\,\sqrt{b\sin\right.}{21d^4\sqrt{\cdot}}$$

$$= -\frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/2}} + \frac{2b\sqrt{b\tan(e+fx)}}{21d^2f(d\sec(e+fx))^{3/2}} + \frac{\left(2b^2\sqrt{d\sec(e+fx)}\,\sqrt{\sin(\right.}{21d^4\sqrt{b}}$$

$$= \frac{4b^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}}{21d^4f\sqrt{b\tan(e+fx)}} - \frac{2b\sqrt{b\tan(e+fx)}}{7f(d\sec(e+fx))^{7/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.42, size = 105, normalized size = 0.80

$$-\frac{b\sqrt{b\tan(e+fx)}\left(4\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\sec^2(e+fx)\right)\sec^2(e+fx)+(1+3\cos(2(e+fx)))\sqrt[4]{-\tan^2(e+fx)}\right)}{21d^2f(d\sec(e+fx))^{3/2}\sqrt[4]{-\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] -1/21*(b*Sqrt[b*Tan[e + f*x]]*(4*Hypergeometric2F1[1/4, 3/4, 5/4, Sec[e + f
*x]^2]*Sec[e + f*x]^2 + (1 + 3*Cos[2*(e + f*x)])*(-Tan[e + f*x]^2)^(1/4)))/
(d^2*f*(d*Sec[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(1/4))
```

Maple [C] Result contains complex when optimal does not.
time = 0.44, size = 241, normalized size = 1.84

method	result
default	$-\frac{\left(2i\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(f}{\sin(fx+e)}}\right.\right.}{21f(\cos(fx+e)-1)\sin(fx-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/f*(2*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+
e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)
+3*2^(1/2)*cos(f*x+e)^4-3*2^(1/2)*cos(f*x+e)^3-cos(f*x+e)^2*2^(1/2)+cos(f*x
+e)*2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(3/2)/(cos(f*x+e)-1)/sin(f*x+e)/cos(
f*x+e)^2/(d/cos(f*x+e))^(7/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 125, normalized size = 0.95

$$\frac{2\left(\sqrt{-2i\,bd}\,b\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+\sqrt{2i\,bd}\,b\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)-i\,\sin(fx+e))-\left(3\,b\cos(fx+e)^4-b\cos(fx+e)^2\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\right)}{21\,d^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/21*(sqrt(-2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x +
 e)) + sqrt(2*I*b*d)*b*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
```

```
e)) - (3*b*cos(f*x + e)^4 - b*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x
+ e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^{3/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(7/2),x)
[Out] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(7/2), x)
```

3.306 $\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}}+\frac{8b\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}$$

```
[Out] -2/9*b*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(9/2)+2/45*b*(b*tan(f*x+e))^(1
/2)/d^2/f/(d*sec(f*x+e))^(5/2)+8/45*b*(b*tan(f*x+e))^(1/2)/d^4/f/(d*sec(f*x
+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2690, 2692, 2685}

$$\frac{8b\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}+\frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}}-\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (-2*b*Sqrt[b*Tan[e + f*x]])/(9*f*(d*Sec[e + f*x])^(9/2)) + (2*b*Sqrt[b*Tan[
e + f*x]])/(45*d^2*f*(d*Sec[e + f*x])^(5/2)) + (8*b*Sqrt[b*Tan[e + f*x]])/(
45*d^4*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
&& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\tan(e+fx))^{3/2}}{(d\sec(e+fx))^{9/2}}\,dx &= -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}+\frac{b^2\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx}{9d^2}\\ &= -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}}+\frac{(4b^2)\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b}}}{45d^4}\\ &= -\frac{2b\sqrt{b\tan(e+fx)}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b\sqrt{b\tan(e+fx)}}{45d^2f(d\sec(e+fx))^{5/2}}+\frac{8b\sqrt{b\tan(e+fx)}}{45d^4f\sqrt{d\sec(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 3.50, size = 158, normalized size = 1.53

$$-\frac{b\sqrt{\sec(e+fx)}\left(16\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{\sec(e+fx)}+\sqrt{\frac{1}{1+\cos(e+fx)}}\left(21\cos(3(e+fx))+5\cos(5(e+fx))\right)\sec^{\frac{3}{2}}(e+fx)-21\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{1+\sec(e+fx)}\right)\sqrt{b\tan(e+fx)}}{360d^3f\sqrt{\frac{1}{1+\cos(e+fx)}}\,(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(3/2)/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] -1/360*(b*Sqrt[Sec[e + f*x]]*(16*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[Sec[e +
 f*x]] + Sqrt[(1 + Cos[e + f*x])^(-1)]*(21*Cos[3*(e + f*x)] + 5*Cos[5*(e +
f*x)])*Sec[e + f*x]^(3/2) - 21*Sec[(e + f*x)/2]^2*Sqrt[1 + Sec[e + f*x]])*S
qrt[b*Tan[e + f*x]])/(d^3*f*Sqrt[(1 + Cos[e + f*x])^(-1)]*(d*Sec[e + f*x])^
(3/2))
```

Maple [A]
time = 0.36, size = 62, normalized size = 0.60

method	result	size
default	$\frac{2\left(5\left(\cos^2(fx+e)\right)+4\right)\sin(fx+e)\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}}{45f\left(\frac{d}{\cos(fx+e)}\right)^{\frac{9}{2}}\cos(fx+e)^3}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45/f*(5*cos(f*x+e)^2+4)*sin(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(3/2)/(d/cos
(f*x+e))^(9/2)/cos(f*x+e)^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(9/2), x)
```

Fricas [**A**]
time = 0.42, size = 76, normalized size = 0.74

$$-\frac{2\left(5\,b\cos\left(fx+e\right)^{5}-b\cos\left(fx+e\right)^{3}-4\,b\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{45\,d^{5}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
[Out] -2/45*(5*b*cos(f*x + e)^5 - b*cos(f*x + e)^3 - 4*b*cos(f*x + e))*sqrt(b*sin
(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(d^5*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(3/2)/(d*sec(f*x+e))**(9/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(3/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(3/2)/(d*sec(f*x + e))^(9/2), x)
```

Mupad [**B**]
time = 3.52, size = 78, normalized size = 0.76

$$-\frac{b\,\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}\,\left(21\,\cos\left(3\,e+3\,f\,x\right)-26\,\cos\left(e+f\,x\right)+5\,\cos\left(5\,e+5\,f\,x\right)\right)}{360\,d^5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(3/2)/(d/cos(e + f*x))^(9/2),x)
```

```
[Out] -(b*(d/cos(e + f*x))^(1/2)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1
/2)*(21*cos(3*e + 3*f*x) - 26*cos(e + f*x) + 5*cos(5*e + 5*f*x)))/(360*d^5*
f)
```

3.307 $\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=208

$$\frac{3b^{5/2}d^3\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}-\frac{3b^{5/2}d^3\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}-$$

```
[Out] 3/32*b^(5/2)*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/
f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)-3/32*b^(5/2)*d^3*arctanh((b*sin
(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(
f*x+e))^(1/2)+1/4*b*(d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(3/2)/f-3/16*b*d^2*
(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.16, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2691, 2693, 2696, 2644, 335, 304, 209, 212}

$$\frac{3b^{5/2}d^3\sqrt{b\tan(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{32f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{3b^{5/2}d^3\sqrt{b\tan(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{32f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{3bd^2(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{16f}+\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (3*b^(5/2)*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(
32*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) - (3*b^(5/2)*d^3*ArcTanh[Sq
rt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(32*f*Sqrt[d*Sec[e + f*x]
]*Sqrt[b*Sin[e + f*x]]) - (3*b*d^2*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3
/2))/(16*f) + (b*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2))/(4*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}\,dx = \frac{b(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}{4f} - \frac{1}{8}(3b^2)\int (d\sec(e+fx))$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b}{4f}$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b}{4f}$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b}{4f}$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b}{4f}$$

$$= -\frac{3bd^2\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{16f} + \frac{b(d\sec(e+fx))^{5/2}(b}{4f}$$

$$= \frac{3b^{5/2}d^3\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{32f\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d^3\tan}{32}$$

Mathematica [A]
time = 3.41, size = 189, normalized size = 0.91

$$\frac{b^3(d\sec(e+fx))^{5/2}\left(12\sqrt{\sec(e+fx)} - 28\sec^{\frac{5}{2}}(e+fx) + 16\sec^{\frac{9}{2}}(e+fx) - 6\text{ArcTan}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\tan^2(e+fx)} + 3\left(\log\left(1-\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right) - \log\left(1+\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\right)\sqrt[4]{\tan^2(e+fx)}\right)}{64f\sec^{\frac{5}{2}}(e+fx)\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (b^3*(d*Sec[e + f*x])^(5/2)*(12*Sqrt[Sec[e + f*x]] - 28*Sec[e + f*x]^(5/2)
+ 16*Sec[e + f*x]^(9/2) - 6*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4
)]*(Tan[e + f*x]^2)^(1/4) + 3*(Log[1 - Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^
(1/4)] - Log[1 + Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)])*(Tan[e + f*x]^
2)^(1/4)))/(64*f*Sec[e + f*x]^(5/2)*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.38, size = 628, normalized size = 3.02

method	result

default	$-\frac{\left(3i\left(\cos^4(fx+e)\right)\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\cos}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/f*(3*I*cos(f*x+e)^4*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-
(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e)
)^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I
,1/2*2^(1/2))-3*I*cos(f*x+e)^4*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/
2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f
*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+
1/2*I,1/2*2^(1/2))-3*cos(f*x+e)^4*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^
(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/si
n(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1
/2-1/2*I,1/2*2^(1/2))-3*cos(f*x+e)^4*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e
))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)
/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2
),1/2+1/2*I,1/2*2^(1/2))+6*2^(1/2)*cos(f*x+e)^3-6*cos(f*x+e)^2*2^(1/2)-8*co
s(f*x+e)*2^(1/2)+8*2^(1/2))*cos(f*x+e)*(d/cos(f*x+e))^(5/2)*(b*sin(f*x+e)/c
os(f*x+e))^(5/2)/(cos(f*x+e)-1)/sin(f*x+e)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(180) = 360$.
time = 0.63, size = 924, normalized size = 4.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/256*(6*sqrt(-b*d)*b^2*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2
- (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*
```

```
sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(
f*x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e)^3 +
 3*sqrt(-b*d)*b^2*d^2*cos(f*x + e)^3*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f
*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x
 + e) - 8*cos(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d
/cos(f*x + e)) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(co
s(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8))
 - 16*(3*b^2*d^2*cos(f*x + e)^2 - 4*b^2*d^2)*sqrt(b*sin(f*x + e)/cos(f*x +
e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3), 1/256*(6*sqrt(b*
d)*b^2*d^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2
+ 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*d)*sqrt(b*s
in(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^2 - b*d +
(b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e)^3 + 3*sqrt(b*d)*b^2*d^
2*cos(f*x + e)^3*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 + 8*(7*cos
(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x +
e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)) + 72*b
*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4 - 8*cos(
f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*(3*b^2*d^2*cos(
f*x + e)^2 - 4*b^2*d^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x +
e))*sin(f*x + e))/(f*cos(f*x + e)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)*(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (b\tan(e+f\,x))^{5/2}\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2), x)
```

3.308 $\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=131

$$\frac{b^2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}-\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}}+\frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f}$$

```
[Out] -1/2*b^2*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*
EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(
f*x+e))^(1/2)/sin(f*x+e)^(1/2)+1/3*b*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(3
/2)/f-1/2*b*d^2*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2691, 2693, 2696, 2721, 2719}

$$\frac{b^2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}}+\frac{b(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (b^2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(2*f*Sqrt[d
*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (b*d^2*(b*Tan[e + f*x])^(3/2))/(2*f*Sq
rt[d*Sec[e + f*x]]) + (b*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2))/(3*
f)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}\,dx &= \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f} - \frac{1}{2}b^2\int (d\sec(e+fx))^{3/}\\ &= -\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f}\\ &= -\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f}\\ &= -\frac{bd^2(b\tan(e+fx))^{3/2}}{2f\sqrt{d\sec(e+fx)}} + \frac{b(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}{3f}\\ &= \frac{b^2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} - \frac{bd^2(b\tan(e+fx}{2f\sqrt{d\sec(e+f}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 12.57, size = 93, normalized size = 0.71

$$\frac{b^3d^2\left(3-5\sec^2(e+fx)+2\sec^4(e+fx)-3\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{6f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (b^3*d^2*(3 - 5*Sec[e + f*x]^2 + 2*Sec[e + f*x]^4 - 3*Hypergeometric2F1[-1/
4, 1/4, 3/4, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(1/4)))/(6*f*Sqrt[d*Sec[e +
f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.42, size = 581, normalized size = 4.44

method	result
default	$\frac{\left(3\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{\frac{-i\cos(fx+e)+\sin(fx+e)+i}{\sin(fx+e)}}\,\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/f*(3*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*((-I*cos(f*x+e)+si
n(f*x+e)+I)/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*Elliptic
F(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^4-
6*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*((-I*cos(f*x+e)+sin(f*x+e)
+I)/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE(((I*co
s(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^4+3*((I*co
s(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*((-I*cos(f*x+e)+sin(f*x+e)+I)/sin(
f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)
-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^3-6*((I*cos(f*x+e)
-I+sin(f*x+e))/sin(f*x+e))^(1/2)*((-I*cos(f*x+e)+sin(f*x+e)+I)/sin(f*x+e))^
(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f
*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^3+3*2^(1/2)*cos(f*x+e)^3-5
*cos(f*x+e)^2*2^(1/2)+2*2^(1/2))*cos(f*x+e)*(d/cos(f*x+e))^(3/2)*(b*sin(f*x
+e)/cos(f*x+e))^(5/2)/sin(f*x+e)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 167, normalized size = 1.27

$$\frac{3i\sqrt{-2i\,bd}\,b^2d\cos(fx+e)^2\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e)))-3i\sqrt{2i\,bd}\,b^2d\cos(fx+e)^2\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)-i\,\sin(fx+e)))-2\left(3\,b^2d\cos(fx+e)^2-2\,b^2d\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\,\sqrt{\frac{d}{\cos(fx+e)}}\,\sin(fx+e)}{12\,f\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*I*sqrt(-2*I*b*d)*b^2*d*cos(f*x + e)^2*weierstrassZeta(4, 0, weierst
rassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 3*I*sqrt(2*I*b*d)*b^2*
d*cos(f*x + e)^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x +
e) - I*sin(f*x + e))) - 2*(3*b^2*d*cos(f*x + e)^2 - 2*b^2*d)*sqrt(b*sin(f*x
 + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\tan(e+f\,x))^{5/2}\left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2),x)
```

```
[Out] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2), x)
```

3.309 $\int \sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{5/2}\,dx$

Optimal. Leaf size=169

$$\frac{3b^{5/2}d\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}-\frac{3b^{5/2}d\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}+\frac{b}{}$$

```
[Out] 3/4*b^(5/2)*d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(
d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)-3/4*b^(5/2)*d*arctanh((b*sin(f*x+e
))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e)
)^(1/2)+1/2*b*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(3/2)/f
```

Rubi [A]
time = 0.11, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2691, 2696, 2644, 335, 304, 209, 212}

$$\frac{3b^{5/2}d\sqrt{b\tan(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\,\sqrt{d\sec(e+fx)}}-\frac{3b^{5/2}d\sqrt{b\tan(e+fx)}\,\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4f\sqrt{b\sin(e+fx)}\,\sqrt{d\sec(e+fx)}}+\frac{b(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (3*b^(5/2)*d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*
f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]]) - (3*b^(5/2)*d*ArcTanh[Sqrt[b*
Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(4*f*Sqrt[d*Sec[e + f*x]]*Sqrt
[b*Sin[e + f*x]]) + (b*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2))/(2*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{5/2}\,dx = \frac{b\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{2f} - \frac{1}{4}(3b^2)\int \sqrt{d\sec(e+fx)}$$

$$= \frac{b\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3b^2d\sqrt{b\tan(e+fx)}\right)\int}{4\sqrt{d\sec(e+f}}$$

$$= \frac{b\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3bd\sqrt{b\tan(e+fx)}\right)\text{Su}}{4f\sqrt{d\sec(e+}}$$

$$= \frac{b\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3bd\sqrt{b\tan(e+fx)}\right)\text{Su}}{2f\sqrt{d\sec(e}}$$

$$= \frac{b\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}{2f} - \frac{\left(3b^3d\sqrt{b\tan(e+fx)}\right)\text{S}}{4f\sqrt{d\sec(e}}$$

$$= \frac{3b^{5/2}d\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{4f\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}} - \frac{3b^{5/2}d\tanh}{4f}$$

Mathematica [A]
time = 1.70, size = 182, normalized size = 1.08

$$\frac{\csc^3(e+fx)\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{5/2}\left(-4\sqrt{\sec(e+fx)}+4\sec^{\frac{5}{2}}(e+fx)-6\,\text{ArcTan}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\tan^2(e+fx)}+3\left(\log\left(1-\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)-\log\left(1+\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\right)\sqrt[4]{\tan^2(e+fx)}\right)}{8f\sec^{\frac{7}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (Csc[e + f*x]^3*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)*(-4*Sqrt[Sec[e
+ f*x]] + 4*Sec[e + f*x]^(5/2) - 6*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^
2)^(1/4)]*(Tan[e + f*x]^2)^(1/4) + 3*(Log[1 - Sqrt[Sec[e + f*x]]/(Tan[e + f
*x]^2)^(1/4)] - Log[1 + Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)])*(Tan[e
+ f*x]^2)^(1/4)))/(8*f*Sec[e + f*x]^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 602, normalized size = 3.56

method	result
default	$\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{5}{2}}\sqrt{\frac{d}{\cos(fx+e)}}\,\cos(fx+e)\left(3i\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i(\cos(fx+e)-}{\sin(fx+e)}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/8/f*(b*sin(f*x+e)/cos(f*x+e))^(5/2)*(d/cos(f*x+e))^(1/2)*cos(f*x+e)*(3*I*cos(f*x+e)^2*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*I*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(f*x+e)^2+3*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)^2+3*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(f*x+e)^2+2*cos(f*x+e)*2^(1/2)-2*2^(1/2))/(cos(f*x+e)-1)/sin(f*x+e)*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(145) = 290$.
time = 0.53, size = 858, normalized size = 5.08

$$\left[\frac{6\sqrt{-bd}\,b^2\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{-bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{4\left(bd\cos(fx+e)^2-bd-(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\right)\cos(fx+e)+3\sqrt{-bd}\,b^2\cos(fx+e)\log\left(\frac{bd\cos(fx+e)^4-72bd\cos(fx+e)^2+8\left(7\cos(fx+e)^3-\left(\cos(fx+e)^3-8\cos(fx+e)\right)\sin(fx+e)-8\cos(fx+e)\right)\sqrt{-bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}+72bd+28\left(bd\cos(fx+e)^2-2bd\right)\sin(fx+e)}{\cos(fx+e)^4-8\cos(fx+e)^2-4\left(\cos(fx+e)^2-2\right)\sin(fx+e)+8}\right)+16b^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin(fx+e)}{32f\cos(fx+e)}, \frac{6\sqrt{bd}\,b^2\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2+\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{4\left(bd\cos(fx+e)^2-bd+(bd\cos(fx+e)+bd)\sin(fx+e)\right)}\right)\cos(fx+e)+3\sqrt{bd}\,b^2\cos(fx+e)\log\left(\frac{bd\cos(fx+e)^4-72bd\cos(fx+e)^2+8\left(7\cos(fx+e)^3+\left(\cos(fx+e)^3-8\cos(fx+e)\right)\sin(fx+e)-8\cos(fx+e)\right)\sqrt{bd}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}+72bd-28\left(bd\cos(fx+e)^2-2bd\right)\sin(fx+e)}{\cos(fx+e)^4-8\cos(fx+e)^2+4\left(\cos(fx+e)^2-2\right)\sin(fx+e)+8}\right)+16b^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\sin(fx+e)}{32f\cos(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [1/32*(6*sqrt(-b*d)*b^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos(f*x + e)^2 - b*d - (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) + 3*sqrt(-b*d)*b^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^2 + 8

```
*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos
(f*x + e))*sqrt(-b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)
) + 72*b*d + 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e)^4
- 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*b^2*sqr
t(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*cos(f*
x + e)), 1/32*(6*sqrt(b*d)*b^2*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^
2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4
)*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b*d*cos
(f*x + e)^2 - b*d + (b*d*cos(f*x + e) + b*d)*sin(f*x + e)))*cos(f*x + e) +
3*sqrt(b*d)*b^2*cos(f*x + e)*log((b*d*cos(f*x + e)^4 - 72*b*d*cos(f*x + e)^
2 + 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) -
8*cos(f*x + e))*sqrt(b*d)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x
+ e)) + 72*b*d - 28*(b*d*cos(f*x + e)^2 - 2*b*d)*sin(f*x + e))/(cos(f*x + e
)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) + 16*b^2
*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x + e))/(f*co
s(f*x + e))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(b*tan(f*x+e))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\tan(e+fx))^{5/2}\sqrt{\frac{d}{\cos(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2),x)
[Out] int((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2), x)
```

3.310 $\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=88

$$-\frac{3b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}+\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}$$

```
[Out] 3*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipti
cE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/f/(d*sec(f*x+e))
^(1/2)/sin(f*x+e)^(1/2)+b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2691, 2696, 2721, 2719}

$$\frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}-\frac{3b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(5/2)/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-3*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(f*Sqrt[d*Se
c[e + f*x]]*Sqrt[Sin[e + f*x]]) + (b*(b*Tan[e + f*x])^(3/2))/(f*Sqrt[d*Sec[
e + f*x]])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\tan(e+fx))^{5/2}}{\sqrt{d\sec(e+fx)}}\,dx &= \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}} - \frac{1}{2}\left(3b^2\right)\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx \\
&= \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}} - \frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{b\sin(e+fx)}\,dx}{2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} \\
&= \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}} - \frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{\sin(e+fx)}\,dx}{2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{3b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} + \frac{b(b\tan(e+fx))^{3/2}}{f\sqrt{d\sec(e+fx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.83, size = 74, normalized size = 0.84

$$\frac{b^3\left(\tan^2(e+fx)+3\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (b^3*(Tan[e + f*x]^2 + 3*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]*
(-Tan[e + f*x]^2)^(1/4)))/(f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 585, normalized size = 6.65

method	result
default	$\frac{\left(6\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\text{EllipticE}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(6*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/
sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE
(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2-3
*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+
e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos
(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)^2+6*(-I*(co
s(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2
)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-
I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)-3*(-I*(cos(f*x+e)-1
)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos
(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+
e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2*cos(f*x+e)^2*2^(1/2)-3*cos(
f*x+e)*2^(1/2)+2^(1/2))*cos(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(5/2)/(d/cos(f
*x+e))^(1/2)/sin(f*x+e)^3*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/sqrt(d*sec(f*x + e)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 122, normalized size = 1.39

$$\frac{2\,b^2\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{d}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)-3i\,\sqrt{-2i\,bd}\,b^2\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+3i\,\sqrt{2i\,bd}\,b^2\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))}{2\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*sin(f*x +
 e) - 3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0
, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^2*weierstrassZeta(4
, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/sqrt(d*sec(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^{5/2}}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(1/2),x)
```

```
[Out] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(1/2), x)
```

3.311 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=168

$$-\frac{b^{5/2}\mathrm{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}-\frac{2b(}{3f}$$

```
[Out] -b^(5/2)*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/d/f/(d*s
ec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)+b^(5/2)*arctanh((b*sin(f*x+e))^(1/2)/
b^(1/2))*(b*tan(f*x+e))^(1/2)/d/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)
-2/3*b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A]

time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2690, 2696, 2644, 335, 304, 209, 212}

$$-\frac{b^{5/2}\sqrt{b\tan(e+fx)}\,\mathrm{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{b^{5/2}\sqrt{b\tan(e+fx)}\,\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{df\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(d*f*
Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])) + (b^(5/2)*ArcTanh[Sqrt[b*Sin[e
 + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(d*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Si
n[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}} + \frac{b^2\int \sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}\,dx}{d^2}$$

$$= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}} + \frac{\left(b^2\sqrt{b\tan(e+fx)}\right)\int \sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{d\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}} + \frac{\left(b\sqrt{b\tan(e+fx)}\right)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx, x, b\sin(e+fx)\right)}{df\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}} + \frac{\left(2b\sqrt{b\tan(e+fx)}\right)\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{df\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2b(b\tan(e+fx))^{3/2}}{3f(d\sec(e+fx))^{3/2}} + \frac{\left(b^3\sqrt{b\tan(e+fx)}\right)\mathrm{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{df\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{df\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}} + \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{df\sqrt{d\sec(e+fx)}\,\sqrt{}}$$

Mathematica [A]
time = 0.99, size = 181, normalized size = 1.08

$$\frac{\csc^3(e+fx)\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{5/2}\left(-4\sqrt{\sec(e+fx)}\,\sin^2(e+fx)+6\,\mathrm{ArcTan}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\tan^2(e+fx)}+3\left(-\log\left(1-\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)+\log\left(1+\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\right)\sqrt[4]{\tan^2(e+fx)}\right)}{6d^2f\sec^{\frac{7}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Csc[e + f*x]^3*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)*(-4*Sqrt[Sec[e
+ f*x]]*Sin[e + f*x]^2 + 6*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)
]*(Tan[e + f*x]^2)^(1/4) + 3*(-Log[1 - Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^
(1/4)] + Log[1 + Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)])*(Tan[e + f*x]^
2)^(1/4)))/(6*d^2*f*Sec[e + f*x]^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.39, size = 558, normalized size = 3.32

method	result

default	$\dfrac{\left(3i\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{\frac{-i\cos(fx+e)+\sin(fx+e)+i}{\sin(fx+e)}}\ \text{EllipticPi}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(fx-}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/f*(3*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-I*cos(f*x+e)+sin(f*x+e)+I)/sin(f*x+e))^(1/2)*Ellipti
cPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3
*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-I*cos(f*x+e)+sin(f*x+e)+I)/sin(f*x+e))^(1/2)*EllipticPi(((I*
cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*(-I*(co
s(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-I*cos(f*x+e)+sin(f*x+e)+I)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)
-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*(-I*(cos(f*x+e)-1
)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*((-I*cos
(f*x+e)+sin(f*x+e)+I)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x
+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*cos(f*x+e)*2^(1/2)+2*2^(1/2
))*cos(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(5/2)/(cos(f*x+e)-1)/(d/cos(f*x+e))
^(3/2)/sin(f*x+e)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(148) = 296$.
time = 0.75, size = 832, normalized size = 4.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(16*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f
*x + e)*sin(f*x + e) + 6*b^2*d*sqrt(-b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*co
```

```
s(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f
*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x +
e))/(b*cos(f*x + e)^2 - (b*cos(f*x + e) + b)*sin(f*x + e) - b)) - 3*b^2*d*s
qrt(-b/d)*log((b*cos(f*x + e)^4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3
 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*
sin(f*x + e)/cos(f*x + e))*sqrt(-b/d)*sqrt(d/cos(f*x + e)) + 28*(b*cos(f*x
+ e)^2 - 2*b)*sin(f*x + e) + 72*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(
cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/(d^2*f), -1/24*(16*b^2*sqrt(b*sin(f
*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + 6*b^
2*d*sqrt(b/d)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)
^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x
+ e)/cos(f*x + e))*sqrt(b/d)*sqrt(d/cos(f*x + e))/(b*cos(f*x + e)^2 + (b*co
s(f*x + e) + b)*sin(f*x + e) - b)) - 3*b^2*d*sqrt(b/d)*log((b*cos(f*x + e)^
4 - 72*b*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x
 + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(b/d)*sqrt(d/cos(f*x + e)) - 28*(b*cos(f*x + e)^2 - 2*b)*sin(f*x + e) + 72
*b)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e
) + 8)))/(d^2*f)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,\tan\left(e+f\,x\right)\right)}^{5/2}}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(3/2),x)
[Out] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(3/2), x)
```

3.312 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=96

$$\frac{6b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

```
[Out] -6/5*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^2/f/(d*sec(
f*x+e))^(1/2)/sin(f*x+e)^(1/2)-2/5*b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))^
(5/2)
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2690, 2696, 2721, 2719}

$$\frac{6b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (6*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*d^2*f*Sqrt
[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(5*f*(d
*Sec[e + f*x])^(5/2))
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{5/2}}\,dx &= -\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}+\frac{(3b^2)\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{5d^2}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}+\frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{b\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}+\frac{\left(3b^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{\sin(e+fx)}\,dx}{5d^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}\\
&= \frac{6b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{5f(d\sec(e+fx))^{5/2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.85, size = 81, normalized size = 0.84

$$
\frac{b^3\left(-1+\cos(2(e+fx))-6\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{5d^2f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b^3*(-1 + Cos[2*(e + f*x)] - 6*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f
*x]^2]*(-Tan[e + f*x]^2)^(1/4)))/(5*d^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e
 + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.40, size = 565, normalized size = 5.89

method	result
default	$-\frac{\left(6\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(f}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/f*(6*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))
/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
E(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)-3*
(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e
))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(
f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+6*(-I*(cos(f
*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(
-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+s
in(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*(-I*(cos(f*x+e)-1)/sin(f*x+e))^
(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(
f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2*2^(1/2))-2^(1/2)*cos(f*x+e)^3+4*cos(f*x+e)*2^(1/2)-3*2^(1/2))*(
b*sin(f*x+e)/cos(f*x+e))^(5/2)/(d/cos(f*x+e))^(5/2)/sin(f*x+e)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 131, normalized size = 1.36

$$-\frac{2\,b^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2\sin(fx+e) - 3i\sqrt{-2i\,bd}\,b^2\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))) + 3i\sqrt{2i\,bd}\,b^2\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)-i\sin(fx+e)))}{5\,d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*b^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x
+ e)^2*sin(f*x + e) - 3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weierstr
assPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^2*w
```

```
eierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e
))))/(d^3*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^{5/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(5/2),x)
[Out] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(5/2), x)
```

3.313 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=34

$$\frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

```
[Out] 2/7*(b*tan(f*x+e))^(7/2)/b/f/(d*sec(f*x+e))^(7/2)
```

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2685}

$$\frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*(b*Tan[e + f*x])^(7/2))/(7*b*f*(d*Sec[e + f*x])^(7/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2(b\tan(e+fx))^{7/2}}{7bf(d\sec(e+fx))^{7/2}}$$

Mathematica [A]
time = 0.17, size = 45, normalized size = 1.32

$$\frac{2b^2\sin^3(e+fx)\sqrt{b\tan(e+fx)}}{7d^3f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*b^2*Sin[e + f*x]^3*Sqrt[b*Tan[e + f*x]])/(7*d^3*f*Sqrt[d*Sec[e + f*x]])
```

Maple [A]
time = 0.34, size = 50, normalized size = 1.47

method	result	size
default	$\frac{2 \sin(fx+e) \left(\frac{b \sin(fx+e)}{\cos(fx+e)}\right)^{\frac{5}{2}}}{7 f \cos(fx+e) \left(\frac{d}{\cos(fx+e)}\right)^{\frac{7}{2}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7/f*sin(f*x+e)*(b*sin(f*x+e)/cos(f*x+e))^(5/2)/cos(f*x+e)/(d/cos(f*x+e))^
(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(7/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(30) = 60.
time = 0.38, size = 74, normalized size = 2.18

$$-\frac{2 \left(b^{2} \cos \left(fx+e\right)^{3}-b^{2} \cos \left(fx+e\right)\right) \sqrt{\frac{b \sin \left(fx+e\right)}{\cos \left(fx+e\right)}} \sqrt{\frac{d}{\cos \left(fx+e\right)}} \sin \left(fx+e\right)}{7 d^{4} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/7*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x +
e))*sqrt(d/cos(f*x + e))*sin(f*x + e)/(d^4*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(7/2), x)
```

Mupad [**B**]
time = 3.21, size = 72, normalized size = 2.12

$$\frac{b^2 \sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(2\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}{28\,d^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(7/2),x)
```

```
[Out] (b^2*(d/cos(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) - sin(4*e + 4*f*x))*((b*sin
(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))/(28*d^4*f)
```

3.314 $\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=131

$$\frac{4b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}$$

```
[Out] -4/15*b^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ell
ipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/d^4/f/(d*sec
(f*x+e))^(1/2)/sin(f*x+e)^(1/2)-2/9*b*(b*tan(f*x+e))^(3/2)/f/(d*sec(f*x+e))
^(9/2)+2/15*b*(b*tan(f*x+e))^(3/2)/d^2/f/(d*sec(f*x+e))^(5/2)
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2690, 2692, 2696, 2721, 2719}

$$\frac{4b^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (4*b^2*EllipticE[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(15*d^4*f*Sqr
t[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]) - (2*b*(b*Tan[e + f*x])^(3/2))/(9*f*(
d*Sec[e + f*x])^(9/2)) + (2*b*(b*Tan[e + f*x])^(3/2))/(15*d^2*f*(d*Sec[e +
f*x])^(5/2))
```

Rule 2690

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*m))
, x] - Dist[b^2*((n - 1)/(a^2*m)), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[n, 1] && (LtQ[m, -1]
 || (EqQ[m, -1] && EqQ[n, 3/2])) && IntegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\tan(e+fx))^{5/2}}{(d\sec(e+fx))^{9/2}}\,dx &= -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}+\frac{b^2\int\frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}\,dx}{3d^2}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}+\frac{(2b^2)\int\frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{15d^4}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}+\frac{\left(2b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{b}}{15d^4\sqrt{d\sec(e+fx)}\sqrt{b}}\\
&= -\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b(b\tan(e+fx))^{3/2}}{15d^2f(d\sec(e+fx))^{5/2}}+\frac{\left(2b^2\sqrt{b\tan(e+fx)}\right)\int\sqrt{s}}{15d^4\sqrt{d\sec(e+fx)}\sqrt{si}}\\
&= \frac{4b^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}-\frac{2b(b\tan(e+fx))^{3/2}}{9f(d\sec(e+fx))^{9/2}}+\frac{2b(b\tan(e}{15d^2f(d\sec(}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 11.08, size = 99, normalized size = 0.76

$$-\frac{b^2\sin(2(e+fx))\sqrt{b\tan(e+fx)}\left(-1+5\cos(2(e+fx))+12\csc^2(e+fx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{90d^4f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Tan[e + f*x])^(5/2)/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] -1/90*(b^2*Sin[2*(e + f*x)]*Sqrt[b*Tan[e + f*x]]*(-1 + 5*Cos[2*(e + f*x)] +
 12*Csc[e + f*x]^2*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]*(-Tan[
e + f*x]^2)^(1/4)))/(d^4*f*Sqrt[d*Sec[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 586, normalized size = 4.47

method	result
default	$\frac{\left(5\sqrt{2}\,(\cos^5(fx+e))-12\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\,\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticE}\Big(\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/45/f*(5*2^(1/2)*cos(f*x+e)^5-12*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*
cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/si
n(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/
2*2^(1/2))*cos(f*x+e)+6*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)
-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^
(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))
*cos(f*x+e)-8*2^(1/2)*cos(f*x+e)^3-12*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*
((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2
),1/2*2^(1/2))+6*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(
f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*E
llipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*cos(
f*x+e)*2^(1/2)+6*2^(1/2))*(b*sin(f*x+e)/cos(f*x+e))^(5/2)/sin(f*x+e)^3/(d/c
os(f*x+e))^(9/2)/cos(f*x+e)^2*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 147, normalized size = 1.12

$$-\frac{2\left(-3i\sqrt{-2i\,bd}\,b^2\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e)))+3i\sqrt{2i\,bd}\,b^2\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)-i\,\sin(fx+e)))+(5\,b^2\cos(fx+e)^4-3\,b^2\cos(fx+e)^2)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\,\sin(fx+e)\right)}{45\,d^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] -2/45*(-3*I*sqrt(-2*I*b*d)*b^2*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2*I*b*d)*b^2*weierstrassZeta
(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))) + (5*b^2*c
os(f*x + e)^4 - 3*b^2*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(d/cos(f*x + e))*sin(f*x + e))/(d^5*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))**(5/2)/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tan(f*x+e))^(5/2)/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e))^(5/2)/(d*sec(f*x + e))^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\tan(e+f\,x))^{5/2}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(9/2),x)
```

```
[Out] int((b*tan(e + f*x))^(5/2)/(d/cos(e + f*x))^(9/2), x)
```

3.315 $\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=178

$$\frac{3d^3\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4\sqrt{b}\,f\sqrt{b\tan(e+fx)}}+\frac{3d^3\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(}}{4\sqrt{b}\,f\sqrt{b\tan(e+}}$$

```
[Out] 3/4*d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*
x+e))^(1/2)/f/b^(1/2)/(b*tan(f*x+e))^(1/2)+3/4*d^3*arctanh((b*sin(f*x+e))^(
1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/f/b^(1/2)/(b*tan(f*
x+e))^(1/2)+1/2*d^2*(d*sec(f*x+e))^(3/2)*(b*tan(f*x+e))^(1/2)/b/f
```

Rubi [A]
time = 0.12, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2693, 2696, 2644, 335, 218, 212, 209}

$$\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,f\sqrt{b\tan(e+fx)}}+\frac{3d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{b\tan(e+fx)}\,(d\sec(e+fx))^{3/2}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (3*d^3*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin
[e + f*x]])/(4*Sqrt[b]*f*Sqrt[b*Tan[e + f*x]]) + (3*d^3*ArcTanh[Sqrt[b*Sin[
e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(4*Sqrt[b]*f*
Sqrt[b*Tan[e + f*x]]) + (d^2*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]])/(
2*b*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{7/2}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{1}{4}(3d^2)\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)}{4\sqrt{b\tan(e+fx)}}$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)}{4bf\sqrt{b\text{ta}}}$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)}{2bf\sqrt{b\tan}}$$

$$= \frac{d^2(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{2bf} + \frac{\left(3d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)}{4f\sqrt{b\tan(}}$$

$$= \frac{3d^3\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{4\sqrt{b}\,f\sqrt{b\tan(e+fx)}} + \frac{3d^3\tanh^{-1}\Big(}{}$$

Mathematica [A]
time = 6.82, size = 136, normalized size = 0.76

$$\frac{d^3\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\left(-3\text{ArcTan}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)+3\tanh^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)+2\sec^{\frac{3}{2}}(e+fx)\sqrt[4]{\tan^2(e+fx)}\right)}{4bf\sqrt{\sec(e+fx)}\sqrt[4]{\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d^3*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]*(-3*ArcTan[Sqrt[Sec[e + f*x]
]/(Tan[e + f*x]^2)^(1/4)] + 3*ArcTanh[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(
1/4)] + 2*Sec[e + f*x]^(3/2)*(Tan[e + f*x]^2)^(1/4)))/(4*b*f*Sqrt[Sec[e + f
*x]]*(Tan[e + f*x]^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 758, normalized size = 4.26

method	result

default	$-\frac{\left(3i\left(\cos^2(fx+e)\right)\sin(fx+e)\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\ \text{EllipticPi}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/f*(3*I*cos(f*x+e)^2*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e)
)^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/
sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))+3*I*cos(f*x+e)^2*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*
x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*(-I
*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/s
in(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*I*cos(f*x+e)^2*sin(f*x+e)*((I*cos
(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f
*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-
I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*cos(f*x+e)^2*sin(f*x+e)*((I*
cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/si
n(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x
+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*cos(f*x+e)^2*s
in(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-
sin(f*x+e))/sin(f*x+e))^(1/2)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*Elliptic
Pi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*
cos(f*x+e)*2^(1/2)+2*2^(1/2))*(d/cos(f*x+e))^(7/2)*sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)-1)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(154) = 308$.
time = 0.59, size = 850, normalized size = 4.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(6*b*d^3*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 -
(cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sq
rt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x
+ e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))*cos(f*x + e) - 3*b*d^3*sqr
t(-d/b)*cos(f*x + e)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 + 8*(7*cos
(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x +
e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*
(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x +
 e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)) - 16*d^3*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b*f*cos(f*x + e)), 1/32*(6*b*d^3*s
qrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 +
6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/
cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2 + (d*cos(f*x
 + e) + d)*sin(f*x + e) - d))*cos(f*x + e) + 3*b*d^3*sqrt(d/b)*cos(f*x + e)
*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f
*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x +
 e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2
*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x +
e)^2 - 2)*sin(f*x + e) + 8)) + 16*d^3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(d/cos(f*x + e)))/(b*f*cos(f*x + e))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(7/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{7/2}}{\sqrt{b\,\tan\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2),x)
[Out] int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(1/2), x)
```

3.316 $\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=92

$$\frac{d^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}{bf}$$

```
[Out] -d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/
f/(b*tan(f*x+e))^(1/2)+d^2*(d*sec(f*x+e))^(1/2)*(b*tan(f*x+e))^(1/2)/b/f
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2693, 2696, 2721, 2720}

$$\frac{d^2\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}{bf}+\frac{d^2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d^2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x
]])/(f*Sqrt[b*Tan[e + f*x]]) + (d^2*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x
]])/(b*f)
```

Rule 2693

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(m + n - 1))), x] + Dist[a^2*((m - 2)/(m + n - 1)), Int[(a*Sec[e +
f*x])^(m - 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (G
tQ[m, 1] || (EqQ[m, 1] && EqQ[n, 1/2])) && NeQ[m + n - 1, 0] && IntegersQ[2
*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{5/2}}{\sqrt{b\tan(e+fx)}}\,dx &= \frac{d^2\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}}{bf}+\frac{1}{2}d^2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx\\ &= \frac{d^2\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}}{bf}+\frac{\left(d^2\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}\right)\int \frac{}{\sqrt{}}}{2\sqrt{b\tan(e+fx)}}\\ &= \frac{d^2\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}}{bf}+\frac{\left(d^2\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}\right)\int \frac{}{\sqrt{s}}}{2\sqrt{b\tan(e+fx)}}\\ &= \frac{d^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}+\frac{d^2\sqrt{d\sec(e+fx)}\,\sqrt{b}}{bf}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.43, size = 83, normalized size = 0.90

$$\frac{d^2\sqrt{d\sec(e+fx)}\,\left(\cos(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\tan^2(e+fx)\right)\sec^2(e+fx)^{3/4}\sin(e+fx)+\tan(e+fx)\right)}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d^2*Sqrt[d*Sec[e + f*x]]*(Cos[e + f*x]*Hypergeometric2F1[1/4, 3/4, 5/4, -T
an[e + f*x]^2]*(Sec[e + f*x]^2)^(3/4)*Sin[e + f*x] + Tan[e + f*x]))/(f*Sqrt
[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 208, normalized size = 2.26

method	result

default	$-\frac{\left(i\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(}{\sin(fx+e)}}\right.\right.}{2f(\cos(fx+e)-1)\sqrt{\frac{b\sin(f}{\cos(f}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))
/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
F(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*co
s(f*x+e)-cos(f*x+e)*2^(1/2)+2^(1/2))*(d/cos(f*x+e))^(5/2)*sin(f*x+e)*cos(f*
x+e)/(cos(f*x+e)-1)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 107, normalized size = 1.16

$$\frac{\sqrt{-2i\,bd}\,d^2\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{2i\,bd}\,d^2\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)-i\sin(fx+e))+2\,d^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}}{2\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-2*I*b*d)*d^2*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
+ e)) + sqrt(2*I*b*d)*d^2*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*
x + e)) + 2*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b*
f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2),x)
[Out] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(1/2), x)
```

3.317 $\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=131

$$\frac{d\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{\sqrt{b}\,f\sqrt{b\tan(e+fx)}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+ \ldots}}{\sqrt{b}\,f\sqrt{b\tan(e+fx)}}$$

```
[Out] d*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^
(1/2)/f/b^(1/2)/(b*tan(f*x+e))^(1/2)+d*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2)
)*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/f/b^(1/2)/(b*tan(f*x+e))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2696, 2644, 335, 218, 212, 209}

$$\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}\,f\sqrt{b\tan(e+fx)}}+\frac{d\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}\,f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (d*ArcTan[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e +
 f*x]])/(Sqrt[b]*f*Sqrt[b*Tan[e + f*x]]) + (d*ArcTanh[Sqrt[b*Sin[e + f*x]]/
Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(Sqrt[b]*f*Sqrt[b*Tan[e
 + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx &= \frac{\left(d\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b\sin(e+fx)}}\,dx}{\sqrt{b\tan(e+fx)}}\\ &= \frac{\left(d\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\sin(e+fx)\right)}{bf\sqrt{b\tan(e+fx)}}\\ &= \frac{\left(2d\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{bf\sqrt{b\tan(e+fx)}}\\ &= \frac{\left(d\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+fx)}\right)}{f\sqrt{b\tan(e+fx)}} + \frac{(d\cdots}{\cdots}\\ &= \frac{d\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}}{\sqrt{b}\,f\sqrt{b\tan(e+fx)}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b\sin\cdots}}{\cdots}\right)}{\cdots}\end{aligned}$$

Mathematica [A]
time = 5.01, size = 105, normalized size = 0.80

$$-\frac{\left(\operatorname{ArcTan}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)-\tanh^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\right)(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}{bf\sec^{\frac{3}{2}}(e+fx)\sqrt[4]{\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] -(((ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)] - ArcTanh[Sqrt[Sec[e
+ f*x]]/(Tan[e + f*x]^2)^(1/4)])*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]
])/(b*f*Sec[e + f*x]^(3/2)*(Tan[e + f*x]^2)^(1/4)))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 344, normalized size = 2.63

method	result
default	$\sqrt{2}\left(2i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{Ellipti}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f*2^(1/2)*(2*I*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2*2^(1/2))-I*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1
/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))
^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f
*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))
/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*cos(f*x+e)*(-I*(cos(f*x+e)-1)/si
n(f*x+e))^(1/2)*(d/cos(f*x+e))^(3/2)*sin(f*x+e)^2*((I*cos(f*x+e)-I+sin(f*x+
e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)/(cos(
f*x+e)-1)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(115) = 230$.
time = 0.57, size = 709, normalized size = 5.41

$$\left[-\frac{2\,d\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{d}{b}}\sqrt{\frac{d}{\cos(fx+e)}}}{4\left(d\cos(fx+e)^2-\left(d\cos(fx+e)+d\right)\sin(fx+e)-d\right)}\right)-d\sqrt{-\frac{d}{b}}\log\left(\frac{d\cos(fx+e)^4-72\,d\cos(fx+e)^2+8\left(7\cos(fx+e)^3-\left(\cos(fx+e)^3-8\cos(fx+e)\right)\sin(fx+e)-8\cos(fx+e)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{d}{b}}\sqrt{\frac{d}{\cos(fx+e)}}+28\left(d\cos(fx+e)^2-2\,d\right)\sin(fx+e)+72\,d}{\cos(fx+e)^4-8\cos(fx+e)^2-4\left(\cos(fx+e)^2-2\right)\sin(fx+e)+8}\right)}{8\,f},\ \frac{2\,d\sqrt{\frac{d}{b}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2+\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{b}}\sqrt{\frac{d}{\cos(fx+e)}}}{4\left(d\cos(fx+e)^2+\left(d\cos(fx+e)+d\right)\sin(fx+e)-d\right)}\right)+d\sqrt{\frac{d}{b}}\log\left(\frac{d\cos(fx+e)^4-72\,d\cos(fx+e)^2-8\left(7\cos(fx+e)^3+\left(\cos(fx+e)^3-8\cos(fx+e)\right)\sin(fx+e)-8\cos(fx+e)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{b}}\sqrt{\frac{d}{\cos(fx+e)}}-28\left(d\cos(fx+e)^2-2\,d\right)\sin(fx+e)+72\,d}{\cos(fx+e)^4-8\cos(fx+e)^2+4\left(\cos(fx+e)^2-2\right)\sin(fx+e)+8}\right)}{8\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(2*d*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(
f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*
sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^
2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d)) - d*sqrt(-d/b)*log((d*cos(f*x +
 e)^4 - 72*d*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos
(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))
*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e)
 + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*
x + e) + 8)))/f, 1/8*(2*d*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x
+ e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e
) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*
cos(f*x + e)^2 + (d*cos(f*x + e) + d)*sin(f*x + e) - d)) + d*sqrt(d/b)*log(
(d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x +
e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/c
os(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2*d)*s
in(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2
- 2)*sin(f*x + e) + 8)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sec(e+fx))^{\frac{3}{2}}}{\sqrt{b\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)/sqrt(b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/sqrt(b*tan(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{b\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2),x)
```

```
[Out] int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(1/2), x)
```

3.318 $\int \frac{\sqrt{d \sec(e+fx)}}{\sqrt{b \tan(e+fx)}} \, dx$

Optimal. Leaf size=55

$$\frac{2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f/
(b*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2696, 2721, 2720}

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]
)/(f*Sqrt[b*Tan[e + f*x]])
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx = \frac{\left(\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{\sqrt{b\tan(e+fx)}}$$
$$= \frac{\left(\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{\sqrt{b\tan(e+fx)}}$$
$$= \frac{2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}}{f\sqrt{b\tan(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.53, size = 89, normalized size = 1.62

$$\frac{2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}}{bf\sqrt{\sec(e+fx)}\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/Sqrt[b*Tan[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[(e + f*x)/2]^2]*Sqrt[d*Sec[e + f*x
]]*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[Sec[e + f*x]]*Sqrt[Cos[(e + f*x)/2]^2*Se
c[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.62, size = 175, normalized size = 3.18

method	result
default	$-\frac{i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{\frac{i\cos(fx+e)-}{\sin(f}}}{f\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\,(\cos(fx+e)-1)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I/f*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*
(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+
e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(d/cos(f*x+e))^(1/
2)*sin(f*x+e)^2*2^(1/2)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)/(cos(f*x+e)-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 62, normalized size = 1.13

$$\frac{\sqrt{-2i\,bd}\ \text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+\sqrt{2i\,bd}\ \text{weierstrassPInverse}(4,0,\cos(fx+e)-i\,\sin(fx+e))}{bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] (sqrt(-2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
sqrt(2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b*
f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*sec(e + f*x))/sqrt(b*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))/sqrt(b*tan(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{\sqrt{b\tan(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2),x)
[Out] int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(1/2), x)
```

3.319 $\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=32

$$\frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

```
[Out] 2*(b*tan(f*x+e))^(1/2)/b/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2685}

$$\frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (2*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

Mathematica [A]
time = 0.44, size = 32, normalized size = 1.00

$$\frac{2\sqrt{b\tan(e+fx)}}{bf\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (2*Sqrt[b*Tan[e + f*x]])/(b*f*Sqrt[d*Sec[e + f*x]])
```

Maple [A]
time = 0.34, size = 50, normalized size = 1.56

method	result	size
default	$\frac{2 \sin(fx+e)}{f \sqrt{\frac{d}{\cos(fx+e)}}\, \sqrt{\frac{b \sin(fx+e)}{\cos(fx+e)}}\, \cos(fx+e)}$	50
risch	$-\frac{i\sqrt{2}\, (\mathrm{e}^{2i(fx+e)}-1)}{\sqrt{\frac{d\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\, (\mathrm{e}^{2i(fx+e)}+1) \sqrt{-\frac{ib(\mathrm{e}^{2i(fx+e)}-1)}{\mathrm{e}^{2i(fx+e)}+1}}\, f}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*sin(f*x+e)/(d/cos(f*x+e))^(1/2)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)/cos(f*x
+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

Fricas [A]
time = 0.39, size = 51, normalized size = 1.59

$$\frac{2 \sqrt{\frac{b \sin(fx+e)}{\cos(fx+e)}} \sqrt{\frac{d}{\cos(fx+e)}} \cos(fx+e)}{bdf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(b*d*
f)
```

Sympy [A]
time = 10.99, size = 51, normalized size = 1.59

$$\begin{cases} \frac{2\tan(e+fx)}{f\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}} & \text{for } f\neq 0 \\ \frac{x}{\sqrt{b\tan(e)}\sqrt{d\sec(e)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Piecewise((2*tan(e + f*x)/(f*sqrt(b*tan(e + f*x))*sqrt(d*sec(e + f*x))), Ne
(f, 0)), (x/(sqrt(b*tan(e))*sqrt(d*sec(e))), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*sqrt(b*tan(f*x + e))), x)
```

Mupad [B]
time = 2.90, size = 52, normalized size = 1.62

$$\frac{2\,\sin\left(e+f\,x\right)\,\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}{d\,f\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(1/2)),x)
```

```
[Out] (2*sin(e + f*x)*(d/cos(e + f*x))^(1/2))/(d*f*((b*sin(2*e + 2*f*x))/(cos(2*e
 + 2*f*x) + 1))^(1/2))
```

3.320 $\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{4F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

```
[Out] -4/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/
d^2/f/(b*tan(f*x+e))^(1/2)+2/3*(b*tan(f*x+e))^(1/2)/b/f/(d*sec(f*x+e))^(3/2
)
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2692, 2696, 2721, 2720}

$$\frac{4\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (4*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]
)/(3*d^2*f*Sqrt[b*Tan[e + f*x]]) + (2*Sqrt[b*Tan[e + f*x]])/(3*b*f*(d*Sec[e
 + f*x])^(3/2))
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}\,dx = \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}} + \frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3d^2}$$

$$= \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}} + \frac{\left(2\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}\right)\int \cdots}{3d^2\sqrt{b\tan(e+fx)}}$$

$$= \frac{2\sqrt{b\tan(e+fx)}}{3bf(d\sec(e+fx))^{3/2}} + \frac{\left(2\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}\right)\int \cdots}{3d^2\sqrt{b\tan(e+fx)}}$$

$$= \frac{4F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}}{3d^2f\sqrt{b\tan(e+fx)}} + \frac{2\sqrt{b\cdots}}{3bf(d\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.12, size = 91, normalized size = 0.96

$$\frac{2\sqrt{b\tan(e+fx)}\left(-2\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\sec^2(e+fx)\right)\sec^2(e+fx)+\sqrt[4]{-\tan^2(e+fx)}\right)}{3bf(d\sec(e+fx))^{3/2}\sqrt[4]{-\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (2*Sqrt[b*Tan[e + f*x]]*(-2*Hypergeometric2F1[1/4, 3/4, 5/4, Sec[e + f*x]^2
]*Sec[e + f*x]^2 + (-Tan[e + f*x]^2)^(1/4)))/(3*b*f*(d*Sec[e + f*x])^(3/2)*
(-Tan[e + f*x]^2)^(1/4))
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 213, normalized size = 2.24

method	result
default	$-\frac{\left(2i\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin}{\sin(fx+e)}}\right.\right.}{3f(\cos(fx+e)-1)\left(\frac{d}{\cos(fx+e)}\right)^{\frac{3}{2}}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\,\mathrm{co}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(2*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e
))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)-
cos(f*x+e)^2*2^(1/2)+cos(f*x+e)*2^(1/2))*sin(f*x+e)/(cos(f*x+e)-1)/(d/cos(f
*x+e))^(3/2)/(b*sin(f*x+e)/cos(f*x+e))^(1/2)/cos(f*x+e)^2*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 109, normalized size = 1.15

$$\frac{2\left(\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2+\sqrt{-2i\,bd}\,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))+\sqrt{2i\,bd}\,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)-i\sin(fx+e))\right)}{3\,bd^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
+ sqrt(-2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
 sqrt(2*I*b*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b
*d^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tan(e+fx)}\,(d\sec(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*sqrt(b*tan(f*x + e))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\tan\left(e+f\,x\right)}\,{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(3/2)), x)
```

3.321 $\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}+\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}$$

```
[Out] 2/5*(b*tan(f*x+e))^(1/2)/b/f/(d*sec(f*x+e))^(5/2)+8/5*(b*tan(f*x+e))^(1/2)/
b/d^2/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2692, 2685}

$$\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}+\frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
```

```
[Out] (2*Sqrt[b*Tan[e + f*x]])/(5*b*f*(d*Sec[e + f*x])^(5/2)) + (8*Sqrt[b*Tan[e +
 f*x]])/(5*b*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}\,dx &= \frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}+\frac{4\int \frac{1}{\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}\,dx}{5d^2}\\ &= \frac{2\sqrt{b\tan(e+fx)}}{5bf(d\sec(e+fx))^{5/2}}+\frac{8\sqrt{b\tan(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 1.27, size = 112, normalized size = 1.56

$$\frac{9\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)+\sqrt{\frac{1}{1+\cos(e+fx)}}\cos(2(e+fx))\tan(e+fx)}{5d^2f\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]),x]
[Out] (9*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2] + Sqrt[(1 + C
os[e + f*x])^(-1)]*Cos[2*(e + f*x)]*Tan[e + f*x])/(5*d^2*f*Sqrt[(1 + Cos[e
+ f*x])^(-1)]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [A]
time = 0.34, size = 60, normalized size = 0.83

method	result	size
default	$\frac{2\sin(fx+e)\left(\cos^2(fx+e)+4\right)}{5f\cos(fx+e)^3\left(\frac{d}{\cos(fx+e)}\right)^{\frac{5}{2}}\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5/f*sin(f*x+e)*(cos(f*x+e)^2+4)/cos(f*x+e)^3/(d/cos(f*x+e))^(5/2)/(b*sin(
f*x+e)/cos(f*x+e))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

Fricas [A]
time = 0.37, size = 63, normalized size = 0.88

$$\frac{2\left(\cos\left(fx+e\right)^3+4\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{5\,bd^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/5*(cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(d/cos(f*x + e))/(b*d^3*f)
```

Sympy [A]
time = 72.09, size = 88, normalized size = 1.22

$$\begin{cases} \frac{8\tan^3(e+fx)}{5f\sqrt{b\tan(e+fx)}\,(d\sec(e+fx))^{\frac{5}{2}}} + \frac{2\tan(e+fx)}{f\sqrt{b\tan(e+fx)}\,(d\sec(e+fx))^{\frac{5}{2}}} & \text{for } f \neq 0 \\ \frac{x}{\sqrt{b\tan(e)}\,(d\sec(e))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(1/2),x)
```

```
[Out] Piecewise((8*tan(e + f*x)**3/(5*f*sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(5
/2)) + 2*tan(e + f*x)/(f*sqrt(b*tan(e + f*x))*(d*sec(e + f*x))**(5/2)), Ne(
f, 0)), (x/(sqrt(b*tan(e))*(d*sec(e))**(5/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*sqrt(b*tan(f*x + e))), x)
```

Mupad [B]
time = 3.02, size = 64, normalized size = 0.89

$$\frac{(17\sin(e+f\,x)+\sin(3\,e+3\,f\,x))\,\sqrt{\frac{d}{\cos(e+f\,x)}}}{10\,d^3\,f\,\sqrt{\frac{b\sin(2\,e+2\,f\,x)}{\cos(2\,e+2\,f\,x)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*tan(e + f*x))^(1/2)*(d/cos(e + f*x))^(5/2)),x)
```

```
[Out] ((17*sin(e + f*x) + sin(3*e + 3*f*x))*(d/cos(e + f*x))^(1/2))/(10*d^3*f*((b
*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.322 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}-\frac{d^3\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

```
[Out] -2*d^2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)-d^3*arctan((b*sin(f*x+
e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(1/2)/b^(3/2)/f/(d*sec(f*x+e))^(1/2)/(b*s
in(f*x+e))^(1/2)+d^3*arctanh((b*sin(f*x+e))^(1/2)/b^(1/2))*(b*tan(f*x+e))^(
1/2)/b^(3/2)/f/(d*sec(f*x+e))^(1/2)/(b*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2688, 2696, 2644, 335, 304, 209, 212}

$$-\frac{d^3\sqrt{b\tan(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{d^3\sqrt{b\tan(e+fx)}\,\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*d^2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]]) - (d^3*ArcTan[Sqrt
[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[b*Tan[e + f*x]])/(b^(3/2)*f*Sqrt[d*Sec[e + f
*x]]*Sqrt[b*Sin[e + f*x]]) + (d^3*ArcTanh[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqr
t[b*Tan[e + f*x]])/(b^(3/2)*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{d^2\int \sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}\,dx}{b^2}$$

$$= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\int \sec(e+fx)\sqrt{b\sin(e+fx)}\,dx}{b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{b^2}}\,dx, x, b\sin(e+fx)\right)}{b^3f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(2d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sin(e+}\right.}{b^3f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} + \frac{\left(d^3\sqrt{b\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sin(e+f}\right.}{bf\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}$$

$$= -\frac{2d^2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}} - \frac{d^3\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tan(e+fx)}}{b^{3/2}f\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} + \frac{d^3\tanh}{b^3}$$

Mathematica [A]
time = 1.21, size = 211, normalized size = 1.23

$$-\frac{d^3\sin(e+fx)\left(-4\csc^2(e+fx)+16\csc^2(2(e+fx))-2\text{ArcTan}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\sec^{\frac{3}{2}}(e+fx)\sqrt[4]{\tan^2(e+fx)}+\log\left(1-\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\sec^{\frac{3}{2}}(e+fx)\sqrt[4]{\tan^2(e+fx)}-\log\left(1+\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\sec^{\frac{3}{2}}(e+fx)\sqrt[4]{\tan^2(e+fx)}\right)}{2f\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(d^3*Sin[e + f*x]*(-4*Csc[e + f*x]^2 + 16*Csc[2*(e + f*x)]^2 - 2*ArcTa
n[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)]*Sec[e + f*x]^(3/2)*(Tan[e + f*
x]^2)^(1/4) + Log[1 - Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)^(1/4)]*Sec[e + f*
x]^(3/2)*(Tan[e + f*x]^2)^(1/4) - Log[1 + Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^
2)^(1/4)]*Sec[e + f*x]^(3/2)*(Tan[e + f*x]^2)^(1/4)))/(f*Sqrt[d*Sec[e + f*x
]]*(b*Tan[e + f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 1061, normalized size = 6.20

method	result	size
default	Expression too large to display	1061

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(I*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-
sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f
*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*co
s(f*x+e)-I*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I
-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(
f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*c
os(f*x+e)+I*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-
I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin
(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)-
((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/
2),1/2-1/2*I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*cos(f*x+e)-I
*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e
))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)-((I*cos(f*x
+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e
))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*cos(f*x+e)-((I*cos(f*x+
e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I
,1/2*2^(1/2))*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)-((I*cos(f*x+e)-I+sin(f*x
+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)+2*2^(1/2))*(d/cos(f*x+e))^(5/2)*sin(
f*x+e)*cos(f*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(153) = 306.
time = 0.53, size = 864, normalized size = 5.05

$$\left[-\frac{2\,bd^2\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2-\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{d}{b}}\sqrt{\frac{d}{\cos(fx+e)}}}{4\left(d\cos(fx+e)^2-\left(d\cos(fx+e)+d\right)\sin(fx+e)-d\right)}\right)\sin(fx+e)-bd^2\sqrt{-\frac{d}{b}}\log\left(\frac{d\cos(fx+e)^4-72\,d\cos(fx+e)^2-8\left(7\cos(fx+e)^3-\left(\cos(fx+e)^3-8\cos(fx+e)\right)\sin(fx+e)-8\cos(fx+e)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{d}{b}}\sqrt{\frac{d}{\cos(fx+e)}}+28\left(d\cos(fx+e)^2-2\,d\right)\sin(fx+e)+72\,d}{\cos(fx+e)^4-8\cos(fx+e)^2-4\left(\cos(fx+e)^2-2\right)\sin(fx+e)+8}\right)\sin(fx+e)+16\,d^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)}{8\,b^2f\sin(fx+e)},\ -\frac{2\,bd^2\sqrt{\frac{d}{b}}\arctan\left(\frac{\left(\cos(fx+e)^3-5\cos(fx+e)^2+\left(\cos(fx+e)^2+6\cos(fx+e)+4\right)\sin(fx+e)-2\cos(fx+e)+4\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{b}}\sqrt{\frac{d}{\cos(fx+e)}}}{4\left(d\cos(fx+e)^2+\left(d\cos(fx+e)+d\right)\sin(fx+e)-d\right)}\right)\sin(fx+e)-bd^2\sqrt{\frac{d}{b}}\log\left(\frac{d\cos(fx+e)^4-72\,d\cos(fx+e)^2-8\left(7\cos(fx+e)^3+\left(\cos(fx+e)^3-8\cos(fx+e)\right)\sin(fx+e)-8\cos(fx+e)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{b}}\sqrt{\frac{d}{\cos(fx+e)}}-28\left(d\cos(fx+e)^2-2\,d\right)\sin(fx+e)+72\,d}{\cos(fx+e)^4-8\cos(fx+e)^2+4\left(\cos(fx+e)^2-2\right)\sin(fx+e)+8}\right)\sin(fx+e)+16\,d^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)}{8\,b^2f\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [-1/8*(2*b*d^2*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d))*sin(f*x + e) - b*d^2*sqrt(-d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8))*sin(f*x + e) + 16*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e)), -1/8*(2*b*d^2*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2 + (d*cos(f*x + e) + d)*sin(f*x + e) - d))*sin(f*x + e) - b*d^2*sqrt(d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8))*sin(f*x + e) + 16*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e))/(b^2*f*sin(f*x + e))]

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{\left(b\tan\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2),x)
[Out] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(3/2), x)
```

3.323 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2*d^2/b/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+2*d^2*(sin(1/2*e+1/4*P
i+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/
2*f*x),2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^
(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2688, 2696, 2721, 2719}

$$-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2d^2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*d^2)/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (2*d^2*EllipticE
[(e - Pi/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]*S
qrt[Sin[e + f*x]])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{3/2}}\,dx &= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{d^2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{b^2} \\
&= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{b\sin(e+fx)}}{b^2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}} \\
&= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{\left(d^2\sqrt{b\tan(e+fx)}\right)\int \sqrt{\sin(e+fx)}}{b^2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}} \\
&= -\frac{2d^2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}} - \frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+f}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.70, size = 70, normalized size = 0.72

$$\frac{2d^2\left(-1+{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*d^2*(-1 + Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]*(-Tan[e + f*
x]^2)^(1/4)))/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 535, normalized size = 5.52

method	result

default	$-\frac{\left(-2\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\,\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(-2*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/
sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE
(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+(-I
*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^
(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x
+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)-2*(-I*(cos(f*x+
e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I
*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(
f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)
*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e
))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/
2),1/2*2^(1/2))+2^(1/2))*(d/cos(f*x+e))^(3/2)*sin(f*x+e)/(b*sin(f*x+e)/cos(
f*x+e))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 141, normalized size = 1.45

$$-\frac{2\,d\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+i\sqrt{-2i\,bd}\,d\sin\left(fx+e\right)\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\right)-i\sqrt{2i\,bd}\,d\sin\left(fx+e\right)\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)\right)}{b^{2}f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
 + I*sqrt(-2*I*b*d)*d*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*d*sin(f*x + e)*we
ierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)
))/(b^2*f*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}{\left(b\tan\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)/(b*tan(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{{\left(b\,\tan\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2),x)
```

```
[Out] int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(3/2), x)
```

3.324 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

```
[Out] -2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2685}

$$-\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

Mathematica [A]
time = 0.12, size = 32, normalized size = 1.00

$$-\frac{2\sqrt{d\sec(e+fx)}}{bf\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]])/(b*f*Sqrt[b*Tan[e + f*x]])
```

Maple [A]
time = 0.33, size = 50, normalized size = 1.56

method	result	size
default	$-\frac{2\sin(fx+e)\sqrt{\frac{d}{\cos(fx+e)}}}{f\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}\cos(fx+e)}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/f*sin(f*x+e)*(d/cos(f*x+e))^(1/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/cos(f*
x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

Fricas [A]
time = 0.35, size = 57, normalized size = 1.78

$$-\frac{2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)}{b^2 f\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(b^2
*f*sin(f*x + e))
```

Sympy [A]
time = 2.19, size = 53, normalized size = 1.66

$$\begin{cases} -\frac{2\sqrt{d\sec(e+fx)}\tan(e+fx)}{f(b\tan(e+fx))^{\frac{3}{2}}} & \text{for } f \neq 0 \\ \frac{x\sqrt{d\sec(e)}}{(b\tan(e))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(d*sec(e + f*x))*tan(e + f*x)/(f*(b*tan(e + f*x))**(3/2))
, Ne(f, 0)), (x*sqrt(d*sec(e))/(b*tan(e))**(3/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(3/2), x)
```

Mupad [B]
time = 2.95, size = 46, normalized size = 1.44

$$-\frac{2\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}{b\,f\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(3/2),x)
```

```
[Out] -(2*(d/cos(e + f*x))^(1/2))/(b*f*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) +
1))^(1/2))
```

3.325 $\int \frac{1}{\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2}{bf\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}-\frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}$$

```
[Out] -2/b/f/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(1/2)+4*(sin(1/2*e+1/4*Pi+1/2*f*
x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2
^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2689, 2696, 2721, 2719}

$$-\frac{4E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{b^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -2/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]]) - (4*EllipticE[(e - Pi/2
 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(b^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e
+ f*x]])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}\,dx = -\frac{2}{bf\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}} - \frac{2\int \frac{\sqrt{b\tan(e+fx)}}{\sqrt{d\sec(e+fx)}}\,dx}{b^2}$$

$$= -\frac{2}{bf\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}} - \frac{\left(2\sqrt{b\tan(e+fx)}\right)\int}{b^2\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2}{bf\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}} - \frac{\left(2\sqrt{b\tan(e+fx)}\right)\int}{b^2\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2}{bf\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}} - \frac{4E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{b^2 f\sqrt{d\sec(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.64, size = 67, normalized size = 0.74

$$\frac{-2+4\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{bf\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)),x]
[Out] (-2 + 4*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)
^(1/4))/(b*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.38, size = 556, normalized size = 6.11

method	result
default	$\left(4\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(4*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/si
n(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((
(I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)-2*(-I
*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^
(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x
+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+4*(-I*(cos(f*x+
e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I
*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)-I+sin(
f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/
2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x
+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(
1/2),1/2*2^(1/2))+cos(f*x+e)*2^(1/2)-2*2^(1/2))*sin(f*x+e)/cos(f*x+e)^2/(d/
cos(f*x+e))^(1/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 140, normalized size = 1.54

$$-\frac{2\left(\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+i\sqrt{-2i\,bd}\,\sin\left(fx+e\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))-i\sqrt{2i\,bd}\,\sin\left(fx+e\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))\right)}{b^{2}df\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -2*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2 +
 I*sqrt(-2*I*b*d)*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2*I*b*d)*sin(f*x + e)*weierstr
assZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b
^2*d*f*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\tan(e+fx))^{\frac{3}{2}}\sqrt{d\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral(1/((b*tan(e + f*x))**(3/2)*sqrt(d*sec(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\tan(e+f\,x))^{3/2}\,\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(1/2)), x)
```

3.326 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2}{3bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}-\frac{8\sqrt{d\sec(e+fx)}}{3bd^2f\sqrt{b\tan(e+fx)}}$$

```
[Out] 2/3/b/f/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(1/2)-8/3*(d*sec(f*x+e))^(1/2)/
b/d^2/f/(b*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 67, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2689, 2685}

$$-\frac{8(b\tan(e+fx))^{3/2}}{3b^3f(d\sec(e+fx))^{3/2}}-\frac{2}{bf\sqrt{b\tan(e+fx)}\,(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -2/(b*f*(d*Sec[e + f*x])^(3/2)*Sqrt[b*Tan[e + f*x]]) - (8*(b*Tan[e + f*x])^
(3/2))/(3*b^3*f*(d*Sec[e + f*x])^(3/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2}{bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}-\frac{4\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{3/2}}}{b^2}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}-\frac{8(b\tan(e+fx))^{3/}}{3b^3f(d\sec(e+fx))}$$

Mathematica [A]
time = 0.21, size = 52, normalized size = 0.72

$$\frac{(-7+\cos(2(e+fx)))\sec^2(e+fx)}{3bf(d\sec(e+fx))^{3/2}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] ((-7 + Cos[2*(e + f*x)])*Sec[e + f*x]^2)/(3*b*f*(d*Sec[e + f*x])^(3/2)*Sqrt
[b*Tan[e + f*x]])
```

Maple [A]
time = 0.31, size = 60, normalized size = 0.83

method	result	size
default	$\frac{2\sin(fx+e)(\cos^2(fx+e)-4)}{3f\cos(fx+e)^3\left(\frac{d}{\cos(fx+e)}\right)^{\frac{3}{2}}\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{3}{2}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/f*sin(f*x+e)*(cos(f*x+e)^2-4)/cos(f*x+e)^3/(d/cos(f*x+e))^(3/2)/(b*sin(
f*x+e)/cos(f*x+e))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [A]
time = 0.39, size = 72, normalized size = 1.00

$$\frac{2\left(\cos\left(fx+e\right)^3-4\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{3\,b^2d^2f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] 2/3*(cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqr
t(d/cos(f*x + e))/(b^2*d^2*f*sin(f*x + e))
```

Sympy [A]
time = 37.90, size = 90, normalized size = 1.25

$$\begin{cases} -\frac{8\tan^3(e+fx)}{3f(b\tan(e+fx))^{\frac{3}{2}}(d\sec(e+fx))^{\frac{3}{2}}} - \frac{2\tan(e+fx)}{f(b\tan(e+fx))^{\frac{3}{2}}(d\sec(e+fx))^{\frac{3}{2}}} & \text{for } f \neq 0 \\ \frac{x}{(b\tan(e))^{\frac{3}{2}}(d\sec(e))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Piecewise((-8*tan(e + f*x)**3/(3*f*(b*tan(e + f*x))**(3/2)*(d*sec(e + f*x))
**(3/2)) - 2*tan(e + f*x)/(f*(b*tan(e + f*x))**(3/2)*(d*sec(e + f*x))**(3/2
)), Ne(f, 0)), (x/((b*tan(e))**(3/2)*(d*sec(e))**(3/2)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(3/2)), x)
```

Mupad [B]
time = 3.20, size = 60, normalized size = 0.83

$$\frac{\left(\cos\left(2\,e+2\,f\,x\right)-7\right)\,\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}{3\,b\,d^2\,f\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(3/2)),x)
```

```
[Out] ((cos(2*e + 2*f*x) - 7)*(d/cos(e + f*x))^(1/2))/(3*b*d^2*f*((b*sin(2*e + 2*
f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.327 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}}-\frac{24E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5b^2d^2f\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}-\frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}}$$

```
[Out] -2/b/f/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(1/2)+24/5*(sin(1/2*e+1/4*Pi+1/2
*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x
),2^(1/2))*(b*tan(f*x+e))^(1/2)/b^2/d^2/f/(d*sec(f*x+e))^(1/2)/sin(f*x+e)^(
1/2)-12/5*(b*tan(f*x+e))^(3/2)/b^3/f/(d*sec(f*x+e))^(5/2)
```

Rubi [A]
time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2689, 2692, 2696, 2721, 2719}

$$-\frac{12(b\tan(e+fx))^{3/2}}{5b^3f(d\sec(e+fx))^{5/2}}-\frac{24E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\tan(e+fx)}}{5b^2d^2f\sqrt{\sin(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2}{bf\sqrt{b\tan(e+fx)}(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] -2/(b*f*(d*Sec[e + f*x])^(5/2)*Sqrt[b*Tan[e + f*x]]) - (24*EllipticE[(e - P
i/2 + f*x)/2, 2]*Sqrt[b*Tan[e + f*x]])/(5*b^2*d^2*f*Sqrt[d*Sec[e + f*x]]*Sq
rt[Sin[e + f*x]]) - (12*(b*Tan[e + f*x])^(3/2))/(5*b^3*f*(d*Sec[e + f*x])^(
5/2))
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}\,dx = -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{6\int \frac{\sqrt{b\tan(e+fx)}}{(d\sec(e+fx))^{5/2}}}{b^2}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{12(b\tan(e+fx))^{3/}}{5b^3f(d\sec(e+fx))}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{12(b\tan(e+fx))^{3/}}{5b^3f(d\sec(e+fx))}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{12(b\tan(e+fx))^{3/}}{5b^3f(d\sec(e+fx))}$$

$$= -\frac{2}{bf(d\sec(e+fx))^{5/2}\sqrt{b\tan(e+fx)}} - \frac{24E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\right.}{5b^2d^2f\sqrt{d\sec(e+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.78, size = 81, normalized size = 0.62

$$\frac{-11+\cos(2(e+fx))+24\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{3}{4};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{5bd^2f\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)),x]
```

```
[Out] (-11 + Cos[2*(e + f*x)] + 24*Hypergeometric2F1[-1/4, 1/4, 3/4, Sec[e + f*x]
^2]*(-Tan[e + f*x]^2)^(1/4))/(5*b*d^2*f*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e +
 f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.36, size = 570, normalized size = 4.38

method	result
default	$\frac{\left(24\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\,\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(fx}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/f*(24*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))
/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
E(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)-12
*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+
e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos
(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)+2^(1/2)*cos
(f*x+e)^3+24*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+
e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticE(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-12*(-I*(co
s(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2
)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-
I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+6*cos(f*x+e)*2^(1/2)-12*2^(1/2
))*sin(f*x+e)/(d/cos(f*x+e))^(5/2)/(b*sin(f*x+e)/cos(f*x+e))^(3/2)/cos(f*x+
e)^4*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 153, normalized size = 1.18

$$-\frac{2\left(6i\sqrt{-2i\,bd}\,\sin(fx+e)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e)))-6i\sqrt{2i\,bd}\,\sin(fx+e)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)-i\,\sin(fx+e)))-\left(\cos(fx+e)^4-6\,\cos(fx+e)^2\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\right)}{5\,b^2d^3f\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] -2/5*(6*I*sqrt(-2*I*b*d)*sin(f*x + e)*weierstrassZeta(4, 0, weierstrassPInv
erse(4, 0, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2*I*b*d)*sin(f*x + e)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
 e))) - (cos(f*x + e)^4 - 6*cos(f*x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e
))*sqrt(d/cos(f*x + e)))/(b^2*d^3*f*sin(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4372 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\tan\left(e+f\,x\right))^{3/2}\,{\left(\frac{d}{\cos(e+f\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/((b*tan(e + f*x))^(3/2)*(d/cos(e + f*x))^(5/2)), x)
```

3.328 $\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}+\frac{d^3\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}}{b^{5/2}f\sqrt{b\tan(e+fx)}}+d^3\tanh^{-1}\Big(\sqrt{b\,\mathrm{s}}$$

```
[Out] d^3*arctan((b*sin(f*x+e))^(1/2)/b^(1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e)
)^(1/2)/b^(5/2)/f/(b*tan(f*x+e))^(1/2)+d^3*arctanh((b*sin(f*x+e))^(1/2)/b^(
1/2))*(d*sec(f*x+e))^(1/2)*(b*sin(f*x+e))^(1/2)/b^(5/2)/f/(b*tan(f*x+e))^(1
/2)-2/3*d^2*(d*sec(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2688, 2696, 2644, 335, 218, 212, 209}

$$\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\,\text{ArcTan}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f\sqrt{b\tan(e+fx)}}+\frac{d^3\sqrt{b\sin(e+fx)}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f\sqrt{b\tan(e+fx)}}-\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(7/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*d^2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (d^3*ArcTa
n[Sqrt[b*Sin[e + f*x]]/Sqrt[b]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/
(b^(5/2)*f*Sqrt[b*Tan[e + f*x]]) + (d^3*ArcTanh[Sqrt[b*Sin[e + f*x]]/Sqrt[b
]]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Sin[e + f*x]])/(b^(5/2)*f*Sqrt[b*Tan[e + f*x
]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{7/2}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{d^2\int \frac{(d\sec(e+fx))^{3/2}}{\sqrt{b\tan(e+fx)}}\,dx}{b^2}$$

$$= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{\left(d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b\sin(e+f}}}{b^2\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{\left(d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1}\right.}{b^3f\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{\left(2d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{b^2}}\right.}{b^3f\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{\left(d^3\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{b-x^2}\,dx\right.}{b^2f\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}} + \frac{d^3\tan^{-1}\left(\frac{\sqrt{b\sin(e+fx)}}{\sqrt{b}}\right)\sqrt{d\sec(e+fx)}\sqrt{b\sin(e}}{b^{5/2}f\sqrt{b\tan(e+fx)}}$$

Mathematica [A]
time = 1.29, size = 144, normalized size = 0.84

$$-\frac{d^4\sqrt{b\tan(e+fx)}\left(3\text{ArcTan}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt{\sec(e+fx)} - 3\tanh^{-1}\left(\frac{\sqrt{\sec(e+fx)}}{\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt{\sec(e+fx)} + 2\csc^2(e+fx)\sqrt[4]{\tan^2(e+fx)}\right)}{3b^3f\sqrt{d\sec(e+fx)}\sqrt[4]{\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] -1/3*(d^4*Sqrt[b*Tan[e + f*x]]*(3*ArcTan[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2
)^(1/4)]*Sqrt[Sec[e + f*x]] - 3*ArcTanh[Sqrt[Sec[e + f*x]]/(Tan[e + f*x]^2)
^(1/4)]*Sqrt[Sec[e + f*x]] + 2*Csc[e + f*x]^2*(Tan[e + f*x]^2)^(1/4)))/(b^3
*f*Sqrt[d*Sec[e + f*x]]*(Tan[e + f*x]^2)^(1/4))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 1367, normalized size = 7.95

method	result	size
default	Expression too large to display	1367

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/f*(6*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e
))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*
cos(f*x+e)-3*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*
x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*cos(f*x+e)*sin(f*x+e)-3*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f
*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/
2*I,1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)+6*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(
1/2)*sin(f*x+e)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x
+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/
sin(f*x+e))^(1/2),1/2*2^(1/2))-3*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I
*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/s
in(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*sin(f*x+e)-3*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*
((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/
2),1/2-1/2*I,1/2*2^(1/2))*sin(f*x+e)-3*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)
*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e
))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1
/2),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)+3*(-I*(cos(f*x+e)-1)/sin(f
*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)
-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+sin(f*x+e))/si
n(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(f*x+e)*sin(f*x+e)-3*(-I*(cos(f*x
+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(
I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+si
n(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(f*x+e)+3*(-I*(cos(f*
x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-
(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((I*cos(f*x+e)-I+s
in(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(f*x+e)+2*2^(1/2))*(
d/cos(f*x+e))^(7/2)*sin(f*x+e)*cos(f*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(5/2)*2
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e))^(5/2), x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(152) = 304$.
time = 0.59, size = 920, normalized size = 5.35

$$\left[\frac{16 d^{3} \sqrt{\frac{b \sin(fx+e)}{\cos(fx+e)}} \sqrt{\frac{d}{\cos(fx+e)}} \cos(fx+e) - 6 \left(b d^{3} \cos(fx+e)^{2} - b d^{3}\right) \sqrt{-\frac{d}{b}} \arctan\left(\frac{\left(\cos(fx+e)^{3} - 5\cos(fx+e)^{2} - \left(\cos(fx+e)^{2} + 6\cos(fx+e) + 4\right)\sin(fx+e) - 2\cos(fx+e) + 4\right)\sqrt{\frac{b \sin(fx+e)}{\cos(fx+e)}} \sqrt{-\frac{d}{b}} \sqrt{\frac{d}{\cos(fx+e)}}}{4\left(d\cos(fx+e)^{2} - \left(d\cos(fx+e) + d\right)\sin(fx+e) - d\right)}\right) + 3\left(b d^{3}\cos(fx+e)^{2} - b d^{3}\right)\sqrt{-\frac{d}{b}} \log\left(\frac{d\cos(fx+e)^{4} - 72 d\cos(fx+e)^{2} + 8\left(7\cos(fx+e)^{3} - \left(\cos(fx+e)^{3} - 8\cos(fx+e)\right)\sin(fx+e) - 8\cos(fx+e)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{-\frac{d}{b}}\sqrt{\frac{d}{\cos(fx+e)}} + 28\left(d\cos(fx+e)^{2} - 2d\right)\sin(fx+e) + 72d}{\cos(fx+e)^{4} - 8\cos(fx+e)^{2} - 4\left(\cos(fx+e)^{2} - 2\right)\sin(fx+e) + 8}\right)}{24\left(b^{3} f\cos(fx+e)^{2} - b^{3} f\right)}, \frac{16 d^{3} \sqrt{\frac{b \sin(fx+e)}{\cos(fx+e)}} \sqrt{\frac{d}{\cos(fx+e)}} \cos(fx+e) + 6 \left(b d^{3} \cos(fx+e)^{2} - b d^{3}\right) \sqrt{\frac{d}{b}} \arctan\left(\frac{\left(\cos(fx+e)^{3} - 5\cos(fx+e)^{2} + \left(\cos(fx+e)^{2} + 6\cos(fx+e) + 4\right)\sin(fx+e) - 2\cos(fx+e) + 4\right)\sqrt{\frac{b \sin(fx+e)}{\cos(fx+e)}} \sqrt{\frac{d}{b}} \sqrt{\frac{d}{\cos(fx+e)}}}{4\left(d\cos(fx+e)^{2} + \left(d\cos(fx+e) + d\right)\sin(fx+e) - d\right)}\right) + 3\left(b d^{3}\cos(fx+e)^{2} - b d^{3}\right)\sqrt{\frac{d}{b}} \log\left(\frac{d\cos(fx+e)^{4} - 72 d\cos(fx+e)^{2} - 8\left(7\cos(fx+e)^{3} + \left(\cos(fx+e)^{3} - 8\cos(fx+e)\right)\sin(fx+e) - 8\cos(fx+e)\right)\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{b}}\sqrt{\frac{d}{\cos(fx+e)}} - 28\left(d\cos(fx+e)^{2} - 2d\right)\sin(fx+e) + 72d}{\cos(fx+e)^{4} - 8\cos(fx+e)^{2} + 4\left(\cos(fx+e)^{2} - 2\right)\sin(fx+e) + 8}\right)}{24\left(b^{3} f\cos(fx+e)^{2} - b^{3} f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [1/24*(16*d^3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e) - 6*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(-d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 - (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2 - (d*cos(f*x + e) + d)*sin(f*x + e) - d)) + 3*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(-d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 + 8*(7*cos(f*x + e)^3 - (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(-d/b)*sqrt(d/cos(f*x + e)) + 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/(b^3*f*cos(f*x + e)^2 - b^3*f), 1/24*(16*d^3*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e) + 6*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(d/b)*arctan(1/4*(cos(f*x + e)^3 - 5*cos(f*x + e)^2 + (cos(f*x + e)^2 + 6*cos(f*x + e) + 4)*sin(f*x + e) - 2*cos(f*x + e) + 4)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e))/(d*cos(f*x + e)^2 + (d*cos(f*x + e) + d)*sin(f*x + e) - d)) + 3*(b*d^3*cos(f*x + e)^2 - b*d^3)*sqrt(d/b)*log((d*cos(f*x + e)^4 - 72*d*cos(f*x + e)^2 - 8*(7*cos(f*x + e)^3 + (cos(f*x + e)^3 - 8*cos(f*x + e))*sin(f*x + e) - 8*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/b)*sqrt(d/cos(f*x + e)) - 28*(d*cos(f*x + e)^2 - 2*d)*sin(f*x + e) + 72*d)/(cos(f*x + e)^4 - 8*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2)*sin(f*x + e) + 8)))/(b^3*f*cos(f*x + e)^2 - b^3*f)]

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sec(f*x+e))**(7/2)/(b*tan(f*x+e))**(5/2),x)

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{\left(b\tan\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(5/2),x)
```

```
[Out] int((d/cos(e + f*x))^(7/2)/(b*tan(e + f*x))^(5/2), x)
```

3.329 $\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{2d^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}$$

```
[Out] -2/3*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elli
pticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1
/2)/b^2/f/(b*tan(f*x+e))^(1/2)-2/3*d^2*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+
e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2688, 2696, 2721, 2720}

$$\frac{2d^2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}-\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*d^2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) + (2*d^2*Ellip
ticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2
*f*Sqrt[b*Tan[e + f*x]])
```

Rule 2688

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a^2*(a*Sec[e + f*x])^(m - 2)*((b*Tan[e + f*x])^(n +
1)/(b*f*(n + 1))), x] - Dist[a^2*((m - 2)/(b^2*(n + 1))), Int[(a*Sec[e + f*
x])^(m - 2)*(b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && L
tQ[n, -1] && (GtQ[m, 1] || (EqQ[m, 1] && EqQ[n, -3/2])) && IntegersQ[2*m, 2
*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{5/2}}{(b\tan(e+fx))^{5/2}}\,dx &= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{d^2\int\frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3b^2}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{\left(d^2\sqrt{d\sec(e+fx)}\,\sqrt{b\sin(e+fx)}\right)\int\frac{1}{\sqrt{b\sin(e+fx)}}\,dx}{3b^2\sqrt{b\tan(e+fx)}}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{\left(d^2\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}\right)\int\frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3b^2\sqrt{b\tan(e+fx)}}\\
&= -\frac{2d^2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}+\frac{2d^2F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\,\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.49, size = 116, normalized size = 1.15

$$
\frac{2d^3\left(\sqrt{2}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{\sec(e+fx)}-\cot(e+fx)\csc(e+fx)\sqrt{1+\sec(e+fx)}\right)\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (2*d^3*(Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, -Tan[(e + f*x)/2]^2]*Sqrt[
Sec[e + f*x]] - Cot[e + f*x]*Csc[e + f*x]*Sqrt[1 + Sec[e + f*x]])*Sqrt[b*Ta
n[e + f*x]])/(3*b^3*f*Sqrt[d*Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.35, size = 314, normalized size = 3.11

method	result
default	$-\frac{\left(-i\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(-I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e)
)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*c
os(f*x+e)-I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*sin(f*x+e)*((I*cos(f*x+e)-
I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(
1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+
cos(f*x+e)*2^(1/2))*(d/cos(f*x+e))^(5/2)*sin(f*x+e)/(b*sin(f*x+e)/cos(f*x+e
))^(5/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 165, normalized size = 1.63

$$\frac{2\,d^2\sqrt{\frac{b\sin(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d}{\cos(fx+e)}}\cos(fx+e)^2+\left(d^2\cos(fx+e)^2-d^2\right)\sqrt{-2i\,bd}\,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))+\left(d^2\cos(fx+e)^2-d^2\right)\sqrt{2i\,bd}\,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)-i\sin(fx+e))}{3\left(b^3f\cos(fx+e)^2-b^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*d^2*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x +
 e)^2 + (d^2*cos(f*x + e)^2 - d^2)*sqrt(-2*I*b*d)*weierstrassPInverse(4, 0,
 cos(f*x + e) + I*sin(f*x + e)) + (d^2*cos(f*x + e)^2 - d^2)*sqrt(2*I*b*d)*
weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b^3*f*cos(f*x +
e)^2 - b^3*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{\left(b\tan\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(5/2),x)
```

```
[Out] int((d/cos(e + f*x))^(5/2)/(b*tan(e + f*x))^(5/2), x)
```

3.330 $\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}} dx$

Optimal. Leaf size=34

$$-\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

```
[Out] -2/3*(d*sec(f*x+e))^(3/2)/b/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2685}

$$-\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(5/2),x]
[Out] (-2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{3/2}}{(b\tan(e+fx))^{5/2}} dx = -\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

Mathematica [A]
time = 0.17, size = 34, normalized size = 1.00

$$-\frac{2(d\sec(e+fx))^{3/2}}{3bf(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(b*Tan[e + f*x])^(5/2),x]
[Out] (-2*(d*Sec[e + f*x])^(3/2))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

Maple [A]
time = 0.30, size = 50, normalized size = 1.47

method	result	size
default	$-\frac{2\left(\frac{d}{\cos(fx+e)}\right)^{\frac{3}{2}}\sin(fx+e)}{3f\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{5}{2}}\cos(fx+e)}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/f*(d/cos(f*x+e))^(3/2)*sin(f*x+e)/(b*sin(f*x+e)/cos(f*x+e))^(5/2)/cos(
f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(30) = 60.
time = 0.37, size = 66, normalized size = 1.94

$$\frac{2\,d\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{d}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)}{3\,\left(b^3f\cos\left(fx+e\right)^2-b^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*d*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)/(
b^3*f*cos(f*x + e)^2 - b^3*f)
```

Sympy [A]
time = 53.83, size = 54, normalized size = 1.59

$$\begin{cases} -\frac{2(d\sec{(e+fx)})^{\frac{3}{2}}\tan{(e+fx)}}{3f(b\tan{(e+fx)})^{\frac{5}{2}}} & \text{for}\ f\neq 0 \\ \frac{x(d\sec{(e)})^{\frac{3}{2}}}{(b\tan{(e)})^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Piecewise((-2*(d*sec(e + f*x))**(3/2)*tan(e + f*x)/(3*f*(b*tan(e + f*x))**(
5/2)), Ne(f, 0)), (x*(d*sec(e))**(3/2)/(b*tan(e))**(5/2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e))^(5/2), x)
```

Mupad [B]
time = 3.17, size = 55, normalized size = 1.62

$$-\frac{2\,d\,\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}{3\,b^2\,f\,\sin\left(e+f\,x\right)\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(3/2)/(b*tan(e + f*x))^(5/2),x)
```

```
[Out] -(2*d*(d/cos(e + f*x))^(1/2))/(3*b^2*f*sin(e + f*x)*((b*sin(2*e + 2*f*x))/(
cos(2*e + 2*f*x) + 1))^(1/2))
```

3.331 $\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}-\frac{4F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}$$

```
[Out] 4/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/b
^2/f/(b*tan(f*x+e))^(1/2)-2/3*(d*sec(f*x+e))^(1/2)/b/f/(b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2689, 2696, 2721, 2720}

$$-\frac{4\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}-\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]])/(3*b*f*(b*Tan[e + f*x])^(3/2)) - (4*EllipticF[(e
- Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2*f*Sqrt[
b*Tan[e + f*x]])
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sqrt{d\sec(e+fx)}}{(b\tan(e+fx))^{5/2}}\,dx = -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{2\int \frac{\sqrt{d\sec(e+fx)}}{\sqrt{b\tan(e+fx)}}\,dx}{3b^2}$$

$$= -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{\left(2\sqrt{d\sec(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{1}{\sqrt{b\sin(e+fx}}}{3b^2\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{\left(2\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,d}{3b^2\sqrt{b\tan(e+fx)}}$$

$$= -\frac{2\sqrt{d\sec(e+fx)}}{3bf(b\tan(e+fx))^{3/2}} - \frac{4F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2f\sqrt{b\tan(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.81, size = 70, normalized size = 0.74

$$-\frac{2\sqrt{d\sec(e+fx)}\left(1+2\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\sec^2(e+fx)\right)\left(-\tan^2(e+fx)\right)^{3/4}\right)}{3bf(b\tan(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(b*Tan[e + f*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]]*(1 + 2*Hypergeometric2F1[1/4, 3/4, 5/4, Sec[e + f*
x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(3*b*f*(b*Tan[e + f*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.36, size = 322, normalized size = 3.39

method	result
default	$-\frac{\left(2i\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\operatorname{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(f}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(2*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e
))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*
cos(f*x+e)+2*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*
x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+
e)+cos(f*x+e)*2^(1/2))*(d/cos(f*x+e))^(1/2)*sin(f*x+e)/(b*sin(f*x+e)/cos(f*
x+e))^(5/2)/cos(f*x+e)^2*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 147, normalized size = 1.55

$$\frac{2\left(\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}-\sqrt{-2i\,bd}\left(\cos\left(fx+e\right)^{2}-1\right)\mathrm{weierstrassPInverse}\left(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)-\sqrt{2i\,bd}\left(\cos\left(fx+e\right)^{2}-1\right)\mathrm{weierstrassPInverse}\left(4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)\right)}{3\left(b^{3}f\cos\left(fx+e\right)^{2}-b^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))*cos(f*x + e)^2
- sqrt(-2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInverse(4, 0, cos(f*x + e
) + I*sin(f*x + e)) - sqrt(2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInvers
e(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b^3*f*cos(f*x + e)^2 - b^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{d\sec\left(e+fx\right)}}{\left(b\tan\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))/(b*tan(e + f*x))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}{{\left(b\,\tan\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(5/2),x)
[Out] int((d/cos(e + f*x))^(1/2)/(b*tan(e + f*x))^(5/2), x)
```

3.332 $\int \frac{1}{\sqrt{d \sec(e+fx)}\,(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=69

$$-\frac{2}{3bf\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}-\frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}$$

```
[Out] -8/3*(b*tan(f*x+e))^(1/2)/b^3/f/(d*sec(f*x+e))^(1/2)-2/3/b/f/(d*sec(f*x+e))
^(1/2)/(b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2689, 2685}

$$-\frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -2/(3*b*f*Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(3/2)) - (8*Sqrt[b*Tan[e +
f*x]])/(3*b^3*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}-\frac{4\int \frac{\ldots}{\sqrt{d\sec(e+fx)}\ldots}}{3\ldots}$$

$$= -\frac{2}{3bf\sqrt{d\sec(e+fx)}\,(b\tan(e+fx))^{3/2}}-\frac{8\sqrt{b\tan(e+fx)}}{3b^3f\sqrt{d\sec(e+fx)}}$$

Mathematica [A]
time = 0.91, size = 110, normalized size = 1.59

$$-\frac{2\left(\sqrt{\frac{1}{1+\cos(e+fx)}}\csc(e+fx)\sec(e+fx)+3\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)\right)}{3b^2f\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{d\sec(e+fx)}\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (-2*(Sqrt[(1 + Cos[e + f*x])^(-1)]*Csc[e + f*x]*Sec[e + f*x] + 3*Sqrt[Sec[e
 + f*x]]*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]))/(3*b^2*f*Sqrt[(1 + Cos[e
 + f*x])^(-1)]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e + f*x]])
```

Maple [A]
time = 0.34, size = 62, normalized size = 0.90

method	result	size
default	$\frac{2\sin(fx+e)\left(3\left(\cos^2(fx+e)\right)-4\right)}{3f\cos(fx+e)^3\sqrt{\frac{d}{\cos(fx+e)}}\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{5}{2}}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/f*sin(f*x+e)*(3*cos(f*x+e)^2-4)/cos(f*x+e)^3/(d/cos(f*x+e))^(1/2)/(b*si
n(f*x+e)/cos(f*x+e))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2)), x)
```

Fricas [A]
time = 0.43, size = 81, normalized size = 1.17

$$-\frac{2\left(3\cos\left(fx+e\right)^3-4\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{3\left(b^3df\cos\left(fx+e\right)^2-b^3df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*(3*cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(b*sin(f*x + e)/cos(f*x + e))*
sqrt(d/cos(f*x + e))/(b^3*d*f*cos(f*x + e)^2 - b^3*d*f)
```

Sympy [A]
time = 72.88, size = 92, normalized size = 1.33

$$\begin{cases} -\frac{8\tan^{3}(e+fx)}{3f(b\tan(e+fx))^{\frac{5}{2}}\sqrt{d\sec(e+fx)}} - \frac{2\tan(e+fx)}{3f(b\tan(e+fx))^{\frac{5}{2}}\sqrt{d\sec(e+fx)}} & \text{for}\ f \neq 0 \\ \frac{x}{(b\tan(e))^{\frac{5}{2}}\sqrt{d\sec(e)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Piecewise((-8*tan(e + f*x)**3/(3*f*(b*tan(e + f*x))**(5/2)*sqrt(d*sec(e + f
*x))) - 2*tan(e + f*x)/(3*f*(b*tan(e + f*x))**(5/2)*sqrt(d*sec(e + f*x))),
Ne(f, 0)), (x/((b*tan(e))**(5/2)*sqrt(d*sec(e))), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e))^(5/2)), x)
```

Mupad [B]
time = 3.56, size = 81, normalized size = 1.17

$$\frac{\left(\frac{13\sin(e+f\,x)}{3}-\sin\left(3\,e+3\,f\,x\right)\right)\,\sqrt{\frac{d}{\cos\left(e+f\,x\right)}}}{b^2\,d\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(1/2)),x)
```

```
[Out] (((13*sin(e + f*x))/3 - sin(3*e + 3*f*x))*(d/cos(e + f*x))^(1/2))/(b^2*d*f*
(cos(2*e + 2*f*x) - 1)*((b*sin(2*e + 2*f*x))/(cos(2*e + 2*f*x) + 1))^(1/2))
```

3.333 $\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}}-\frac{8F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{d\sec(e+fx)}\sqrt{\sin(e+fx)}}{3b^2d^2f\sqrt{b\tan(e+fx)}}-\frac{4\sqrt{b\,\mathrm{t}}}{3b^3f(d\,\mathrm{s}}$$

```
[Out] 8/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF
(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*sec(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/b
^2/d^2/f/(b*tan(f*x+e))^(1/2)-4/3*(b*tan(f*x+e))^(1/2)/b^3/f/(d*sec(f*x+e))
^(3/2)-2/3/b/f/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2689, 2692, 2696, 2721, 2720}

$$-\frac{4\sqrt{b\tan(e+fx)}}{3b^3f(d\sec(e+fx))^{3/2}}-\frac{8\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\sec(e+fx)}}{3b^2d^2f\sqrt{b\tan(e+fx)}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -2/(3*b*f*(d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(3/2)) - (8*EllipticF[(e
- Pi/2 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(3*b^2*d^2*f*S
qrt[b*Tan[e + f*x]]) - (4*Sqrt[b*Tan[e + f*x]])/(3*b^3*f*(d*Sec[e + f*x])^(
3/2))
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
 && EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rule 2696

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[a^(m + n)*((b*Tan[e + f*x])^n/((a*Sec[e + f*x])^n*(b*
Sin[e + f*x])^n)), Int[(b*Sin[e + f*x])^n/Cos[e + f*x]^(m + n), x], x] /; F
reeQ[{a, b, e, f, m, n}, x] && IntegerQ[n + 1/2] && IntegerQ[m + 1/2]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{2\int \frac{}{(d\sec(e+fx))^{3/2}\sqrt{\ldots}}}{b\ldots}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{4\sqrt{b\tan(e+f\ldots}}{3b^3f(d\sec(e+f\ldots}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{4\sqrt{b\tan(e+f\ldots}}{3b^3f(d\sec(e+f\ldots}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{4\sqrt{b\tan(e+f\ldots}}{3b^3f(d\sec(e+f\ldots}$$

$$= -\frac{2}{3bf(d\sec(e+fx))^{3/2}(b\tan(e+fx))^{3/2}} - \frac{8F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+f\ldots\right.\right.}{3b\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.55, size = 112, normalized size = 0.85

$$\frac{\csc^2(e+fx)\sqrt{b\tan(e+fx)}\left(-\tan^2(e+fx)\right)^{3/4}\left(-8\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\sec^2(e+fx)\right)+\left(-1+\cos(2(e+fx))+2\csc^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{3b^3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (Csc[e + f*x]^2*Sqrt[b*Tan[e + f*x]]*(-Tan[e + f*x]^2)^(3/4)*(-8*Hypergeome
tric2F1[1/4, 3/4, 5/4, Sec[e + f*x]^2] + (-1 + Cos[2*(e + f*x)] + 2*Csc[e +
 f*x]^2)*(-Tan[e + f*x]^2)^(1/4)))/(3*b^3*f*(d*Sec[e + f*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 336, normalized size = 2.55

method	result
default	$-\frac{\left(4i\sqrt{-\frac{i(\cos(fx+e)-1)}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)-i+\sin(fx+e)}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-i-\sin(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(fx+e)-i+\sin(}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*(4*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)-I+sin(f*x+e
))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*
cos(f*x+e)+4*I*(-I*(cos(f*x+e)-1)/sin(f*x+e))^(1/2)*sin(f*x+e)*((I*cos(f*x+
e)-I+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-I-sin(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticF(((I*cos(f*x+e)-I+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2
))-2^(1/2)*cos(f*x+e)^3+2*cos(f*x+e)*2^(1/2))*sin(f*x+e)/(d/cos(f*x+e))^(3/
2)/(b*sin(f*x+e)/cos(f*x+e))^(5/2)/cos(f*x+e)^4*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 165, normalized size = 1.25

$$-\frac{2\left(2\sqrt{-2i\,bd}\left(\cos\left(fx+e\right)^{2}-1\right)\mathrm{weierstrassPInverse}\left(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)+2\sqrt{2i\,bd}\left(\cos\left(fx+e\right)^{2}-1\right)\mathrm{weierstrassPInverse}\left(4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)+\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\right)}{3\left(b^{3}d^{2}f\cos\left(fx+e\right)^{2}-b^{3}d^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/3*(2*sqrt(-2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstrassPInverse(4, 0, cos(f
*x + e) + I*sin(f*x + e)) + 2*sqrt(2*I*b*d)*(cos(f*x + e)^2 - 1)*weierstras
sPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)) + (cos(f*x + e)^4 - 2*cos(f*
x + e)^2)*sqrt(b*sin(f*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e)))/(b^3*d^2*
f*cos(f*x + e)^2 - b^3*d^2*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4372 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e))^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\tan(e+f\,x))^{5/2}\,\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(3/2)), x)
```

3.334 $\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}}-\frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}}-\frac{64\sqrt{b\tan(e+fx)}}{15b^3d^2f\sqrt{d\sec(e+fx)}}$$

```
[Out] -16/15*(b*tan(f*x+e))^(1/2)/b^3/f/(d*sec(f*x+e))^(5/2)-64/15*(b*tan(f*x+e))
^(1/2)/b^3/d^2/f/(d*sec(f*x+e))^(1/2)-2/3/b/f/(d*sec(f*x+e))^(5/2)/(b*tan(f
*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2689, 2692, 2685}

$$-\frac{64\sqrt{b\tan(e+fx)}}{15b^3d^2f\sqrt{d\sec(e+fx)}}-\frac{16\sqrt{b\tan(e+fx)}}{15b^3f(d\sec(e+fx))^{5/2}}-\frac{2}{3bf(b\tan(e+fx))^{3/2}(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] -2/(3*b*f*(d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(3/2)) - (16*Sqrt[b*Tan[e
 + f*x]])/(15*b^3*f*(d*Sec[e + f*x])^(5/2)) - (64*Sqrt[b*Tan[e + f*x]])/(15
*b^3*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rule 2685

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*
f*m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 1, 0]
```

Rule 2689

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n
 + 1))), x] - Dist[(m + n + 1)/(b^2*(n + 1)), Int[(a*Sec[e + f*x])^m*(b*Tan
[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] && In
tegersQ[2*m, 2*n]
```

Rule 2692

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(a*Sec[e + f*x])^m)*((b*Tan[e + f*x])^(n + 1)/(b*f*
m)), x] + Dist[(m + n + 1)/(a^2*m), Int[(a*Sec[e + f*x])^(m + 2)*(b*Tan[e +
 f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (LtQ[m, -1] || (EqQ[m, -1]
```

```
&& EqQ[n, -2^(-1)])) && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{5/2}}\,dx = -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} - \frac{8\int \frac{}{(d\sec(e+fx))^{5/2}\sqrt{}}}{3}$$
$$= -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} - \frac{16\sqrt{b\tan(e+}}{15b^3f(d\sec(e+f}$$
$$= -\frac{2}{3bf(d\sec(e+fx))^{5/2}(b\tan(e+fx))^{3/2}} - \frac{16\sqrt{b\tan(e+}}{15b^3f(d\sec(e+f}$$

Mathematica [A]
time = 3.32, size = 159, normalized size = 1.50

$$\frac{\sqrt{\frac{1}{1+\cos(e+fx)}}\,(-43+3\cos(2(e+fx)))\csc(e+fx)\sec(e+fx)-228\sqrt{\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}\,\tan\left(\frac{1}{2}(e+fx)\right)-6\sqrt{\frac{1}{1+\cos(e+fx)}}\,(-1+2\cos(2(e+fx)))\tan(e+fx)}{60b^2d^2f\sqrt{\frac{1}{1+\cos(e+fx)}}\,\sqrt{d\sec(e+fx)}\,\sqrt{b\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(b*Tan[e + f*x])^(5/2)),x]
```

```
[Out] (Sqrt[(1 + Cos[e + f*x])^(-1)]*(-43 + 3*Cos[2*(e + f*x)])*Csc[e + f*x]*Sec[
e + f*x] - 228*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2] -
 6*Sqrt[(1 + Cos[e + f*x])^(-1)]*(-1 + 2*Cos[2*(e + f*x)])*Tan[e + f*x])/(6
0*b^2*d^2*f*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[d*Sec[e + f*x]]*Sqrt[b*Tan[e
 + f*x]])
```

Maple [A]
time = 0.32, size = 72, normalized size = 0.68

method	result	size
default	$\frac{2\sin(fx+e)\left(3(\cos^4(fx+e))+24(\cos^2(fx+e))-32\right)}{15f\left(\frac{d}{\cos(fx+e)}\right)^{\frac{5}{2}}\left(\frac{b\sin(fx+e)}{\cos(fx+e)}\right)^{\frac{5}{2}}\cos(fx+e)^5}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/f*sin(f*x+e)*(3*cos(f*x+e)^4+24*cos(f*x+e)^2-32)/(d/cos(f*x+e))^(5/2)/
(b*sin(f*x+e)/cos(f*x+e))^(5/2)/cos(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2)), x)
```

Fricas [A]
time = 0.43, size = 96, normalized size = 0.91

$$-\frac{2\left(3\cos\left(fx+e\right)^{5}+24\cos\left(fx+e\right)^{3}-32\cos\left(fx+e\right)\right)\sqrt{\frac{b\sin\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}}{15\left(b^{3}d^{3}f\cos\left(fx+e\right)^{2}-b^{3}d^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] -2/15*(3*cos(f*x + e)^5 + 24*cos(f*x + e)^3 - 32*cos(f*x + e))*sqrt(b*sin(f
*x + e)/cos(f*x + e))*sqrt(d/cos(f*x + e))/(b^3*d^3*f*cos(f*x + e)^2 - b^3*
d^3*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(b*tan(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(b*tan(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e))^(5/2)), x)
```

Mupad [B]
time = 4.35, size = 93, normalized size = 0.88

$$-\frac{\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(105\,\sin\left(3\,e+3\,f\,x\right)-410\,\sin\left(e+f\,x\right)+3\,\sin\left(5\,e+5\,f\,x\right)\right)}{60\,b^2\,d^3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)\,\sqrt{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{\cos\left(2\,e+2\,f\,x\right)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*tan(e + f*x))^(5/2)*(d/cos(e + f*x))^(5/2)),x)
```

```
[Out] -((d/cos(e + f*x))^(1/2)*(105*sin(3*e + 3*f*x) - 410*sin(e + f*x) + 3*sin(5
*e + 5*f*x)))/(60*b^2*d^3*f*(cos(2*e + 2*f*x) - 1)*((b*sin(2*e + 2*f*x))/(c
os(2*e + 2*f*x) + 1))^(1/2))
```

3.335 $\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{7}{4};\sin^2(e+fx)\right)(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] 2/3*(cos(f*x+e)^2)^(17/12)*hypergeom([3/4, 17/12],[7/4],sin(f*x+e)^2)*(b*se
c(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{2697\}$

$$\frac{2\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{7}{4};\sin^2(e+fx)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[3/4, 17/12, 7/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{3}{4},\frac{17}{12};\frac{7}{4};\sin^2(e+fx)\right)(b\sec(e+fx))^{4/3}(d}{3df}$$

Mathematica [A]
time = 0.12, size = 64, normalized size = 1.00

$$\frac{3d\,{}_2F_1\left(\frac{1}{4},\frac{2}{3};\frac{5}{3};\sec^2(e+fx)\right)(b\sec(e+fx))^{4/3}\sqrt[4]{-\tan^2(e+fx)}}{4f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (3*d*Hypergeometric2F1[1/4, 2/3, 5/3, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(4/3
)*(-Tan[e + f*x]^2)^(1/4))/(4*f*Sqrt[d*Tan[e + f*x]])
```

Maple [**F**]
time = 0.29, size = 0, normalized size = 0.00

$$\int (b\sec (fx+e))^{\frac{4}{3}} \sqrt{d\tan (fx+e)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*sec(f*x + e), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(4/3)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(4/3)*sqrt(d*tan(f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{d\tan\left(e+f\,x\right)}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(4/3),x)
[Out] int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(4/3), x)
```

3.336 $\int \sqrt[3]{b\sec(e+fx)}\,\sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{11/12}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{7}{4};\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\,(d\tan(e+fx))^{3/2}}{3df}$$

```
[Out] 2/3*(cos(f*x+e)^2)^(11/12)*hypergeom([3/4, 11/12],[7/4],sin(f*x+e)^2)*(b*se
c(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2)/d/f
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{2\cos^2(e+fx)^{11/12}\sqrt[3]{b\sec(e+fx)}\,(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{7}{4};\sin^2(e+fx)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[3/4, 11/12, 7/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2))/(3*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt[3]{b\sec(e+fx)}\,\sqrt{d\tan(e+fx)}\,dx = \frac{2\cos^2(e+fx)^{11/12}\,{}_2F_1\left(\frac{3}{4},\frac{11}{12};\frac{7}{4};\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\,(d$$

Mathematica [A]
time = 0.09, size = 62, normalized size = 0.97

$$\frac{3d\,{}_2F_1\left(\frac{1}{6},\frac{1}{4};\frac{7}{6};\sec^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\,\sqrt[4]{-\tan^2(e+fx)}}{f\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (3*d*Hypergeometric2F1[1/6, 1/4, 7/6, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(1/3
)*(-Tan[e + f*x]^2)^(1/4))/(f*Sqrt[d*Tan[e + f*x]])
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(f x+e\right)\right)^{\frac{1}{3}} \sqrt{d \tan \left(f x+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b \sec \left(e+f x\right)} \sqrt{d \tan \left(e+f x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/3)*(d*tan(f*x+e))**(1/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(1/3)*sqrt(d*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{d\tan\left(e+f\,x\right)}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/3),x)
[Out] int((d*tan(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/3), x)
```

3.337 $\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{7/12}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df\sqrt[3]{b\sec(e+fx)}}$$

```
[Out] 2/3*(cos(f*x+e)^2)^(7/12)*hypergeom([7/12, 3/4],[7/4],sin(f*x+e)^2)*(d*tan(
f*x+e))^(3/2)/d/f/(b*sec(f*x+e))^(1/3)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{3/2}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)}{3df\sqrt[3]{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(1/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 3/4, 7/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{7/12}\,{}_2F_1\left(\frac{7}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df\sqrt[3]{b\sec(e+fx)}}$$

Mathematica [A]
time = 0.11, size = 62, normalized size = 0.97

$$-\frac{3d\,{}_2F_1\left(-\frac{1}{6},\frac{1}{4};\frac{5}{6};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{f\sqrt[3]{b\sec(e+fx)}\,\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(1/3),x]
```

```
[Out] (-3*d*Hypergeometric2F1[-1/6, 1/4, 5/6, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(
1/4))/(f*(b*Sec[e + f*x])^(1/3)*Sqrt[d*Tan[e + f*x]])
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\tan\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x)
```

```
[Out] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b*sec(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\tan\left(e+fx\right)}}{\sqrt[3]{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(b*sec(f*x+e))**(1/3),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))/(b*sec(e + f*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(1/3),x)
[Out] int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(1/3), x)
```

3.338 $\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=64

$$\frac{2\sqrt[12]{\cos^2(e+fx)}\;{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df(b\sec(e+fx))^{4/3}}$$

```
[Out] 2/3*(cos(f*x+e)^2)^(1/12)*hypergeom([1/12, 3/4],[7/4],sin(f*x+e)^2)*(d*tan(
f*x+e))^(3/2)/d/f/(b*sec(f*x+e))^(4/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{2\sqrt[12]{\cos^2(e+fx)}\,(d\tan(e+fx))^{3/2}\;{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)}{3df(b\sec(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(4/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[1/12, 3/4, 7/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(3/2))/(3*d*f*(b*Sec[e + f*x])^(4/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt{d\tan(e+fx)}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{2\sqrt[12]{\cos^2(e+fx)}\;{}_2F_1\left(\frac{1}{12},\frac{3}{4};\frac{7}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{3df(b\sec(e+fx))^{4/3}}$$

Mathematica [A]
time = 0.14, size = 64, normalized size = 1.00

$$-\frac{3d\;{}_2F_1\left(-\frac{2}{3},\frac{1}{4};\frac{1}{3};\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{4f(b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Tan[e + f*x]]/(b*Sec[e + f*x])^(4/3),x]
```

```
[Out] (-3*d*Hypergeometric2F1[-2/3, 1/4, 1/3, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(
1/4))/(4*f*(b*Sec[e + f*x])^(4/3)*Sqrt[d*Tan[e + f*x]])
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \tan \left(f x +e \right)}}{\left(b \sec \left(f x +e \right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x)
```

```
[Out] int((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))/(b^2*sec(f*x + e)^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \tan \left(e + f x \right)}}{\left(b \sec \left(e + f x \right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(1/2)/(b*sec(f*x+e))**(4/3),x)
```

```
[Out] Integral(sqrt(d*tan(e + f*x))/(b*sec(e + f*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(1/2)/(b*sec(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))/(b*sec(f*x + e))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\tan\left(e+f\,x\right)}}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(4/3),x)
```

```
[Out] int((d*tan(e + f*x))^(1/2)/(b/cos(e + f*x))^(4/3), x)
```

3.339 $\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{23/12}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{9}{4};\sin^2(e+fx)\right)(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{5/2}}{5df}$$

```
[Out] 2/5*(cos(f*x+e)^2)^(23/12)*hypergeom([5/4, 23/12],[9/4],sin(f*x+e)^2)*(b*se
c(f*x+e))^(4/3)*(d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{2\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{4/3}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{9}{4};\sin^2(e+fx)\right)}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[5/4, 23/12, 9/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^{4/3}(d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{23/12}\,{}_2F_1\left(\frac{5}{4},\frac{23}{12};\frac{9}{4};\sin^2(e+fx)\right)(b\sec(e+fx))^{4/3}(d}{5df}$$

Mathematica [A]
time = 0.17, size = 64, normalized size = 1.00

$$\frac{3d\,{}_2F_1\left(-\frac{1}{4},\frac{2}{3};\frac{5}{3};\sec^2(e+fx)\right)(b\sec(e+fx))^{4/3}\sqrt{d\tan(e+fx)}}{4f\sqrt[4]{-\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(4/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (3*d*Hypergeometric2F1[-1/4, 2/3, 5/3, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(4/
3)*Sqrt[d*Tan[e + f*x]])/(4*f*(-Tan[e + f*x]^2)^(1/4))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (b \sec (fx+e))^{\frac{4}{3}} (d \tan (fx+e))^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*b*d*sec(f*x + e)*tan(f
*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(4/3)*(d*tan(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(4/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(4/3)*(d*tan(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\tan(e+f\,x))^{3/2}\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(4/3),x)
[Out] int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(4/3), x)
```

3.340 $\int \sqrt[3]{b\sec(e+fx)}\,(d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{9}{4};\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\,(d\tan(e+fx))^{5/2}}{5df}$$

```
[Out] 2/5*(cos(f*x+e)^2)^(17/12)*hypergeom([5/4, 17/12],[9/4],sin(f*x+e)^2)*(b*se
c(f*x+e))^(1/3)*(d*tan(f*x+e))^(5/2)/d/f
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{2\cos^2(e+fx)^{17/12}\sqrt[3]{b\sec(e+fx)}\,(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{9}{4};\sin^2(e+fx)\right)}{5df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[5/4, 17/12, 9/4, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(5/2))/(5*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt[3]{b\sec(e+fx)}\,(d\tan(e+fx))^{3/2}\,dx = \frac{2\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{5}{4},\frac{17}{12};\frac{9}{4};\sin^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\,(d$$

Mathematica [A]
time = 0.15, size = 62, normalized size = 0.97

$$\frac{3d\,{}_2F_1\left(-\frac{1}{4},\frac{1}{6};\frac{7}{6};\sec^2(e+fx)\right)\sqrt[3]{b\sec(e+fx)}\,\sqrt{d\tan(e+fx)}}{f\sqrt[4]{-\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(1/3)*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (3*d*Hypergeometric2F1[-1/4, 1/6, 7/6, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(1/
3)*Sqrt[d*Tan[e + f*x]])/(f*(-Tan[e + f*x]^2)^(1/4))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{1}{3}} \left(d\tan\left(fx+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

```
[Out] int((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(1/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\sec\left(e+fx\right)} \left(d\tan\left(e+fx\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/3)*(d*tan(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(1/3)*(d*tan(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/3)*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(1/3)*(d*tan(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\tan(e+f\,x))^{3/2}\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/3),x)
[Out] int((d*tan(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/3), x)
```

3.341 $\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{13/12}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df\sqrt[3]{b\sec(e+fx)}}$$

```
[Out] 2/5*(cos(f*x+e)^2)^(13/12)*hypergeom([13/12, 5/4],[9/4],sin(f*x+e)^2)*(d*ta
n(f*x+e))^(5/2)/d/f/(b*sec(f*x+e))^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{2\cos^2(e+fx)^{13/12}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)}{5df\sqrt[3]{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(1/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[13/12, 5/4, 9/4, Sin[e + f*x]
^2]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{\sqrt[3]{b\sec(e+fx)}}\,dx = \frac{2\cos^2(e+fx)^{13/12}\,{}_2F_1\left(\frac{13}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df\sqrt[3]{b\sec(e+fx)}}$$

Mathematica [A]
time = 0.08, size = 69, normalized size = 1.08

$$\frac{3\cot^3(e+fx)\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{6};\frac{5}{6};\sec^2(e+fx)\right)(d\tan(e+fx))^{3/2}\left(-\tan^2(e+fx)\right)^{3/4}}{f\sqrt[3]{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(1/3),x]
```

```
[Out] (3*Cot[e + f*x]^3*Hypergeometric2F1[-1/4, -1/6, 5/6, Sec[e + f*x]^2]*(d*Tan
[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(3/4))/(f*(b*Sec[e + f*x])^(1/3))
```

Maple [**F**]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(d \tan (fx+e))^{\frac{3}{2}}}{(b \sec (fx+e))^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x)
```

```
[Out] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(1/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b*sec(
f*x + e)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \tan (e+fx))^{\frac{3}{2}}}{\sqrt[3]{b \sec (e+fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sec(f*x+e))**(1/3),x)
```

```
[Out] Integral((d*tan(e + f*x))**(3/2)/(b*sec(e + f*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\tan(e+f\,x))^{3/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(1/3),x)
```

```
[Out] int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(1/3), x)
```

3.342 $\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=64

$$\frac{2\cos^2(e+fx)^{7/12}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df(b\sec(e+fx))^{4/3}}$$

```
[Out] 2/5*(cos(f*x+e)^2)^(7/12)*hypergeom([7/12, 5/4],[9/4],sin(f*x+e)^2)*(d*tan(
f*x+e))^(5/2)/d/f/(b*sec(f*x+e))^(4/3)
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{2\cos^2(e+fx)^{7/12}(d\tan(e+fx))^{5/2}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)}{5df(b\sec(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(4/3),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[7/12, 5/4, 9/4, Sin[e + f*x]^2
]*(d*Tan[e + f*x])^(5/2))/(5*d*f*(b*Sec[e + f*x])^(4/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(d\tan(e+fx))^{3/2}}{(b\sec(e+fx))^{4/3}}\,dx = \frac{2\cos^2(e+fx)^{7/12}\,{}_2F_1\left(\frac{7}{12},\frac{5}{4};\frac{9}{4};\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{5df(b\sec(e+fx))^{4/3}}$$

Mathematica [A]
time = 0.08, size = 71, normalized size = 1.11

$$\frac{3\cot^3(e+fx)\,{}_2F_1\left(-\frac{2}{3},-\frac{1}{4};\frac{1}{3};\sec^2(e+fx)\right)(d\tan(e+fx))^{3/2}\left(-\tan^2(e+fx)\right)^{3/4}}{4f(b\sec(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[e + f*x])^(3/2)/(b*Sec[e + f*x])^(4/3),x]
```

```
[Out] (3*Cot[e + f*x]^3*Hypergeometric2F1[-2/3, -1/4, 1/3, Sec[e + f*x]^2]*(d*Tan
[e + f*x])^(3/2)*(-Tan[e + f*x]^2)^(3/4))/(4*f*(b*Sec[e + f*x])^(4/3))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x)
```

```
[Out] int((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^(2/3)*sqrt(d*tan(f*x + e))*d*tan(f*x + e)/(b^2*se
c(f*x + e)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\tan(e+fx))^{\frac{3}{2}}}{(b\sec(e+fx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))**(3/2)/(b*sec(f*x+e))**(4/3),x)
```

```
[Out] Integral((d*tan(e + f*x))**(3/2)/(b*sec(e + f*x))**(4/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(f*x+e))^(3/2)/(b*sec(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*tan(f*x + e))^(3/2)/(b*sec(f*x + e))^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(d\tan\left(e+f\,x\right)\right)}^{3/2}}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(4/3),x)
```

```
[Out] int((d*tan(e + f*x))^(3/2)/(b/cos(e + f*x))^(4/3), x)
```

3.343 $\int \sqrt{b\sec(e+fx)}\,(d\tan(e+fx))^{4/3}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{13}{6};\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}\,(d\tan(e+fx))^{7/3}}{7df}$$

```
[Out] 3/7*(cos(f*x+e)^2)^(17/12)*hypergeom([7/6, 17/12],[13/6],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(1/2)*(d*tan(f*x+e))^(7/3)/d/f
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{3\cos^2(e+fx)^{17/12}\sqrt{b\sec(e+fx)}\,(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{13}{6};\sin^2(e+fx)\right)}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[7/6, 17/12, 13/6, Sin[e + f*x
]^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\,(d\tan(e+fx))^{4/3}\,dx = \frac{3\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{7}{6},\frac{17}{12};\frac{13}{6};\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}\,(d$$

Mathematica [A]
time = 0.16, size = 62, normalized size = 0.97

$$\frac{2d\,{}_2F_1\left(-\frac{1}{6},\frac{1}{4};\frac{5}{4};\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\,\sqrt[3]{d\tan(e+fx)}}{f\sqrt[6]{-\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[-1/6, 1/4, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]
*(d*Tan[e + f*x])^(1/3))/(f*(-Tan[e + f*x]^2)^(1/6))
```

Maple [**F**]
time = 0.26, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(f x+e\right)}\, \left(d \tan \left(f x+e\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*d*tan(f*x + e), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)*(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\tan(e+f\,x))^{4/3}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(1/2),x)
[Out] int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(1/2), x)
```

3.344 $\int \sqrt{b \sec(e+fx)}\,\sqrt[3]{d \tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{11/12}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{5}{3};\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}\,(d\tan(e+fx))^{4/3}}{4df}$$

```
[Out] 3/4*(cos(f*x+e)^2)^(11/12)*hypergeom([2/3, 11/12],[5/3],sin(f*x+e)^2)*(b*se
c(f*x+e))^(1/2)*(d*tan(f*x+e))^(4/3)/d/f
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{3\cos^2(e+fx)^{11/12}\sqrt{b\sec(e+fx)}\,(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{5}{3};\sin^2(e+fx)\right)}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(11/12)*Hypergeometric2F1[2/3, 11/12, 5/3, Sin[e + f*x]
^2]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\,\sqrt[3]{d\tan(e+fx)}\,dx = \frac{3\cos^2(e+fx)^{11/12}\,{}_2F_1\left(\frac{2}{3},\frac{11}{12};\frac{5}{3};\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}\,(d$$

Mathematica [A]
time = 0.10, size = 62, normalized size = 0.97

$$\frac{2d\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{5}{4};\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\,\sqrt[3]{-\tan^2(e+fx)}}{f(d\tan(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[1/4, 1/3, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]*
(-Tan[e + f*x]^2)^(1/3))/(f*(d*Tan[e + f*x])^(2/3))
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(f x +e \right)}\, \left(d \tan \left(f x +e \right)\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(e + f x\right)} \sqrt[3]{d \tan \left(e + f x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)*(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*(d*tan(e + f*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\tan(e+f\,x))^{1/3}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(1/2),x)
[Out] int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(1/2), x)
```

3.345 $\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{7/12}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{4}{3};\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}\,(d\tan(e+fx))^{2/3}}{2df}$$

```
[Out] 3/2*(cos(f*x+e)^2)^(7/12)*hypergeom([1/3, 7/12],[4/3],sin(f*x+e)^2)*(b*sec(
f*x+e))^(1/2)*(d*tan(f*x+e))^(2/3)/d/f
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{3\cos^2(e+fx)^{7/12}\sqrt{b\sec(e+fx)}\,(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{4}{3};\sin^2(e+fx)\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[1/3, 7/12, 4/3, Sin[e + f*x]^2
]*Sqrt[b*Sec[e + f*x]]*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{7/12}\,{}_2F_1\left(\frac{1}{3},\frac{7}{12};\frac{4}{3};\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}\,(d\tan(e+fx))^{2/3}}{2df}$$

Mathematica [A]
time = 0.13, size = 62, normalized size = 0.97

$$\frac{2d\,{}_2F_1\left(\frac{1}{4},\frac{2}{3};\frac{5}{4};\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\,\left(-\tan^2(e+fx)\right)^{2/3}}{f(d\tan(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[1/4, 2/3, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]*
(-Tan[e + f*x]^2)^(2/3))/(f*(d*Tan[e + f*x])^(4/3))
```

Maple [**F**]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec \left(f x +e \right)}}{\left(d \tan \left(f x +e \right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)/(d*tan(f*x + e)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec{\left(e + f x \right)}}}{\sqrt[3]{d \tan{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))/(d*tan(e + f*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{{\left(d\,\tan\left(e+f\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3),x)
[Out] int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(1/3), x)
```

3.346 $\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=62

$$-\frac{3\sqrt[12]{\cos^2(e+fx)}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{5}{6};\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}}{df\sqrt[3]{d\tan(e+fx)}}$$

```
[Out] -3*(cos(f*x+e)^2)^(1/12)*hypergeom([-1/6, 1/12],[5/6],sin(f*x+e)^2)*(b*sec(
f*x+e))^(1/2)/d/f/(d*tan(f*x+e))^(1/3)
```

Rubi [A]

time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$-\frac{3\sqrt[12]{\cos^2(e+fx)}\sqrt{b\sec(e+fx)}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{5}{6};\sin^2(e+fx)\right)}{df\sqrt[3]{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (-3*(Cos[e + f*x]^2)^(1/12)*Hypergeometric2F1[-1/6, 1/12, 5/6, Sin[e + f*x]
^2]*Sqrt[b*Sec[e + f*x]])/(d*f*(d*Tan[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{(d\tan(e+fx))^{4/3}}\,dx = -\frac{3\sqrt[12]{\cos^2(e+fx)}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{12};\frac{5}{6};\sin^2(e+fx)\right)\sqrt{b\sec(e+fx)}}{df\sqrt[3]{d\tan(e+fx)}}$$

Mathematica [A]

time = 0.24, size = 62, normalized size = 1.00

$$\frac{2d\,{}_2F_1\left(\frac{1}{4},\frac{7}{6};\frac{5}{4};\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}\left(-\tan^2(e+fx)\right)^{7/6}}{f(d\tan(e+fx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[1/4, 7/6, 5/4, Sec[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]*
(-Tan[e + f*x]^2)^(7/6))/(f*(d*Tan[e + f*x])^(7/3))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec (f x+e)}}{(d \tan (f x+e))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)/(d^2*tan(f*x + e)^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec (e+f x)}}{(d \tan (e+f x))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))/(d*tan(e + f*x))**(4/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(d*tan(f*x + e))^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{{\left(d\,\tan\left(e+f\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3),x)
```

```
[Out] int((b/cos(e + f*x))^(1/2)/(d*tan(e + f*x))^(4/3), x)
```

3.347 $\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{23/12}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{13}{6};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{7/3}}{7df}$$

```
[Out] 3/7*(cos(f*x+e)^2)^(23/12)*hypergeom([7/6, 23/12],[13/6],sin(f*x+e)^2)*(b*s
ec(f*x+e))^(3/2)*(d*tan(f*x+e))^(7/3)/d/f
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{3\cos^2(e+fx)^{23/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{7/3}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{13}{6};\sin^2(e+fx)\right)}{7df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(23/12)*Hypergeometric2F1[7/6, 23/12, 13/6, Sin[e + f*x
]^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(7/3))/(7*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,dx = \frac{3\cos^2(e+fx)^{23/12}\,{}_2F_1\left(\frac{7}{6},\frac{23}{12};\frac{13}{6};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(}{7df}$$

Mathematica [A]
time = 0.16, size = 64, normalized size = 1.00

$$\frac{2d\,{}_2F_1\left(-\frac{1}{6},\frac{3}{4};\frac{7}{4};\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}}{3f\sqrt[6]{-\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[-1/6, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/
2)*(d*Tan[e + f*x])^(1/3))/(3*f*(-Tan[e + f*x]^2)^(1/6))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(f x+e\right)\right)^{\frac{3}{2}} \left(d \tan \left(f x+e\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

```
[Out] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*b*d*sec(f*x + e)*tan(f
*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*(d*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\tan(e+f\,x))^{4/3}\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(3/2),x)
[Out] int((d*tan(e + f*x))^(4/3)*(b/cos(e + f*x))^(3/2), x)
```

3.348 $\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{5}{3};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}}{4df}$$

```
[Out] 3/4*(cos(f*x+e)^2)^(17/12)*hypergeom([2/3, 17/12],[5/3],sin(f*x+e)^2)*(b*se
c(f*x+e))^(3/2)*(d*tan(f*x+e))^(4/3)/d/f
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{2697\}$

$$\frac{3\cos^2(e+fx)^{17/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{5}{3};\sin^2(e+fx)\right)}{4df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(17/12)*Hypergeometric2F1[2/3, 17/12, 5/3, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(4/3))/(4*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}\sqrt[3]{d\tan(e+fx)}\,dx = \frac{3\cos^2(e+fx)^{17/12}\,{}_2F_1\left(\frac{2}{3},\frac{17}{12};\frac{5}{3};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(}{4df}$$

Mathematica [A]
time = 0.09, size = 64, normalized size = 1.00

$$\frac{2d\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{7}{4};\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\sqrt[3]{-\tan^2(e+fx)}}{3f(d\tan(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[1/3, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2
)*(-Tan[e + f*x]^2)^(1/3))/(3*f*(d*Tan[e + f*x])^(2/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (b\sec(fx+e))^{\frac{3}{2}} (d\tan(fx+e))^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(1/3)*b*sec(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(e+fx))^{\frac{3}{2}} \sqrt[3]{d\tan(e+fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*(d*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral((b*sec(e + f*x))**(3/2)*(d*tan(e + f*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)*(d*tan(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\tan(e+f\,x))^{1/3}\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(3/2),x)
[Out] int((d*tan(e + f*x))^(1/3)*(b/cos(e + f*x))^(3/2), x)
```

3.349 $\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=64

$$\frac{3\cos^2(e+fx)^{13/12}\,{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{4}{3};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{2/3}}{2df}$$

```
[Out] 3/2*(cos(f*x+e)^2)^(13/12)*hypergeom([1/3, 13/12],[4/3],sin(f*x+e)^2)*(b*se
c(f*x+e))^(3/2)*(d*tan(f*x+e))^(2/3)/d/f
```

Rubi [A]
time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$\frac{3\cos^2(e+fx)^{13/12}(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{4}{3};\sin^2(e+fx)\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (3*(Cos[e + f*x]^2)^(13/12)*Hypergeometric2F1[1/3, 13/12, 4/3, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2)*(d*Tan[e + f*x])^(2/3))/(2*d*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(b\sec(e+fx))^{3/2}}{\sqrt[3]{d\tan(e+fx)}}\,dx = \frac{3\cos^2(e+fx)^{13/12}\,{}_2F_1\left(\frac{1}{3},\frac{13}{12};\frac{4}{3};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}(d\tan(e+fx))^{2/3}}{2df}$$

Mathematica [A]
time = 0.12, size = 64, normalized size = 1.00

$$\frac{2d\,{}_2F_1\left(\frac{2}{3},\frac{3}{4};\frac{7}{4};\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\left(-\tan^2(e+fx)\right)^{2/3}}{3f(d\tan(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(1/3),x]
```

```
[Out] (2*d*Hypergeometric2F1[2/3, 3/4, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2
)*(-Tan[e + f*x]^2)^(2/3))/(3*f*(d*Tan[e + f*x])^(4/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

```
[Out] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sec(f*x + e)/(d*tan(
f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(e+fx))^{\frac{3}{2}}}{\sqrt[3]{d\tan(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)/(d*tan(f*x+e))**(1/3),x)
[Out] Integral((b*sec(e + f*x))**(3/2)/(d*tan(e + f*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{{\left(d\,\mathrm{tan}\left(e+f\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3),x)
[Out] int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(1/3), x)
```

3.350 $\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=62

$$-\frac{3\cos^2(e+fx)^{7/12}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{5}{6};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}}{df\sqrt[3]{d\tan(e+fx)}}$$

```
[Out] -3*(cos(f*x+e)^2)^(7/12)*hypergeom([-1/6, 7/12],[5/6],sin(f*x+e)^2)*(b*sec(
f*x+e))^(3/2)/d/f/(d*tan(f*x+e))^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2697}

$$-\frac{3\cos^2(e+fx)^{7/12}(b\sec(e+fx))^{3/2}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{5}{6};\sin^2(e+fx)\right)}{df\sqrt[3]{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
```

```
[Out] (-3*(Cos[e + f*x]^2)^(7/12)*Hypergeometric2F1[-1/6, 7/12, 5/6, Sin[e + f*x]
^2]*(b*Sec[e + f*x])^(3/2))/(d*f*(d*Tan[e + f*x])^(1/3))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(b\sec(e+fx))^{3/2}}{(d\tan(e+fx))^{4/3}}\,dx = -\frac{3\cos^2(e+fx)^{7/12}\,{}_2F_1\left(-\frac{1}{6},\frac{7}{12};\frac{5}{6};\sin^2(e+fx)\right)(b\sec(e+fx))^{3/2}}{df\sqrt[3]{d\tan(e+fx)}}$$

Mathematica [A]
time = 0.22, size = 64, normalized size = 1.03

$$\frac{2d\,{}_2F_1\left(\frac{3}{4},\frac{7}{6};\frac{7}{4};\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}\left(-\tan^2(e+fx)\right)^{7/6}}{3f(d\tan(e+fx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)/(d*Tan[e + f*x])^(4/3),x]
[Out] (2*d*Hypergeometric2F1[3/4, 7/6, 7/4, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2
)*(-Tan[e + f*x]^2)^(7/6))/(3*f*(d*Tan[e + f*x])^(7/3))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(fx+e))^{\frac{3}{2}}}{(d\tan(fx+e))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
[Out] int((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*(d*tan(f*x + e))^(2/3)*b*sec(f*x + e)/(d^2*ta
n(f*x + e)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(e+fx))^{\frac{3}{2}}}{(d\tan(e+fx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)/(d*tan(f*x+e))**(4/3),x)
[Out] Integral((b*sec(e + f*x))**(3/2)/(d*tan(e + f*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)/(d*tan(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)/(d*tan(f*x + e))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(\frac{b}{\cos(e+f\,x)}\right)}^{3/2}}{{\left(d\,\tan\left(e+f\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3),x)
[Out] int((b/cos(e + f*x))^(3/2)/(d*tan(e + f*x))^(4/3), x)
```

3.351 $\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx$

Optimal. Leaf size=67

$$\frac{(b\sec(e+fx))^m}{fm}-\frac{2(b\sec(e+fx))^{2+m}}{b^2f(2+m)}+\frac{(b\sec(e+fx))^{4+m}}{b^4f(4+m)}$$

```
[Out] (b*sec(f*x+e))^m/f/m-2*(b*sec(f*x+e))^(2+m)/b^2/f/(2+m)+(b*sec(f*x+e))^(4+m
)/b^4/f/(4+m)
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2686, 276}

$$\frac{(b\sec(e+fx))^{m+4}}{b^4f(m+4)}-\frac{2(b\sec(e+fx))^{m+2}}{b^2f(m+2)}+\frac{(b\sec(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^5,x]
```

```
[Out] (b*Sec[e + f*x])^m/(f*m) - (2*(b*Sec[e + f*x])^(2 + m))/(b^2*f*(2 + m)) + (
b*Sec[e + f*x])^(4 + m)/(b^4*f*(4 + m))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\int (b\sec(e+fx))^m \tan^5(e+fx)\,dx &= \frac{b\text{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)^2\,dx,x,\sec(e+fx)\right)}{f}\\
&= \frac{b\text{Subst}\left(\int \left((bx)^{-1+m}-\frac{2(bx)^{1+m}}{b^2}+\frac{(bx)^{3+m}}{b^4}\right)dx,x,\sec(e+fx)\right)}{f}\\
&= \frac{(b\sec(e+fx))^m}{fm}-\frac{2(b\sec(e+fx))^{2+m}}{b^2f(2+m)}+\frac{(b\sec(e+fx))^{4+m}}{b^4f(4+m)}
\end{aligned}$$

Mathematica [A]
time = 0.46, size = 47, normalized size = 0.70

$$\frac{(b\sec(e+fx))^m\left(\frac{1}{m}-\frac{2\sec^2(e+fx)}{2+m}+\frac{\sec^4(e+fx)}{4+m}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^5,x]
```

```
[Out] ((b*Sec[e + f*x])^m*(m^(-1) - (2*Sec[e + f*x]^2)/(2 + m) + Sec[e + f*x]^4/(
4 + m)))/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.57, size = 6797, normalized size = 101.45

method	result	size
risch	Expression too large to display	6797

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e)^5,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.29, size = 82, normalized size = 1.22

$$\frac{\frac{b^m\cos(fx+e)^{-m}}{m}-\frac{2\,b^m\cos(fx+e)^{-m}}{(m+2)\cos(fx+e)^2}+\frac{b^m\cos(fx+e)^{-m}}{(m+4)\cos(fx+e)^4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] (b^m*cos(f*x + e)^(-m)/m - 2*b^m*cos(f*x + e)^(-m)/((m + 2)*cos(f*x + e)^2)
 + b^m*cos(f*x + e)^(-m)/((m + 4)*cos(f*x + e)^4))/f
```

Fricas [A]
time = 0.40, size = 84, normalized size = 1.25

$$\frac{\left((m^2+6\,m+8)\cos\left(fx+e\right)^4-2\,(m^2+4\,m)\cos\left(fx+e\right)^2+m^2+2\,m\right)\left(\frac{b}{\cos(fx+e)}\right)^m}{(fm^3+6\,fm^2+8\,fm)\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] ((m^2 + 6*m + 8)*cos(f*x + e)^4 - 2*(m^2 + 4*m)*cos(f*x + e)^2 + m^2 + 2*m)
*(b/cos(f*x + e))^m/((f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^4)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} x(b\sec(e))^m\tan^5(e) & \text{for } f=0\\ \frac{\int \frac{\tan^5(e+fx)}{\sec^4(e+fx)}\,dx}{b^4} & \text{for } m=-4\\ \frac{\int \frac{\tan^5(e+fx)}{\sec^2(e+fx)}\,dx}{b^2} & \text{for } m=-2\\ \frac{\log(\tan^2(e+fx)+1)}{2f}+\frac{\tan^4(e+fx)}{4f}-\frac{\tan^2(e+fx)}{2f} & \text{for } m=0\\ \frac{m^2(b\sec(e+fx))^m\tan^4(e+fx)}{fm^3+6fm^2+8fm}+\frac{2m(b\sec(e+fx))^m\tan^4(e+fx)}{fm^3+6fm^2+8fm}-\frac{4m(b\sec(e+fx))^m\tan^2(e+fx)}{fm^3+6fm^2+8fm}+\frac{8(b\sec(e+fx))^m}{fm^3+6fm^2+8fm} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**5,x)
```

```
[Out] Piecewise((x*(b*sec(e))**m*tan(e)**5, Eq(f, 0)), (Integral(tan(e + f*x)**5/
sec(e + f*x)**4, x)/b**4, Eq(m, -4)), (Integral(tan(e + f*x)**5/sec(e + f*x
)**2, x)/b**2, Eq(m, -2)), (log(tan(e + f*x)**2 + 1)/(2*f) + tan(e + f*x)**
4/(4*f) - tan(e + f*x)**2/(2*f), Eq(m, 0)), (m**2*(b*sec(e + f*x))**m*tan(e
 + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) + 2*m*(b*sec(e + f*x))**m*tan(e + f*
x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) - 4*m*(b*sec(e + f*x))**m*tan(e + f*x)**2
/(f*m**3 + 6*f*m**2 + 8*f*m) + 8*(b*sec(e + f*x))**m/(f*m**3 + 6*f*m**2 + 8
*f*m), True))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^5, x)
```

Mupad [B]

time = 7.81, size = 199, normalized size = 2.97

$$\frac{(\cos(4e+4fx)-\sin(4e+4fx)\,1\mathrm{i})\left(\frac{b}{\cos(e+fx)}\right)^m\left(\frac{2\cos(4e+4fx)(\cos(4e+4fx)+\sin(4e+4fx)\,1\mathrm{i})}{fm}+\frac{(\cos(4e+4fx)+\sin(4e+4fx)\,1\mathrm{i})(6m^2+4m+48)}{fm(m^2+6m+8)}-\frac{2\cos(2e+2fx)(\cos(4e+4fx)+\sin(4e+4fx)\,1\mathrm{i})(4m^2+8m-32)}{fm(m^2+6m+8)}\right)}{16\left(\frac{\cos(2e+2fx)}{2}+\frac{1}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(b/cos(e + f*x))^m,x)
```

```
[Out] ((cos(4*e + 4*f*x) - sin(4*e + 4*f*x)*1i)*(b/cos(e + f*x))^m*((2*cos(4*e +
4*f*x)*(cos(4*e + 4*f*x) + sin(4*e + 4*f*x)*1i))/(f*m) + ((cos(4*e + 4*f*x)
 + sin(4*e + 4*f*x)*1i)*(4*m + 6*m^2 + 48))/(f*m*(6*m + m^2 + 8)) - (2*cos(
2*e + 2*f*x)*(cos(4*e + 4*f*x) + sin(4*e + 4*f*x)*1i)*(8*m + 4*m^2 - 32))/(
f*m*(6*m + m^2 + 8))))/(16*(cos(2*e + 2*f*x)/2 + 1/2)^2)
```

3.352 $\int (b\sec(e+fx))^m \tan^3(e+fx)\,dx$

Optimal. Leaf size=43

$$-\frac{(b\sec(e+fx))^m}{fm}+\frac{(b\sec(e+fx))^{2+m}}{b^2f(2+m)}$$

```
[Out] -(b*sec(f*x+e))^m/f/m+(b*sec(f*x+e))^(2+m)/b^2/f/(2+m)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2686, 14}

$$\frac{(b\sec(e+fx))^{m+2}}{b^2f(m+2)}-\frac{(b\sec(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] -((b*Sec[e + f*x])^m/(f*m)) + (b*Sec[e + f*x])^(2 + m)/(b^2*f*(2 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^m\tan^3(e+fx)\,dx &= \frac{b\text{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{b\text{Subst}\left(\int \left(-(bx)^{-1+m}+\frac{(bx)^{1+m}}{b^2}\right)\,dx,x,\sec(e+fx)\right)}{f}\\ &= -\frac{(b\sec(e+fx))^m}{fm}+\frac{(b\sec(e+fx))^{2+m}}{b^2f(2+m)}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 34, normalized size = 0.79

$$\frac{(b\sec(e+fx))^m\left(-\frac{1}{m}+\frac{\sec^2(e+fx)}{2+m}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] ((b*Sec[e + f*x])^m*(-m^(-1) + Sec[e + f*x]^2/(2 + m)))/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.19, size = 2707, normalized size = 62.95

method	result	size
risch	Expression too large to display	2707

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/(2+m)/f/(exp(2*I*(f*x+e))+1)^2/m*(m/((exp(2*I*(f*x+e))+1)^m)*exp(I*(Re(f
*x)+Re(e)))^m*b^m*2^m*exp(-m*Im(f*x)-m*Im(e))*exp(-1/2*I*Pi*csgn(I*exp(I*(f
*x+e))/(exp(2*I*(f*x+e))+1))^3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2
*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+
e))/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*Pi*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(e
xp(2*I*(f*x+e))+1))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))
+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(
I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*
(f*x+e)))*csgn(I*b)*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f
*x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*c
sgn(I*b)*m)*exp(4*I*f*x)*exp(4*I*e)+2/((exp(2*I*(f*x+e))+1)^m)*exp(I*(Re(f*
x)+Re(e)))^m*b^m*2^m*exp(-m*Im(f*x)-m*Im(e))*exp(-1/2*I*Pi*csgn(I*exp(I*(f*
x+e))/(exp(2*I*(f*x+e))+1))^3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*
I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e
))/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*Pi*cs
gn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(ex
p(2*I*(f*x+e))+1))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+
1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(I
*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(
f*x+e)))*csgn(I*b)*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*
x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*cs
gn(I*b)*m)*exp(4*I*f*x)*exp(4*I*e)-2*m/((exp(2*I*(f*x+e))+1)^m)*exp(I*(Re(f
*x)+Re(e)))^m*b^m*2^m*exp(-m*Im(f*x)-m*Im(e))*exp(-1/2*I*Pi*csgn(I*exp(I*(f
```

```
*x+e))/(exp(2*I*(f*x+e))+1))^3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2
*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+
e))/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*Pi*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(e
xp(2*I*(f*x+e))+1))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))
+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(
I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*
(f*x+e)))*csgn(I*b)*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f
*x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*c
sgn(I*b)*m)*exp(2*I*f*x)*exp(2*I*e)+4/((exp(2*I*(f*x+e))+1)^m)*exp(I*(Re(f*
x)+Re(e)))^m*b^m*2^m*exp(-m*Im(f*x)-m*Im(e))*exp(-1/2*I*Pi*csgn(I*exp(I*(f*
x+e))/(exp(2*I*(f*x+e))+1))^3*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*
I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e
))/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))*m)*exp(-1/2*I*Pi*cs
gn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(ex
p(2*I*(f*x+e))+1))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+
1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(I
*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(
f*x+e)))*csgn(I*b)*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*
x+e)))^3*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*cs
gn(I*b)*m)*exp(2*I*f*x)*exp(2*I*e)+m/((exp(2*I*(f*x+e))+1)^m)*exp(I*(Re(f*x
)+Re(e)))^m*b^m*2^m*exp(-1/2*m*(I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e)
)+1))^3-I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*
x+e)))-I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(
f*x+e))+1))+I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(
f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1))-I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(
f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2+I*Pi*csgn(I*exp
(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+
e)))*csgn(I*b)+I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^3-I*Pi*cs
gn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2*csgn(I*b)+2*Im(f*x)+2*Im(e)))
+2/((exp(2*I*(f*x+e))+1)^m)*exp(I*(Re(f*x)+Re(e)))^m*b^m*2^m*exp(-1/2*m*(I*
Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3-I*Pi*csgn(I*exp(I*(f*x+e))
/(exp(2*I*(f*x+e))+1))^2*csgn(I*exp(I*(f*x+e)))-I*Pi*csgn(I*exp(I*(f*x+e))/
(exp(2*I*(f*x+e))+1))^2*csgn(I/(exp(2*I*(f*x+e))+1))+I*Pi*csgn(I*exp(I*(f*x
+e))/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+
1))-I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b/(exp(2*I*(f*x
+e))+1)*exp(I*(f*x+e)))^2+I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*
csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))*csgn(I*b)+I*Pi*csgn(I*b/(exp(
2*I*(f*x+e))+1)*exp(I*(f*x+e)))^3-I*Pi*csgn(I*b/(exp(2*I*(f*x+e))+1)*exp(I*
(f*x+e)))^2*csgn(I*b)+2*Im(f*x)+2*Im(e))))
```

Maxima [A]

time = 0.27, size = 54, normalized size = 1.26

$$-\frac{\frac{b^m \cos(fx+e)^{-m}}{m} - \frac{b^m \cos(fx+e)^{-m}}{(m+2)\cos(fx+e)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -(b^m*cos(f*x + e)^(-m)/m - b^m*cos(f*x + e)^(-m)/((m + 2)*cos(f*x + e)^2))
/f
```

Fricas [A]
time = 0.48, size = 53, normalized size = 1.23

$$-\frac{\left((m+2)\cos\left(fx+e\right)^2 - m\right)\left(\frac{b}{\cos(fx+e)}\right)^m}{\left(fm^2 + 2\,fm\right)\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -((m + 2)*cos(f*x + e)^2 - m)*(b/cos(f*x + e))^m/((f*m^2 + 2*f*m)*cos(f*x +
 e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} x(b\sec\left(e\right))^m\tan^3\left(e\right) & \text{for}\ f = 0 \\ \frac{\int \frac{\tan^3\left(e+fx\right)}{\sec^2\left(e+fx\right)}\,dx}{b^2} & \text{for}\ m = -2 \\ -\frac{\log\left(\tan^2\left(e+fx\right)+1\right)}{2f} + \frac{\tan^2\left(e+fx\right)}{2f} & \text{for}\ m = 0 \\ \frac{m(b\sec\left(e+fx\right))^m\tan^2\left(e+fx\right)}{fm^2+2fm} - \frac{2(b\sec\left(e+fx\right))^m}{fm^2+2fm} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**3,x)
```

```
[Out] Piecewise((x*(b*sec(e))**m*tan(e)**3, Eq(f, 0)), (Integral(tan(e + f*x)**3/
sec(e + f*x)**2, x)/b**2, Eq(m, -2)), (-log(tan(e + f*x)**2 + 1)/(2*f) + ta
n(e + f*x)**2/(2*f), Eq(m, 0)), (m*(b*sec(e + f*x))**m*tan(e + f*x)**2/(f*m
**2 + 2*f*m) - 2*(b*sec(e + f*x))**m/(f*m**2 + 2*f*m), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^3, x)
```

Mupad [B]
time = 3.39, size = 87, normalized size = 2.02

$$-\frac{\left(\frac{b}{\cos(e+f\,x)}\right)^m\,\left(8\,\cos\left(2\,e+2\,f\,x\right)-m+2\,\cos\left(4\,e+4\,f\,x\right)+m\,\cos\left(4\,e+4\,f\,x\right)+6\right)}{f\,m\,\left(m+2\right)\,\left(4\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(b/cos(e + f*x))^m,x)
```

```
[Out] -((b/cos(e + f*x))^m*(8*cos(2*e + 2*f*x) - m + 2*cos(4*e + 4*f*x) + m*cos(4
*e + 4*f*x) + 6))/(f*m*(m + 2)*(4*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) + 3))
```

3.353 $\int (b\sec(e+fx))^m \tan(e+fx)\,dx$

Optimal. Leaf size=17

$$\frac{(b\sec(e+fx))^m}{fm}$$

```
[Out] (b*sec(f*x+e))^m/f/m
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2686, 32}

$$\frac{(b\sec(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] (b*Sec[e + f*x])^m/(f*m)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^m \tan(e+fx)\,dx &= \frac{b\text{Subst}\left(\int (bx)^{-1+m}\,dx, x, \sec(e+fx)\right)}{f}\\ &= \frac{(b\sec(e+fx))^m}{fm}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 17, normalized size = 1.00

$$\frac{(b\sec(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] (b*Sec[e + f*x])^m/(f*m)
```

Maple [A]
time = 0.07, size = 18, normalized size = 1.06

method	result
derivativedivides	$\frac{(b\sec(fx+e))^m}{fm}$
default	$\frac{(b\sec(fx+e))^m}{fm}$
risch	$e^{\frac{m\left(-i\pi\,\mathrm{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^3+i\pi\,\mathrm{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^2\mathrm{csgn}\left(ie^{i(fx+e)}\right)+i\pi\,\mathrm{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^2\mathrm{csgn}\left(\frac{i}{e^{2i(fx+e)}+1}\right)-i\pi\,\mathrm{csgn}}{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e),x,method=_RETURNVERBOSE)
```

```
[Out] (b*sec(f*x+e))^m/f/m
```

Maxima [A]
time = 0.28, size = 21, normalized size = 1.24

$$\frac{b^m\cos(fx+e)^{-m}}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] b^m*cos(f*x + e)^(-m)/(f*m)
```

Fricas [A]
time = 0.37, size = 20, normalized size = 1.18

$$\frac{\left(\frac{b}{\cos(fx+e)}\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] (b/cos(f*x + e))^m/(f*m)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

time = 0.14, size = 42, normalized size = 2.47

$$\begin{cases} x\tan(e) & \text{for } f = 0 \wedge m = 0 \\ x(b\sec(e))^m \tan(e) & \text{for } f = 0 \\ \frac{\log(\tan^2(e+fx)+1)}{2f} & \text{for } m = 0 \\ \frac{(b\sec(e+fx))^m}{fm} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e),x)
```

```
[Out] Piecewise((x*tan(e), Eq(f, 0) & Eq(m, 0)), (x*(b*sec(e))**m*tan(e), Eq(f, 0
)), (log(tan(e + f*x)**2 + 1)/(2*f), Eq(m, 0)), ((b*sec(e + f*x))**m/(f*m),
 True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e), x)
```

Mupad [B]
time = 0.12, size = 19, normalized size = 1.12

$$\frac{\left(\frac{b}{\cos(e+f\,x)}\right)^m}{f\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(b/cos(e + f*x))^m,x)
```

```
[Out] (b/cos(e + f*x))^m/(f*m)
```

3.354 $\int \cot(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=40

$$-\frac{{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

```
[Out] -hypergeom([1, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2686, 371}

$$-\frac{(b\sec(e+fx))^m\,{}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};\sec^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(b*Sec[e + f*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)
/(f*m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)(b\sec(e+fx))^m\,dx &= \frac{b\text{Subst}\left(\int \frac{(bx)^{-1+m}}{-1+x^2}\,dx,x,\sec(e+fx)\right)}{f}\\ &= -\frac{{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(40) = 80.
time = 0.93, size = 124, normalized size = 3.10

$$\frac{b\sec^2\left(\frac{1}{2}(e+fx)\right)\left((1+\cos(e+fx))\,{}_2F_1(1,1-m;2-m;\cos(e+fx))-2^m\,{}_2F_1\left(1-m,1-m;2-m;\frac{1}{2}\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-m}\right)(b\sec(e+fx))^{-1+m}}{4f(-1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(b*Sec[e + f*x])^m,x]
```

```
[Out] (b*Sec[(e + f*x)/2]^2*((1 + Cos[e + f*x])*Hypergeometric2F1[1, 1 - m, 2 - m
, Cos[e + f*x]] - (2^m*Hypergeometric2F1[1 - m, 1 - m, 2 - m, (Cos[e + f*x]
*Sec[(e + f*x)/2]^2)/2])/(Sec[(e + f*x)/2]^2)^m)*(b*Sec[e + f*x])^(-1 + m))
/(4*f*(-1 + m))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \cot\left(fx+e\right)\left(b\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(b*sec(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)*(b*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec{(e+fx)})^m \cot{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))**m,x)
```

```
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e+f\,x)\,\left(\frac{b}{\cos{(e+f\,x)}}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(b/cos(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)*(b/cos(e + f*x))^m, x)
```

3.355 $\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=39

$$\frac{{}_2F_1\left(2,\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

```
[Out] hypergeom([2, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m
```

Rubi [A]

time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2686, 371}

$$\frac{(b\sec(e+fx))^m\,{}_2F_1\left(2,\frac{m}{2};\frac{m+2}{2};\sec^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(b*Sec[e + f*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)/(
f*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot^3(e+fx)(b\sec(e+fx))^m\,dx &= \frac{b\text{Subst}\left(\int \frac{(bx)^{-1+m}}{(-1+x^2)^2}\,dx,x,\sec(e+fx)\right)}{f}\\ &= \frac{{}_2F_1\left(2,\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 12.54, size = 815, normalized size = 20.90

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^3*(b*Sec[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Sec[(e + f*x)/2]^2*(-((1 + Cos[e + f*x])*Hypergeometric2F1[1,
 1 - m, 2 - m, Cos[e + f*x]]) + (2^m*Hypergeometric2F1[1 - m, 1 - m, 2 - m,
 (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2])/(Sec[(e + f*x)/2]^2)^m)*(b*Sec[e + f
*x])^m)/(4*f*(-1 + m)) - (2*Cot[(e + f*x)/2]*Cot[e + f*x]*Csc[e + f*x]^2*(A
ppellF1[1, m, -m, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)
/2]^4*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^m + Appel
lF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Csc[(e + f*x)/2]
^2)^m*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m)*(b*Sec[e + f*x])^m)/(f*(2*Appell
F1[1, m, -m, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^6
*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^(1 + m) + m*Ap
pellF1[2, m, 1 - m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*
x)/2]^8*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^(1 + m)
 + m*AppellF1[2, 1 + m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot
[(e + f*x)/2]^8*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)
^(1 + m) - 2*AppellF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*(Csc[(e + f*x)/2]^2)^(1 + m)*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m - m*Appel
lF1[2, m, 1 - m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Csc[(e + f*x)
/2]^2)^m*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(1 + m)*Sec[e + f*x] - m*AppellF
1[2, 1 + m, -m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Csc[(e + f*x)/
2]^2)^m*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(1 + m)*Sec[e + f*x]))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(f x +e \right)\right) \left(b \sec \left(f x +e \right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(b*sec(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^3*(b*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec{(e+fx)})^m\cot^3{(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(b*sec(f*x+e))**m,x)
```

```
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \mathrm{cot}(e+f\,x)^3\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(b/cos(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)^3*(b/cos(e + f*x))^m, x)
```

3.356 $\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=40

$$-\frac{{}_2F_1\left(3,\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}$$

[Out] -hypergeom([3, 1/2*m],[1+1/2*m],sec(f*x+e)^2)*(b*sec(f*x+e))^m/f/m

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2686, 371}

$$-\frac{(b\sec(e+fx))^m\,{}_2F_1\left(3,\frac{m}{2};\frac{m+2}{2};\sec^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]^5*(b*Sec[e + f*x])^m,x]

[Out] -((Hypergeometric2F1[3, m/2, (2 + m)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^m)/(f*m))

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot^5(e+fx)(b\sec(e+fx))^m\,dx &= \frac{b\text{Subst}\left(\int \frac{(bx)^{-1+m}}{(-1+x^2)^3}\,dx,x,\sec(e+fx)\right)}{f}\\ &= -\frac{{}_2F_1\left(3,\frac{m}{2};\frac{2+m}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^m}{fm}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 22.17, size = 2138, normalized size = 53.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^5*(b*Sec[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*Sec[(e + f*x)/2]^2*((1 + Cos[e + f*x])*Hypergeometric2F1[1, 1
 - m, 2 - m, Cos[e + f*x]] - (2^m*Hypergeometric2F1[1 - m, 1 - m, 2 - m, (C
os[e + f*x]*Sec[(e + f*x)/2]^2)/2])/(Sec[(e + f*x)/2]^2)^m)*(b*Sec[e + f*x]
)^m)/(4*f*(-1 + m)) + (3*Cot[(e + f*x)/2]*Cot[e + f*x]*Csc[e + f*x]^4*(4*Ap
pellF1[1, m, -m, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/
2]^6*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^m + Appell
F1[2, m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^8
*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^m + (AppellF1[
2, m, -m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*AppellF1[1, m, -m
, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^2)*(Csc[(e +
 f*x)/2]^2)^m*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m)*(b*Sec[e + f*x])^m)/(2*f
*(-6*AppellF1[1, m, -m, 2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e
+ f*x)/2]^8*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^(1
+ m) - 3*m*AppellF1[2, m, 1 - m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2
]*Cot[(e + f*x)/2]^10*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)
/2]^2)^(1 + m) - 3*AppellF1[2, m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)
/2]^2]*Cot[(e + f*x)/2]^10*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e +
 f*x)/2]^2)^(1 + m) - 3*m*AppellF1[2, 1 + m, -m, 3, Cot[(e + f*x)/2]^2, -Co
t[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^10*(-(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^
m*(Sec[(e + f*x)/2]^2)^(1 + m) - m*AppellF1[3, m, 1 - m, 4, Cot[(e + f*x)/2
]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^12*(-(Cos[e + f*x]*Csc[(e + f*x)
/2]^2))^m*(Sec[(e + f*x)/2]^2)^(1 + m) - m*AppellF1[3, 1 + m, -m, 4, Cot[(e
 + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^12*(-(Cos[e + f*x]*Csc[
(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^(1 + m) + 3*m*AppellF1[2, m, 1 - m,
 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Csc[(e + f*x)/2]^2)^(1 + m)*(
Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m + 3*AppellF1[2, m, -m, 3, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*(Csc[(e + f*x)/2]^2)^(1 + m)*(Cos[e + f*x]*Sec[(
e + f*x)/2]^2)^m + 3*m*AppellF1[2, 1 + m, -m, 3, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*(Csc[(e + f*x)/2]^2)^(1 + m)*(Cos[e + f*x]*Sec[(e + f*x)/2]^
2)^m + 6*AppellF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cot
[(e + f*x)/2]^2*(Csc[(e + f*x)/2]^2)^(1 + m)*(Cos[e + f*x]*Sec[(e + f*x)/2]
^2)^m + m*AppellF1[3, m, 1 - m, 4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*(Csc[(e + f*x)/2]^2)^m*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2
)^m + m*AppellF1[3, 1 + m, -m, 4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
(Csc[(e + f*x)/2]^2)^m*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)
^m)) + (4*Cot[(e + f*x)/2]*Cot[e + f*x]*Csc[e + f*x]^2*(AppellF1[1, m, -m,
```

```
2, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^4*(-(Cos[e + f
*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^m + AppellF1[1, m, -m, 2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Csc[(e + f*x)/2]^2)^m*(Cos[e + f*x
]*Sec[(e + f*x)/2]^2)^m)*(b*Sec[e + f*x])^m)/(f*(2*AppellF1[1, m, -m, 2, Co
t[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^6*(-(Cos[e + f*x]*C
sc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^(1 + m) + m*AppellF1[2, m, 1 - m
, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^8*(-(Cos[e +
 f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^(1 + m) + m*AppellF1[2, 1
 + m, -m, 3, Cot[(e + f*x)/2]^2, -Cot[(e + f*x)/2]^2]*Cot[(e + f*x)/2]^8*(-
(Cos[e + f*x]*Csc[(e + f*x)/2]^2))^m*(Sec[(e + f*x)/2]^2)^(1 + m) - 2*Appel
lF1[1, m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Csc[(e + f*x)/2]
^2)^(1 + m)*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m - m*AppellF1[2, m, 1 - m, 3
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Csc[(e + f*x)/2]^2)^m*(Cos[e +
f*x]*Sec[(e + f*x)/2]^2)^(1 + m)*Sec[e + f*x] - m*AppellF1[2, 1 + m, -m, 3,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Csc[(e + f*x)/2]^2)^m*(Cos[e + f
*x]*Sec[(e + f*x)/2]^2)^(1 + m)*Sec[e + f*x]))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\cot^{5}\left(fx+e\right)\right) \left(b\sec\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(b*sec(f*x+e))^m,x)
[Out] int(cot(f*x+e)^5*(b*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(e+fx))^m \cot^5(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(b*sec(f*x+e))**m,x)
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*sec(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e+f\,x)^5\left(\frac{b}{\cos(e+f\,x)}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(b/cos(e + f*x))^m,x)
[Out] int(cot(e + f*x)^5*(b/cos(e + f*x))^m, x)
```

3.357 $\int (b\sec(e+fx))^m \tan^4(e+fx)\,dx$

Optimal. Leaf size=63

$$\frac{\cos^2(e+fx)^{\frac{5+m}{2}}\,{}_2F_1\left(\frac{5}{2},\frac{5+m}{2};\frac{7}{2};\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^5(e+fx)}{5f}$$

```
[Out] 1/5*(cos(f*x+e)^2)^(5/2+1/2*m)*hypergeom([5/2, 5/2+1/2*m],[7/2],sin(f*x+e)^
2)*(b*sec(f*x+e))^m*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$\frac{\tan^5(e+fx)\cos^2(e+fx)^{\frac{m+5}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(\frac{5}{2},\frac{m+5}{2};\frac{7}{2};\sin^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] ((Cos[e + f*x]^2)^((5 + m)/2)*Hypergeometric2F1[5/2, (5 + m)/2, 7/2, Sin[e
+ f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^5)/(5*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^m\tan^4(e+fx)\,dx = \frac{\cos^2(e+fx)^{\frac{5+m}{2}}\,{}_2F_1\left(\frac{5}{2},\frac{5+m}{2};\frac{7}{2};\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^5(e}{5f}$$

Mathematica [A]
time = 0.30, size = 110, normalized size = 1.75

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(-1+m)}\left({}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3}{2};\sin^2(e+fx)\right)-2\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{3}{2};\sin^2(e+fx)\right)+{}_2F_1\left(\frac{1}{2},\frac{5+m}{2};\frac{3}{2};\sin^2(e+fx)\right)\right)(b\sec(e+fx))^m\sin(2(e+fx))}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^4,x]

[Out] ((Cos[e + f*x]^2)^((-1 + m)/2)*(Hypergeometric2F1[1/2, (1 + m)/2, 3/2, Sin[e + f*x]^2] - 2*Hypergeometric2F1[1/2, (3 + m)/2, 3/2, Sin[e + f*x]^2] + Hypergeometric2F1[1/2, (5 + m)/2, 3/2, Sin[e + f*x]^2])*(b*Sec[e + f*x])^m*Sin[2*(e + f*x)])/(2*f)

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(f x+e\right)\right)^{m} \left(\tan^{4}\left(f x+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(f*x+e))^m*tan(f*x+e)^4,x)

[Out] int((b*sec(f*x+e))^m*tan(f*x+e)^4,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^4, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")

[Out] integral((b*sec(f*x + e))^m*tan(f*x + e)^4, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(e+f x\right)\right)^{m} \tan^{4}\left(e+f x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**4,x)

[Out] Integral((b*sec(e + f*x))**m*tan(e + f*x)**4, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^4\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(b/cos(e + f*x))^m,x)
[Out] int(tan(e + f*x)^4*(b/cos(e + f*x))^m, x)
```

3.358 $\int (b\sec(e+fx))^m \tan^2(e+fx)\,dx$

Optimal. Leaf size=63

$$\frac{\cos^2(e+fx)^{\frac{3+m}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5}{2};\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^3(e+fx)}{3f}$$

```
[Out] 1/3*(cos(f*x+e)^2)^(3/2+1/2*m)*hypergeom([3/2, 3/2+1/2*m],[5/2],sin(f*x+e)^
2)*(b*sec(f*x+e))^m*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{2697\}$

$$\frac{\tan^3(e+fx)\cos^2(e+fx)^{\frac{m+3}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] ((Cos[e + f*x]^2)^((3 + m)/2)*Hypergeometric2F1[3/2, (3 + m)/2, 5/2, Sin[e
+ f*x]^2]*(b*Sec[e + f*x])^m*Tan[e + f*x]^3)/(3*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\sec(e+fx))^m\tan^2(e+fx)\,dx = \frac{\cos^2(e+fx)^{\frac{3+m}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5}{2};\sin^2(e+fx)\right)(b\sec(e+fx))^m\tan^3(e+fx)}{3f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 25.32, size = 6726, normalized size = 106.76

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (b\sec(fx+e))^m \left(\tan^2(fx+e)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^m*tan(f*x+e)^2,x)
```

```
[Out] int((b*sec(f*x+e))^m*tan(f*x+e)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(e+fx))^m \tan^2(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**m*tan(f*x+e)**2,x)
```

```
[Out] Integral((b*sec(e + f*x))**m*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^2\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(b/cos(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)^2*(b/cos(e + f*x))^m, x)
```

3.359 $\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=59

$$-\frac{\cos^2(e+fx)^{\frac{1}{2}(-1+m)}\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1}{2};\sin^2(e+fx)\right)(b\sec(e+fx))^m}{f}$$

```
[Out] -(cos(f*x+e)^2)^(-1/2+1/2*m)*cot(f*x+e)*hypergeom([-1/2, -1/2+1/2*m],[1/2],
sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$-\frac{\cot(e+fx)\cos^2(e+fx)^{\frac{m-1}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{1}{2};\sin^2(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(b*Sec[e + f*x])^m,x]
```

```
[Out] -(((Cos[e + f*x]^2)^((-1 + m)/2)*Cot[e + f*x]*Hypergeometric2F1[-1/2, (-1 +
 m)/2, 1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cot^2(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-1+m)}\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1}{2};\sin^2(e+f$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 21.60, size = 4872, normalized size = 82.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(b*Sec[e + f*x])^m,x]
```

```
[Out] (Cot[(e + f*x)/2]*Cot[e + f*x]^2*(b*Sec[e + f*x])^m*(Cos[(e + f*x)/2]^2*Sec
[e + f*x])^m*(-(AppellF1[-1/2, m, -m, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m) + 3*(Sec[(e + f*x)/2]^2)^m*Ta
n[(e + f*x)/2]^2*((-4*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m, 2 - m, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 1 - m,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + Appel
lF1[1/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1
/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*m*(AppellF1[3/
2, m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2,
1 + m, -m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2))))/(2*f*(-1/4*(Csc[(e + f*x)/2]^2*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*(-
(AppellF1[-1/2, m, -m, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e
 + f*x]*Sec[(e + f*x)/2]^2)^m) + 3*(Sec[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2]^
2*((-4*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + AppellF1[1/2, m, -m,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, m, -m, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*m*(AppellF1[3/2, m, 1 - m, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + m, -m, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))) + (Cot[(e
 + f*x)/2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*(-((Cos[e + f*x]*Sec[(e + f*
x)/2]^2)^m*(-(m*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - m*AppellF1[1/2, 1 + m, -m
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2])) - m*AppellF1[-1/2, m, -m, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + m)*(-(Sec[(e + f*x)/2]^2*Si
n[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + 3*(Sec[(e
 + f*x)/2]^2)^(1 + m)*Tan[(e + f*x)/2]*((-4*AppellF1[1/2, m, 1 - m, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2,
 m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*Appe
llF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*Appel
lF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2) + AppellF1[1/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]/(3*AppellF1[1/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + 2*m*(AppellF1[3/2, m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + AppellF1[3/2, 1 + m, -m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2)) + 3*m*(Sec[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2]^3*(
(-4*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*C
os[(e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan
```

```
[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + AppellF1[1/2, m, -m, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, m, -m, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*m*(AppellF1[3/2, m, 1 - m, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + m, -m, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + 3*(Sec[(e +
f*x)/2]^2)^m*Tan[(e + f*x)/2]^2*((4*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]*Sin[(e + f*x)/2])/(3*Appell
F1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 +
m)*AppellF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
)*Tan[(e + f*x)/2]^2) - (4*Cos[(e + f*x)/2]^2*(-1/3*((1 - m)*AppellF1[3/2,
m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2]) + (m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[
1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*
AppellF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*A
ppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2) + ((m*AppellF1[3/2, m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[3/
2, 1 + m, -m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2])/3)/(3*AppellF1[1/2, m, -m, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + 2*m*(AppellF1[3/2, m, 1 -...
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\cot^2 \left(fx+e\right)\right) \left(b \sec \left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(b*sec(f*x+e))^m,x)
[Out] int(cot(f*x+e)^2*(b*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (e + fx))^m \cot^2 (e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(b*sec(f*x+e))**m,x)
```

```
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e + f\,x)^2 \left(\frac{b}{\cos (e + f\,x)}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(b/cos(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)^2*(b/cos(e + f*x))^m, x)
```

3.360 $\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=63

$$-\frac{\cos^2(e+fx)^{\frac{1}{2}(-3+m)}\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);-\frac{1}{2};\sin^2(e+fx)\right)(b\sec(e+fx))^m}{3f}$$

```
[Out] -1/3*(cos(f*x+e)^2)^(-3/2+1/2*m)*cot(f*x+e)^3*hypergeom([-3/2, -3/2+1/2*m],
[-1/2],sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$-\frac{\cot^3(e+fx)\cos^2(e+fx)^{\frac{m-3}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};-\frac{1}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(b*Sec[e + f*x])^m,x]
```

```
[Out] -1/3*((Cos[e + f*x]^2)^((-3 + m)/2)*Cot[e + f*x]^3*Hypergeometric2F1[-3/2,
(-3 + m)/2, -1/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cot^4(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-3+m)}\cot^3(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);-\frac{1}{2};\sin^2(e+\right.}{3f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 26.70, size = 6532, normalized size = 103.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(b*Sec[e + f*x])^m,x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\cot^4\left(fx+e\right)\right)\left(b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(b*sec(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^4*(b*sec(f*x+e))^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^m\cot^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(b*sec(f*x+e))**m,x)
```

```
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e+f\,x)^4\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(b/cos(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)^4*(b/cos(e + f*x))^m, x)
```

3.361 $\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx$

Optimal. Leaf size=63

$$-\frac{\cos^2(e+fx)^{\frac{1}{2}(-5+m)}\cot^5(e+fx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);-\frac{3}{2};\sin^2(e+fx)\right)(b\sec(e+fx))^m}{5f}$$

```
[Out] -1/5*(cos(f*x+e)^2)^(-5/2+1/2*m)*cot(f*x+e)^5*hypergeom([-5/2, -5/2+1/2*m],
[-3/2],sin(f*x+e)^2)*(b*sec(f*x+e))^m/f
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$-\frac{\cot^5(e+fx)\cos^2(e+fx)^{\frac{m-5}{2}}(b\sec(e+fx))^m\,{}_2F_1\left(-\frac{5}{2},\frac{m-5}{2};-\frac{3}{2};\sin^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*(b*Sec[e + f*x])^m,x]
```

```
[Out] -1/5*((Cos[e + f*x]^2)^((-5 + m)/2)*Cot[e + f*x]^5*Hypergeometric2F1[-5/2,
(-5 + m)/2, -3/2, Sin[e + f*x]^2]*(b*Sec[e + f*x])^m)/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx = -\frac{\cos^2(e+fx)^{\frac{1}{2}(-5+m)}\cot^5(e+fx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);-\frac{3}{2};\sin^2(e+
}{5f}$$

Mathematica [F]
time = 0.69, size = 0, normalized size = 0.00

$$\int \cot^6(e+fx)(b\sec(e+fx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^6*(b*Sec[e + f*x])^m,x]
[Out] Integrate[Cot[e + f*x]^6*(b*Sec[e + f*x])^m, x]
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\cot^{6}\left(fx+e\right)\right)\left(b\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(b*sec(f*x+e))^m,x)
[Out] int(cot(f*x+e)^6*(b*sec(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^{m}\cot^{6}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(b*sec(f*x+e))**m,x)
[Out] Integral((b*sec(e + f*x))**m*cot(e + f*x)**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^m*cot(f*x + e)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e+f\,x)^6\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(b/cos(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)^6*(b/cos(e + f*x))^m, x)
```

3.362 $\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx$

Optimal. Leaf size=82

$$\frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+n)}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1+m+n);\frac{3+n}{2};\sin^2(e+fx)\right)(a\sec(e+fx))^m(b\tan(e+fx))^{1+n}}{bf(1+n)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*m+1/2*n)*hypergeom([1/2+1/2*n, 1/2+1/2*m+1/2*n],[3/
2+1/2*n],sin(f*x+e)^2)*(a*sec(f*x+e))^m*(b*tan(f*x+e))^(1+n)/b/f/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2697}

$$\frac{(a\sec(e+fx))^m(b\tan(e+fx))^{n+1}\cos^2(e+fx)^{\frac{1}{2}(m+n+1)}\,{}_2F_1\left(\frac{n+1}{2},\frac{1}{2}(m+n+1);\frac{n+3}{2};\sin^2(e+fx)\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + m + n)/2)*Hypergeometric2F1[(1 + n)/2, (1 + m + n)/
2, (3 + n)/2, Sin[e + f*x]^2]*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 + n))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (a\sec(e+fx))^m(b\tan(e+fx))^n\,dx=\frac{\cos^2(e+fx)^{\frac{1}{2}(1+m+n)}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1+m+n);\frac{3+n}{2};\sin^2(e+fx)\right)(}{bf(1+n)}$$

Mathematica [A]
time = 0.18, size = 80, normalized size = 0.98

$$\frac{b\,{}_2F_1\left(\frac{m}{2},\frac{1-n}{2};\frac{2+m}{2};\sec^2(e+fx)\right)(a\sec(e+fx))^m(b\tan(e+fx))^{-1+n}\left(-\tan^2(e+fx)\right)^{\frac{1-n}{2}}}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] (b*Hypergeometric2F1[m/2, (1 - n)/2, (2 + m)/2, Sec[e + f*x]^2]*(a*Sec[e +
f*x])^m*(b*Tan[e + f*x])^(-1 + n)*(-Tan[e + f*x]^2)^((1 - n)/2))/(f*m)
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int (a \sec (fx+e))^m (b \tan (fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec (e+fx))^m (b \tan (e+fx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(f*x+e))**m*(b*tan(f*x+e))**n,x)
```

```
[Out] Integral((a*sec(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sec(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*sec(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\tan(e+f\,x))^n\left(\frac{a}{\cos\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^n*(a/cos(e + f*x))^m,x)
[Out] int((b*tan(e + f*x))^n*(a/cos(e + f*x))^m, x)
```

3.363 $\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=74

$$\frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}+\frac{2(d\tan(a+bx))^{3+n}}{bd^3(3+n)}+\frac{(d\tan(a+bx))^{5+n}}{bd^5(5+n)}$$

```
[Out] (d*tan(b*x+a))^(1+n)/b/d/(1+n)+2*(d*tan(b*x+a))^(3+n)/b/d^3/(3+n)+(d*tan(b*
x+a))^(5+n)/b/d^5/(5+n)
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2687, 276}

$$\frac{(d\tan(a+bx))^{n+5}}{bd^5(n+5)}+\frac{2(d\tan(a+bx))^{n+3}}{bd^3(n+3)}+\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n)) + (2*(d*Tan[a + b*x])^(3 + n))/(b*d^
3*(3 + n)) + (d*Tan[a + b*x])^(5 + n)/(b*d^5*(5 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^6(a+bx)(d\tan(a+bx))^n\,dx &= \frac{\text{Subst}\left(\int (dx)^n\left(1+x^2\right)^2\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left((dx)^n+\frac{2(dx)^{2+n}}{d^2}+\frac{(dx)^{4+n}}{d^4}\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}+\frac{2(d\tan(a+bx))^{3+n}}{bd^3(3+n)}+\frac{(d\tan(a+bx))^{5+n}}{bd^5(5+n)}\end{aligned}$$

Mathematica [A]
time = 2.26, size = 101, normalized size = 1.36

$$\frac{d(d\tan(a+bx))^{-1+n}\left((8+6n+n^2+2(3+n)\cos(2(a+bx))+\cos(4(a+bx)))\sec^4(a+bx)\tan^2(a+bx)+8(-\tan^2(a+bx))^{\frac{1-n}{2}}\right)}{b(1+n)(3+n)(5+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*(d*Tan[a + b*x])^(-1 + n)*((8 + 6*n + n^2 + 2*(3 + n)*Cos[2*(a + b*x)] +
 Cos[4*(a + b*x)])*Sec[a + b*x]^4*Tan[a + b*x]^2 + 8*(-Tan[a + b*x]^2)^((1
- n)/2)))/(b*(1 + n)*(3 + n)*(5 + n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.26, size = 10923, normalized size = 147.61

method	result	size
risch	Expression too large to display	10923

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*(d*tan(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.28, size = 77, normalized size = 1.04

$$\frac{\frac{d^n\tan(bx+a)^n\tan(bx+a)^5}{n+5}+\frac{2\,d^n\tan(bx+a)^n\tan(bx+a)^3}{n+3}+\frac{(d\tan(bx+a))^{n+1}}{d(n+1)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (d^n*tan(b*x + a)^n*tan(b*x + a)^5/(n + 5) + 2*d^n*tan(b*x + a)^n*tan(b*x +
 a)^3/(n + 3) + (d*tan(b*x + a))^(n + 1)/(d*(n + 1)))/b
```

Fricas [A]
time = 0.38, size = 85, normalized size = 1.15

$$\frac{\left(8\,\cos\left(bx+a\right)^4+4\,(n+1)\cos\left(bx+a\right)^2+n^2+4\,n+3\right)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n\sin\left(bx+a\right)}{(bn^3+9\,bn^2+23\,bn+15\,b)\cos\left(bx+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] (8*cos(b*x + a)^4 + 4*(n + 1)*cos(b*x + a)^2 + n^2 + 4*n + 3)*(d*sin(b*x +
a)/cos(b*x + a))^n*sin(b*x + a)/((b*n^3 + 9*b*n^2 + 23*b*n + 15*b)*cos(b*x
+ a)^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan{(a+bx)})^n \sec^6{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**6, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Evaluation time: 0.44Unable to divide
, perhaps due to rounding error%%%{1,[0,1,4,0,0]%%%}+%%%{2,[0,1,2,2,0]%%%}+
%%%{1,
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(a+b\,x))^n}{\cos{(a+b\,x)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x)^6,x)
```

```
[Out] int((d*tan(a + b*x))^n/cos(a + b*x)^6, x)
```

3.364 $\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=49

$$\frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}+\frac{(d\tan(a+bx))^{3+n}}{bd^3(3+n)}$$

```
[Out] (d*tan(b*x+a))^(1+n)/b/d/(1+n)+(d*tan(b*x+a))^(3+n)/b/d^3/(3+n)
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2687, 14}

$$\frac{(d\tan(a+bx))^{n+3}}{bd^3(n+3)}+\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n)) + (d*Tan[a + b*x])^(3 + n)/(b*d^3*(3
 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(a+bx)(d\tan(a+bx))^n\,dx &= \frac{\text{Subst}\left(\int (dx)^n\left(1+x^2\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left((dx)^n+\frac{(dx)^{2+n}}{d^2}\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}+\frac{(d\tan(a+bx))^{3+n}}{bd^3(3+n)}\end{aligned}$$

Mathematica [A]
time = 1.26, size = 78, normalized size = 1.59

$$\frac{d(d\tan(a+bx))^{-1+n}\left((2+n+\cos(2(a+bx)))\sec^2(a+bx)\tan^2(a+bx)+2(-\tan^2(a+bx))^{\frac{1-n}{2}}\right)}{b(1+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*(d*Tan[a + b*x])^(-1 + n)*((2 + n + Cos[2*(a + b*x)])*Sec[a + b*x]^2*Tan
[a + b*x]^2 + 2*(-Tan[a + b*x]^2)^((1 - n)/2)))/(b*(1 + n)*(3 + n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.44, size = 5438, normalized size = 110.98

method	result	size
risch	Expression too large to display	5438

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*(d*tan(b*x+a))^n,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.28, size = 51, normalized size = 1.04

$$\frac{\frac{d^n\tan(bx+a)^n\tan(bx+a)^3}{n+3}+\frac{(d\tan(bx+a))^{n+1}}{d(n+1)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] (d^n*tan(b*x + a)^n*tan(b*x + a)^3/(n + 3) + (d*tan(b*x + a))^(n + 1)/(d*(n
 + 1)))/b
```

Fricas [A]
time = 0.38, size = 61, normalized size = 1.24

$$\frac{\left(2\cos\left(bx+a\right)^2+n+1\right)\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n\sin\left(bx+a\right)}{\left(bn^2+4\,bn+3\,b\right)\cos\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] (2*cos(b*x + a)^2 + n + 1)*(d*sin(b*x + a)/cos(b*x + a))^n*sin(b*x + a)/((b
*n^2 + 4*b*n + 3*b)*cos(b*x + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan (a+bx))^n \sec^4 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**4, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,2,0,0]%%%}+%%%{1,[0,1,0,2,0]%%%} / %%%{1,[0,0,3,0,1]%%
%} Err
```

Mupad [B]
time = 3.84, size = 139, normalized size = 2.84

$$-\frac{2\left(-\frac{d\sin (2\,a+2\,b\,x)}{2\sin (a+b\,x)^2-2}\right)^n\left(9\,\sin \left(2\,a+2\,b\,x\right)+6\,\sin \left(4\,a+4\,b\,x\right)+\sin \left(6\,a+6\,b\,x\right)+4\,n\,\sin \left(2\,a+2\,b\,x\right)+2\,n\,\sin \left(4\,a+4\,b\,x\right)\right)}{b\,\left(n^2+4\,n+3\right)\,\left(30\sin \left(a+b\,x\right)^2+12\sin \left(2\,a+2\,b\,x\right)^2+2\sin \left(3\,a+3\,b\,x\right)^2-32\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x)^4,x)
```

```
[Out] -(2*(-(d*sin(2*a + 2*b*x))/(2*sin(a + b*x)^2 - 2))^n*(9*sin(2*a + 2*b*x) +
6*sin(4*a + 4*b*x) + sin(6*a + 6*b*x) + 4*n*sin(2*a + 2*b*x) + 2*n*sin(4*a
+ 4*b*x)))/(b*(4*n + n^2 + 3)*(12*sin(2*a + 2*b*x)^2 + 2*sin(3*a + 3*b*x)^2
 + 30*sin(a + b*x)^2 - 32))
```

3.365 $\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=24

$$\frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] (d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2687, 32}

$$\frac{(d\tan(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Tan[a + b*x])^(1 + n)/(b*d*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)(d\tan(a+bx))^n\,dx &= \frac{\text{Subst}\left(\int (dx)^n\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{(d\tan(a+bx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.04

$$\frac{\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b*x]^2*(d*Tan[a + b*x])^n,x]

[Out] (Tan[a + b*x]*(d*Tan[a + b*x])^n)/(b*(1 + n))

Maple [A]
time = 0.15, size = 25, normalized size = 1.04

method	result	size
derivativedivides	$\frac{(d\tan(bx+a))^{1+n}}{bd(1+n)}$	25
default	$\frac{(d\tan(bx+a))^{1+n}}{bd(1+n)}$	25
risch	Expression too large to display	1783

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)^2*(d*tan(b*x+a))^n,x,method=_RETURNVERBOSE)

[Out] (d*tan(b*x+a))^(1+n)/b/d/(1+n)

Maxima [A]
time = 0.29, size = 24, normalized size = 1.00

$$\frac{(d\tan(bx+a))^{n+1}}{bd(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="maxima")

[Out] (d*tan(b*x + a))^(n + 1)/(b*d*(n + 1))

Fricas [A]
time = 0.39, size = 40, normalized size = 1.67

$$\frac{\left(\frac{d\sin(bx+a)}{\cos(bx+a)}\right)^n\sin(bx+a)}{(bn+b)\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="fricas")

[Out] (d*sin(b*x + a)/cos(b*x + a))^n*sin(b*x + a)/((b*n + b)*cos(b*x + a))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(d\tan(a+bx))^n\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Bad Argument Val
ue
```

Mupad [B]
time = 2.64, size = 49, normalized size = 2.04

$$\frac{\sin\left(2\,a+2\,b\,x\right)\,{\left(\frac{d\,\sin\left(2\,a+2\,b\,x\right)}{2\,{\cos\left(a+b\,x\right)}^2}\right)}^n}{2\,b\,{\cos\left(a+b\,x\right)}^2\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x)^2,x)
```

```
[Out] (sin(2*a + 2*b*x)*((d*sin(2*a + 2*b*x))/(2*cos(a + b*x)^2))^n)/(2*b*cos(a +
 b*x)^2*(n + 1))
```

3.366 $\int (d\tan(a+bx))^n \, dx$

Optimal. Leaf size=50

$$\frac{{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/
d/(1+n)
```

Rubi [A]

time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3557, 371}

$$\frac{(d\tan(a+bx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Tan[a + b*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (d\tan(a+bx))^n \, dx &= \frac{d\text{Subst}\left(\int \frac{x^n}{d^2+x^2}\,dx, x, d\tan(a+bx)\right)}{b}\\ &= \frac{{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 53, normalized size = 1.06

$$\frac{{}_2F_1\left(1,\frac{1+n}{2};1+\frac{1+n}{2};-\tan^2(a+bx)\right)\tan(a+bx)(d\tan(a+bx))^n}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Tan[a + b*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/2, -Tan[a + b*x]^2]*Tan[a + b*
x]*(d*Tan[a + b*x])^n)/(b*(1 + n))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (d\tan(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(b*x+a))^n,x)
```

```
[Out] int((d*tan(b*x+a))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(a+bx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (d\tan(a+b\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^n,x)
```

```
[Out] int((d*tan(a + b*x))^n, x)
```

3.367 $\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=50

$$\frac{{}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] hypergeom([2, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/
d/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2687, 371}

$$\frac{(d\tan(a+bx))^{n+1}\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)(d\tan(a+bx))^n\,dx &= \frac{\text{Subst}\left(\int \frac{(dx)^n}{(1+x^2)^2}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{{}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 4.55, size = 939, normalized size = 18.78

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^2*(d*Tan[a + b*x])^n,x]
```

```
[Out] (2*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)
/2]^2] - 4*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a
 + b*x)/2]^2] + 4*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/2]^2,
-Tan[(a + b*x)/2]^2])*Cos[a + b*x]^2*Tan[(a + b*x)/2]*(d*Tan[a + b*x])^n)/(
b*((AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)
/2]^2] - 4*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a
 + b*x)/2]^2] + 4*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/2]^2,
-Tan[(a + b*x)/2]^2])*Sec[(a + b*x)/2]^2 + n*(AppellF1[(1 + n)/2, n, 1, (3
+ n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(1 + n)/2, n,
 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 4*AppellF1[(1 + n
)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sec[(a + b*
x)/2]^2*Sec[a + b*x] + (2*(1 + n)*(-AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Ta
n[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(3 + n)/2, n, 3, (5 + n
)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*AppellF1[(3 + n)/2, n, 4
, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + n*AppellF1[(3 + n)/
2, 1 + n, 1, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*n*Appe
llF1[(3 + n)/2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^
2] + 4*n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[
(a + b*x)/2]^2])*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2]^2)/(3 + n) - 2*n*(Appe
llF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] -
 4*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/
2]^2] + 4*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a
+ b*x)/2]^2])*Sec[a + b*x]*Tan[(a + b*x)/2]^2))
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(bx+a\right)\right)\left(d\tan\left(bx+a\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

```
[Out] int(cos(b*x+a)^2*(d*tan(b*x+a))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (d\tan(a+bx))^n\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*cos(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos(a+b\,x)^2\,(d\tan(a+b\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(d*tan(a + b*x))^n,x)
```

```
[Out] int(cos(a + b*x)^2*(d*tan(a + b*x))^n, x)
```

3.368 $\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=50

$$\frac{{}_2F_1\left(3,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] hypergeom([3, 1/2+1/2*n],[3/2+1/2*n],-tan(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/
d/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2687, 371}

$$\frac{(d\tan(a+bx))^{n+1}\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};-\tan^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, -Tan[a + b*x]^2]*(d*Tan[a + b*x
])^(1 + n))/(b*d*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \cos^4(a+bx)(d\tan(a+bx))^n\,dx &= \frac{\text{Subst}\left(\int \frac{(dx)^n}{(1+x^2)^3}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{{}_2F_1\left(3,\frac{1+n}{2};\frac{3+n}{2};-\tan^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 13.08, size = 1712, normalized size = 34.24

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^4*(d*Tan[a + b*x])^n,x]
```

```
[Out] (-8*(3 + n)*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[
(a + b*x)/2]^2] - 8*(AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2] - 3*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2] + 4*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan
[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*AppellF1[(1 + n)/2, n, 5, (3 + n)
/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]))*Cos[(a + b*x)/2]^3*Cos[a + b
*x]^5*Sin[(a + b*x)/2]^2*(d*Tan[a + b*x])^n)/(b*(1 + n)*((3 + n)*AppellF1[(
1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Co
s[a + b*x]) + 2*(16*AppellF1[(3 + n)/2, n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2
, -Tan[(a + b*x)/2]^2] - 72*AppellF1[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2] + 128*AppellF1[(3 + n)/2, n, 5, (5 + n)/2, T
an[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 80*AppellF1[(3 + n)/2, n, 6, (5 +
 n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + n*AppellF1[(3 + n)/2, 1 +
 n, 1, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 8*n*AppellF1[(
3 + n)/2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2
4*n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2] - 32*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] + 16*n*AppellF1[(3 + n)/2, 1 + n, 5, (5 + n)/2, Ta
n[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 24*AppellF1[(1 + n)/2, n, 2, (3 +
n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 - 8*n*App
ellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*
Cos[(a + b*x)/2]^2 + 72*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/
2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 24*n*AppellF1[(1 + n)/2, n,
 3, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2
- 96*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x
)/2]^2]*Cos[(a + b*x)/2]^2 - 32*n*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[
(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + AppellF1[(3 + n)/
2, n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(-1 + Cos[a +
b*x]) - 16*AppellF1[(3 + n)/2, n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a
 + b*x)/2]^2]*Cos[a + b*x] + 72*AppellF1[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 128*AppellF1[(3 + n)/2, n
, 5, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] + 80*
AppellF1[(3 + n)/2, n, 6, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^
2]*Cos[a + b*x] - n*AppellF1[(3 + n)/2, 1 + n, 1, (5 + n)/2, Tan[(a + b*x)/
2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] + 8*n*AppellF1[(3 + n)/2, 1 + n, 2,
 (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 24*n*Ap
```

```
pellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2
]^2]*Cos[a + b*x] + 32*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n)/2, Tan[(a +
b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 16*n*AppellF1[(3 + n)/2, 1 +
 n, 5, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] + 8
*(3 + n)*AppellF1[(1 + n)/2, n, 5, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
 b*x)/2]^2]*(1 + Cos[a + b*x]))*(Sin[(a + b*x)/2] - Sin[(3*(a + b*x))/2]))
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int \left(\cos^{4}\left(b x +a \right)\right) \left(d \tan \left(b x +a \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*(d*tan(b*x+a))^n,x)
[Out] int(cos(b*x+a)^4*(d*tan(b*x+a))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \tan{\left(a + b x \right)}\right)^{n} \cos^{4}{\left(a + b x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*cos(a + b*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(a+b\,x\right)^4\,{\left(d\,\tan\left(a+b\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4*(d*tan(a + b*x))^n,x)
[Out] int(cos(a + b*x)^4*(d*tan(a + b*x))^n, x)
```

3.369 $\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=78

$$\frac{\cos^2(a+bx)^{\frac{6+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{6+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)\sec^5(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] (cos(b*x+a)^2)^(3+1/2*n)*hypergeom([3+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*x
+a)^2)*sec(b*x+a)^5*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$\frac{\sec^5(a+bx)\cos^2(a+bx)^{\frac{n+6}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+6}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*(d*Tan[a + b*x])^n,x]
```

```
[Out] ((Cos[a + b*x]^2)^((6 + n)/2)*Hypergeometric2F1[(1 + n)/2, (6 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^5*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n)
)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sec^5(a+bx)(d\tan(a+bx))^n\,dx=\frac{\cos^2(a+bx)^{\frac{6+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{6+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)\sec^5(a+bx)(d\tan(a}{bd(1+n)}$$

Mathematica [A]
time = 0.13, size = 72, normalized size = 0.92

$$\frac{d\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{7}{2};\sec^2(a+bx)\right)\sec^5(a+bx)(d\tan(a+bx))^{-1+n}\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Hypergeometric2F1[5/2, (1 - n)/2, 7/2, Sec[a + b*x]^2]*Sec[a + b*x]^5*(d
*Tan[a + b*x])^(-1 + n)*(-Tan[a + b*x]^2)^((1 - n)/2))/(5*b)
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \left(\sec^{5}\left(bx+a\right)\right)\left(d\tan\left(bx+a\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*(d*tan(b*x+a))^n,x)
```

```
[Out] int(sec(b*x+a)^5*(d*tan(b*x+a))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(a+bx\right)\right)^{n}\sec^{5}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(a+b\,x))^n}{\cos\left(a+b\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x)^5,x)
[Out] int((d*tan(a + b*x))^n/cos(a + b*x)^5, x)
```

3.370 $\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=78

$$\frac{\cos^2(a+bx)^{\frac{4+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{4+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)\sec^3(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] (cos(b*x+a)^2)^(2+1/2*n)*hypergeom([2+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*x
+a)^2)*sec(b*x+a)^3*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{2697\}$

$$\frac{\sec^3(a+bx)\cos^2(a+bx)^{\frac{n+4}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+4}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

```
[Out] ((Cos[a + b*x]^2)^((4 + n)/2)*Hypergeometric2F1[(1 + n)/2, (4 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]^3*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n)
)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sec^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{4+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{4+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)\sec^3(a+bx)(d\tan(a+}{bd(1+n)}$$

Mathematica [A]
time = 0.10, size = 72, normalized size = 0.92

$$\frac{d\,{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{5}{2};\sec^2(a+bx)\right)\sec^3(a+bx)(d\tan(a+bx))^{-1+n}\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

```
[Out] (d*Hypergeometric2F1[3/2, (1 - n)/2, 5/2, Sec[a + b*x]^2]*Sec[a + b*x]^3*(d
*Tan[a + b*x])^(-1 + n)*(-Tan[a + b*x]^2)^((1 - n)/2))/(3*b)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}\left(bx+a\right)\right)\left(d\tan\left(bx+a\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

```
[Out] int(sec(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\tan\left(a+bx\right)\right)^{n}\sec^{3}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(a+b\,x))^n}{\cos\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x)^3,x)
[Out] int((d*tan(a + b*x))^n/cos(a + b*x)^3, x)
```

3.371 $\int \sec(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=76

$$\frac{\cos^2(a+bx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)\sec(a+bx)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] (cos(b*x+a)^2)^(1+1/2*n)*hypergeom([1+1/2*n, 1/2+1/2*n],[3/2+1/2*n],sin(b*x
+a)^2)*sec(b*x+a)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2697}

$$\frac{\sec(a+bx)\cos^2(a+bx)^{\frac{n+2}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*(d*Tan[a + b*x])^n,x]
```

```
[Out] ((Cos[a + b*x]^2)^((2 + n)/2)*Hypergeometric2F1[(1 + n)/2, (2 + n)/2, (3 +
n)/2, Sin[a + b*x]^2]*Sec[a + b*x]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \sec(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^2(a+bx)^{\frac{2+n}{2}}\,{}_2F_1\left(\frac{1+n}{2},\frac{2+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)\sec(a+bx)(d\tan(a+$$
$$bd(1+n)$$

Mathematica [A]
time = 0.08, size = 64, normalized size = 0.84

$$\frac{\csc(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3}{2};\sec^2(a+bx)\right)(d\tan(a+bx))^n\left(-\tan^2(a+bx)\right)^{\frac{1-n}{2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Csc[a + b*x]*Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Sec[a + b*x]^2]*(d*Tan
[a + b*x])^n*(-Tan[a + b*x]^2)^((1 - n)/2))/b
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \sec \left(bx+a\right) \left(d \tan \left(bx+a\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*(d*tan(b*x+a))^n,x)
```

```
[Out] int(sec(b*x+a)*(d*tan(b*x+a))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*sec(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \tan \left(a+bx\right)\right)^{n} \sec \left(a+bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*sec(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*sec(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\tan(a+b\,x))^n}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(a + b*x))^n/cos(a + b*x),x)
[Out] int((d*tan(a + b*x))^n/cos(a + b*x), x)
```

3.372 $\int \cos(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=72

$$\frac{\cos(a+bx)\cos^2(a+bx)^{n/2}\,{}_2F_1\left(\frac{n}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] cos(b*x+a)*(cos(b*x+a)^2)^(1/2*n)*hypergeom([1/2*n, 1/2+1/2*n],[3/2+1/2*n],
sin(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2697\}$

$$\frac{\cos(a+bx)\cos^2(a+bx)^{n/2}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Cos[a + b*x]*(Cos[a + b*x]^2)^(n/2)*Hypergeometric2F1[n/2, (1 + n)/2, (3 +
 n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 + n))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cos(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos(a+bx)\cos^2(a+bx)^{n/2}\,{}_2F_1\left(\frac{n}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 2.54, size = 452, normalized size = 6.28

$$-\frac{2\left(F_1\left(\frac{1+n}{2};n,1;\frac{3+n}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-2F_1\left(\frac{1+n}{2};n,2;\frac{3+n}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\right)\cos\left(\frac{1}{2}(a+bx)\right)\cos(a+bx)\sin\left(\frac{1}{2}(a+bx)\right)(d\tan(a+bx))^n}{b(1+n)\left(-F_1\left(\frac{1+n}{2};n,1;\frac{3+n}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+\frac{\left(-\left(\left(F_1\left(\frac{3+n}{2};n,2;\frac{5+n}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-4F_1\left(\frac{3+n}{2};n,3;\frac{5+n}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-nF_1\left(\frac{3+n}{2};1+n,1;\frac{5+n}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2nF_1\left(\frac{3+n}{2};1+n,2;\frac{5+n}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\right)(-1+\cos(a+bx))\right)+(3+n)F_1\left(\frac{1+n}{2};n,2;\frac{3+n}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)(1+\cos(a+bx))\right)\sec^2\left(\frac{1}{2}(a+bx)\right)}{3+n}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]*(d*Tan[a + b*x])^n,x]
```

```
[Out] (-2*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x
)/2]^2] - 2*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2])*Cos[(a + b*x)/2]*Cos[a + b*x]*Sin[(a + b*x)/2]*(d*Tan[a + b
*x])^n)/(b*(1 + n)*(-AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2] + ((-((AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(a
+ b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(3 + n)/2, n, 3, (5 + n)/2,
Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n, 1,
(5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*n*AppellF1[(3 + n)/
2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos
[a + b*x])) + (3 + n)*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))*Sec[(a + b*x)/2]^2)/(3 + n)))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \cos \left(b x +a \right) \left(d \tan \left(b x +a \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*(d*tan(b*x+a))^n,x)
```

```
[Out] int(cos(b*x+a)*(d*tan(b*x+a))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \tan {\left(a + b x \right)}\right)^{n} \cos {\left(a + b x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))**n,x)
```

```
[Out] Integral((d*tan(a + b*x))**n*cos(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(d*tan(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,{\left(d\,\tan(a+b\,x)\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(d*tan(a + b*x))^n,x)
```

```
[Out] int(cos(a + b*x)*(d*tan(a + b*x))^n, x)
```

3.373 $\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx$

Optimal. Leaf size=78

$$\frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(\frac{1}{2}(-2+n),\frac{1+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)(d\tan(a+bx))^{1+n}}{bd(1+n)}$$

```
[Out] cos(b*x+a)^3*(cos(b*x+a)^2)^(-1+1/2*n)*hypergeom([-1+1/2*n, 1/2+1/2*n],[3/2
+1/2*n],sin(b*x+a)^2)*(d*tan(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$\frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{n-2}{2}}(d\tan(a+bx))^{n+1}\,{}_2F_1\left(\frac{n-2}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

```
[Out] (Cos[a + b*x]^3*(Cos[a + b*x]^2)^((-2 + n)/2)*Hypergeometric2F1[(-2 + n)/2,
 (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*(d*Tan[a + b*x])^(1 + n))/(b*d*(1 +
n))
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cos^3(a+bx)(d\tan(a+bx))^n\,dx = \frac{\cos^3(a+bx)\cos^2(a+bx)^{\frac{1}{2}(-2+n)}\,{}_2F_1\left(\frac{1}{2}(-2+n),\frac{1+n}{2};\frac{3+n}{2};\sin^2(a+bx\right.}{bd(1+n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 8.35, size = 1340, normalized size = 17.18

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^3*(d*Tan[a + b*x])^n,x]
```

```
[Out] (8*(3 + n)*(AppellF1[(1 + n)/2, n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2] - 6*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2,
 -Tan[(a + b*x)/2]^2] + 12*AppellF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*
x)/2]^2, -Tan[(a + b*x)/2]^2] - 8*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[
(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^3*Cos[a + b*x]^4*Sin
[(a + b*x)/2]^2*(d*Tan[a + b*x])^n)/(b*(1 + n)*((3 + n)*AppellF1[(1 + n)/2,
 n, 1, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x
]) - 2*(AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2] - 12*AppellF1[(3 + n)/2, n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] + 36*AppellF1[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a + b*x)/
2]^2, -Tan[(a + b*x)/2]^2] - 32*AppellF1[(3 + n)/2, n, 5, (5 + n)/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - n*AppellF1[(3 + n)/2, 1 + n, 1, (5 + n
)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 6*n*AppellF1[(3 + n)/2, 1 +
 n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*n*AppellF1[
(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] +
8*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2] + 18*AppellF1[(1 + n)/2, n, 2, (3 + n)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 6*n*AppellF1[(1 + n)/2, n, 2, (3 +
n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 8*(3 +
n)*AppellF1[(1 + n)/2, n, 4, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/
2]^2]*Cos[(a + b*x)/2]^2 - AppellF1[(3 + n)/2, n, 2, (5 + n)/2, Tan[(a + b*
x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] + 12*AppellF1[(3 + n)/2, n, 3, (
5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 36*Appell
F1[(3 + n)/2, n, 4, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos
[a + b*x] + 32*AppellF1[(3 + n)/2, n, 5, (5 + n)/2, Tan[(a + b*x)/2]^2, -Ta
n[(a + b*x)/2]^2]*Cos[a + b*x] + n*AppellF1[(3 + n)/2, 1 + n, 1, (5 + n)/2,
 Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 6*n*AppellF1[(3 +
n)/2, 1 + n, 2, (5 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a +
 b*x] + 12*n*AppellF1[(3 + n)/2, 1 + n, 3, (5 + n)/2, Tan[(a + b*x)/2]^2, -
Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 8*n*AppellF1[(3 + n)/2, 1 + n, 4, (5 + n
)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[a + b*x] - 6*(3 + n)*Appe
llF1[(1 + n)/2, n, 3, (3 + n)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(
1 + Cos[a + b*x])))*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))
```

Maple [F]

time = 0.30, size = 0, normalized size = 0.00

$$\int \left(\cos^3\left(bx+a\right)\right)\left(d\tan\left(bx+a\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

```
[Out] int(cos(b*x+a)^3*(d*tan(b*x+a))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="maxima")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="fricas")
[Out] integral((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*(d*tan(b*x+a))**n,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(d*tan(b*x+a))^n,x, algorithm="giac")
[Out] integrate((d*tan(b*x + a))^n*cos(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)^3\,(d\tan(a+b\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(d*tan(a + b*x))^n,x)
[Out] int(cos(a + b*x)^3*(d*tan(a + b*x))^n, x)
```

3.374 $\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx$

Optimal. Leaf size=40

$$-\frac{(b\csc(e+fx))^m\,{}_2F_1\left(2,\frac{m}{2};\frac{2+m}{2};\csc^2(e+fx)\right)}{fm}$$

```
[Out] -(b*csc(f*x+e))^m*hypergeom([2, 1/2*m],[1+1/2*m],csc(f*x+e)^2)/f/m
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2686, 371}

$$-\frac{(b\csc(e+fx))^m\,{}_2F_1\left(2,\frac{m}{2};\frac{m+2}{2};\csc^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] -(((b*Csc[e + f*x])^m*Hypergeometric2F1[2, m/2, (2 + m)/2, Csc[e + f*x]^2])
/(f*m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx &= -\frac{b\text{Subst}\left(\int \frac{(bx)^{-1+m}}{(-1+x^2)^2}\,dx,x,\csc(e+fx)\right)}{f}\\ &= -\frac{(b\csc(e+fx))^m\,{}_2F_1\left(2,\frac{m}{2};\frac{2+m}{2};\csc^2(e+fx)\right)}{fm}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 52, normalized size = 1.30

$$-\frac{(b\csc(e+fx))^m\,{}_2F_1\left(2,2-\frac{m}{2};3-\frac{m}{2};\sin^2(e+fx)\right)\sin^4(e+fx)}{f(-4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] -(((b*Csc[e + f*x])^m*Hypergeometric2F1[2, 2 - m/2, 3 - m/2, Sin[e + f*x]^2
]*Sin[e + f*x]^4)/(f*(-4 + m)))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^m \left(\tan^3(fx+e)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e)^3,x)
```

```
[Out] int((b*csc(f*x+e))^m*tan(f*x+e)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^m \tan^3(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e)**3,x)
```

```
[Out] Integral((b*csc(e + f*x))**m*tan(e + f*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^3\,{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(b/sin(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)^3*(b/sin(e + f*x))^m, x)
```

3.375 $\int (b\csc(e+fx))^m \tan(e+fx)\,dx$

Optimal. Leaf size=39

$$\frac{(b\csc(e+fx))^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};\csc^2(e+fx)\right)}{fm}$$

```
[Out] (b*csc(f*x+e))^m*hypergeom([1, 1/2*m],[1+1/2*m],csc(f*x+e)^2)/f/m
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2686, 371}

$$\frac{(b\csc(e+fx))^m\,{}_2F_1\left(1,\frac{m}{2};\frac{m+2}{2};\csc^2(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[1, m/2, (2 + m)/2, Csc[e + f*x]^2])/(
f*m)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int (b\csc(e+fx))^m \tan(e+fx)\,dx &= -\frac{b\text{Subst}\left(\int \frac{(bx)^{-1+m}}{-1+x^2}\,dx, x, \csc(e+fx)\right)}{f}\\ &= \frac{(b\csc(e+fx))^m\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};\csc^2(e+fx)\right)}{fm}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 52, normalized size = 1.33

$$-\frac{(b\csc(e+fx))^m\,{}_2F_1\left(1,1-\frac{m}{2};2-\frac{m}{2};\sin^2(e+fx)\right)\sin^2(e+fx)}{f(-2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] -(((b*Csc[e + f*x])^m*Hypergeometric2F1[1, 1 - m/2, 2 - m/2, Sin[e + f*x]^2
]*Sin[e + f*x]^2)/(f*(-2 + m)))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^m \tan(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e),x)
```

```
[Out] int((b*csc(f*x+e))^m*tan(f*x+e),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^m \tan(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e),x)
```

```
[Out] Integral((b*csc(e + f*x))**m*tan(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \tan(e+f\,x)\,{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(b/sin(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)*(b/sin(e + f*x))^m, x)
```

3.376 $\int \cot(e+fx)(b\csc(e+fx))^m\,dx$

Optimal. Leaf size=18

$$-\frac{(b\csc(e+fx))^m}{fm}$$

```
[Out] -(b*csc(f*x+e))^m/f/m
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2686, 32}

$$-\frac{(b\csc(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(b*Csc[e + f*x])^m,x]
```

```
[Out] -((b*Csc[e + f*x])^m/(f*m))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(e+fx)(b\csc(e+fx))^m\,dx &= -\frac{b\text{Subst}\left(\int (bx)^{-1+m}\,dx, x, \csc(e+fx)\right)}{f}\\ &= -\frac{(b\csc(e+fx))^m}{fm}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.00

$$-\frac{(b\csc(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(b*Csc[e + f*x])^m,x]
```

```
[Out] -((b*Csc[e + f*x])^m/(f*m))
```

Maple [A]
time = 0.14, size = 19, normalized size = 1.06

method	result
derivativedivides	$-\frac{(b\csc(fx+e))^m}{fm}$
default	$-\frac{(b\csc(fx+e))^m}{fm}$
risch	$-\mathrm{e}^{\frac{m\left(-i\pi\,\mathrm{csgn}\left(\frac{ib\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}-1}\right)\mathrm{csgn}\left(\frac{b\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}-1}\right)-i\pi\mathrm{csgn}\left(\frac{i\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}-1}\right)^3+i\pi\,\mathrm{csgn}\left(\frac{ib\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}-1}\right)\mathrm{csgn}\left(\frac{b\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}-1}\right)^2+}{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*csc(f*x+e))^m/f/m
```

Maxima [A]
time = 0.29, size = 22, normalized size = 1.22

$$-\frac{b^m\sin\left(fx+e\right)^{-m}}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] -b^m*sin(f*x + e)^(-m)/(f*m)
```

Fricas [A]
time = 0.37, size = 21, normalized size = 1.17

$$-\frac{\left(\frac{b}{\sin(fx+e)}\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] -(b/sin(f*x + e))^m/(f*m)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(14) = 28$.

time = 0.14, size = 54, normalized size = 3.00

$$\begin{cases} x\cot(e) & \text{for } f = 0 \wedge m = 0 \\ x(b\csc(e))^m \cot(e) & \text{for } f = 0 \\ -\frac{\log(\tan^2(e+fx)+1)}{2f} + \frac{\log(\tan(e+fx))}{f} & \text{for } m = 0 \\ -\frac{(b\csc(e+fx))^m}{fm} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*csc(f*x+e))**m,x)
```

```
[Out] Piecewise((x*cot(e), Eq(f, 0) & Eq(m, 0)), (x*(b*csc(e))**m*cot(e), Eq(f, 0
)), (-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f*x))/f, Eq(m, 0)), (-(b
*csc(e + f*x))**m/(f*m), True))
```

Giac [A]
time = 0.47, size = 21, normalized size = 1.17

$$-\frac{\left(\frac{b}{\sin(fx+e)}\right)^m}{fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] -(b/sin(f*x + e))^m/(f*m)
```

Mupad [B]
time = 2.84, size = 43, normalized size = 2.39

$$\begin{cases} -\frac{\ln\left(\frac{b}{\sin(e+f\,x)}\right)}{f} & \text{if } m = 0 \\ -\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m}{f\,m} & \text{if } m \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(b/sin(e + f*x))^m,x)
```

```
[Out] piecewise(m == 0, -log(b/sin(e + f*x))/f, m ~= 0, -(b/sin(e + f*x))^m/(f*m)
)
```

3.377 $\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx$

Optimal. Leaf size=43

$$\frac{(b\csc(e+fx))^m}{fm}-\frac{(b\csc(e+fx))^{2+m}}{b^2f(2+m)}$$

```
[Out] (b*csc(f*x+e))^m/f/m-(b*csc(f*x+e))^(2+m)/b^2/f/(2+m)
```

Rubi [A]

time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2686, 14}

$$\frac{(b\csc(e+fx))^m}{fm}-\frac{(b\csc(e+fx))^{m+2}}{b^2f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(b*Csc[e + f*x])^m,x]
```

```
[Out] (b*Csc[e + f*x])^m/(f*m) - (b*Csc[e + f*x])^(2 + m)/(b^2*f*(2 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot^3(e+fx)(b\csc(e+fx))^m\,dx &= -\frac{b\text{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)\,dx,x,\csc(e+fx)\right)}{f}\\ &= -\frac{b\text{Subst}\left(\int \left(-(bx)^{-1+m}+\frac{(bx)^{1+m}}{b^2}\right)\,dx,x,\csc(e+fx)\right)}{f}\\ &= \frac{(b\csc(e+fx))^m}{fm}-\frac{(b\csc(e+fx))^{2+m}}{b^2f(2+m)}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 36, normalized size = 0.84

$$\frac{(b\csc(e+fx))^m\left(2+m-m\csc^2(e+fx)\right)}{fm(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(b*Csc[e + f*x])^m,x]
```

```
[Out] ((b*Csc[e + f*x])^m*(2 + m - m*Csc[e + f*x]^2))/(f*m*(2 + m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.47, size = 3826, normalized size = 88.98

method	result	size
risch	Expression too large to display	3826

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2+m)/f/(exp(2*I*(f*x+e))-1)^2/m*(m/((exp(2*I*(f*x+e))-1)^m)*exp(I*(Re(f*
x)+Re(e)))^m*b^m*2^m*exp(-m*Im(f*x)-m*Im(e))*exp(1/2*I*Pi*csgn(I*exp(I*(f*x
+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I/(exp(2*I*(f*x+e))-1))*m)*exp(1/2*I*Pi*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*ex
p(I*(f*x+e)))^2*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)
))^2*csgn(I*b)*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e))
)*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(I*exp
(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x
+e))-1)*exp(I*(f*x+e)))*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*m)*exp(
-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e))
)*csgn(I/(exp(2*I*(f*x+e))-1))*m)*exp(1/2*I*Pi*csgn(b/(exp(2*I*(f*x+e))-1)*
exp(I*(f*x+e)))^3*m)*exp(-1/2*I*Pi*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e
)))^2*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I*
exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp
(2*I*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b)*m
)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*m)*exp(4*I*
f*x)*exp(4*I*e)+2/((exp(2*I*(f*x+e))-1)^m)*exp(I*(Re(f*x)+Re(e)))^m*b^m*2^m
*exp(-m*Im(f*x)-m*Im(e))*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e
))-1))^2*csgn(I/(exp(2*I*(f*x+e))-1))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))
/(exp(2*I*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*m)*e
xp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*b)*m)*ex
p(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(b/(exp(2*I*(f
*x+e))-1)*exp(I*(f*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I
*(f*x+e))-1))^3*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e
)))*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*m)*exp(-1/2*I*Pi*csgn(I*exp
```

```
(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(f
*x+e))-1))*m)*exp(1/2*I*Pi*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*m)
*exp(-1/2*I*Pi*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*m)*exp(1/2*I*P
i*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*(f*x+e)))*m)*e
xp(1/2*I*Pi*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*cs
gn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b)*m)*exp(-1/2*I*Pi*csgn
(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*m)*exp(4*I*f*x)*exp(4*I*e)+2*m/
((exp(2*I*(f*x+e))-1)^m)*exp(I*(Re(f*x)+Re(e)))^m*b^m*2^m*exp(-m*Im(f*x)-m*
Im(e))*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I/(e
xp(2*I*(f*x+e))-1))*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))
-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*m)*exp(1/2*I*Pi*csgn(I
*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*csgn(I*b)*m)*exp(1/2*I*Pi*csgn(I*
b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f
*x+e)))^2*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3*m)
*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(b/(exp(2*
I*(f*x+e))-1)*exp(I*(f*x+e)))*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2
*I*(f*x+e))-1))*csgn(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))-1))*m)*exp(
1/2*I*Pi*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*m)*exp(-1/2*I*Pi*csg
n(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*
x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*m)*exp
(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f
*x+e))-1)*exp(I*(f*x+e)))*csgn(I*b)*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x
+e))-1)*exp(I*(f*x+e)))^3*m)*exp(2*I*f*x)*exp(2*I*e)-4/((exp(2*I*(f*x+e))-1
)^m)*exp(I*(Re(f*x)+Re(e)))^m*b^m*2^m*exp(-m*Im(f*x)-m*Im(e))*exp(1/2*I*Pi*
csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^2*csgn(I/(exp(2*I*(f*x+e))-1))*
m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*b/(exp(2
*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))
-1)*exp(I*(f*x+e)))^2*csgn(I*b)*m)*exp(1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-
1)*exp(I*(f*x+e)))*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^2*m)*exp(-1/
2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))^3*m)*exp(-1/2*I*Pi*csgn(
I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*
(f*x+e)))*m)*exp(-1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn
(I*exp(I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))-1))*m)*exp(1/2*I*Pi*csgn(b/(exp
(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3*m)*exp(-1/2*I*Pi*csgn(b/(exp(2*I*(f*x+e)
)-1)*exp(I*(f*x+e)))^2*m)*exp(1/2*I*Pi*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+
e))-1))^2*csgn(I*exp(I*(f*x+e)))*m)*exp(1/2*I*Pi*m)*exp(-1/2*I*Pi*csgn(I*ex
p(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x
+e)))*csgn(I*b)*m)*exp(-1/2*I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e
)))^3*m)*exp(2*I*f*x)*exp(2*I*e)+m/((exp(2*I*(f*x+e))-1)^m)*exp(I*(Re(f*x)+
Re(e)))^m*b^m*2^m*exp(-1/2*m*(-I*Pi*csgn(b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+
e)))^3+I*Pi*csgn(I*b/(exp(2*I*(f*x+e))-1)*exp(I*(f*x+e)))^3-I*Pi*csgn(I*exp
(I*(f*x+e))/(exp(2*I*(f*x+e))-1))*csgn(I*b/(exp...
```

Maxima [A]

time = 0.29, size = 53, normalized size = 1.23

$$\frac{\frac{b^m \sin(fx+e)^{-m}}{m} - \frac{b^m \sin(fx+e)^{-m}}{(m+2)\sin(fx+e)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] (b^m*sin(f*x + e)^(-m)/m - b^m*sin(f*x + e)^(-m)/((m + 2)*sin(f*x + e)^2))/
f
```

Fricas [A]
time = 0.37, size = 63, normalized size = 1.47

$$-\frac{\left((m+2)\cos\left(fx+e\right)^2-2\right)\left(\frac{b}{\sin(fx+e)}\right)^m}{fm^2-\left(fm^2+2\,fm\right)\cos\left(fx+e\right)^2+2\,fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] -((m + 2)*cos(f*x + e)^2 - 2)*(b/sin(f*x + e))^m/(f*m^2 - (f*m^2 + 2*f*m)*c
os(f*x + e)^2 + 2*f*m)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} x(b\csc(e))^m \cot^3(e) & \text{for}\ f=0 \\ \frac{\int \frac{\cot^3(e+fx)}{\csc^2(e+fx)}\,dx}{b^2} & \text{for}\ m=-2 \\ \frac{\log(\tan^2(e+fx)+1)}{2f} - \frac{\log(\tan(e+fx))}{f} - \frac{1}{2f\tan^2(e+fx)} & \text{for}\ m=0 \\ -\frac{m(b\csc(e+fx))^m\cot^2(e+fx)}{fm^2+2fm} + \frac{2(b\csc(e+fx))^m}{fm^2+2fm} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(b*csc(f*x+e))**m,x)
```

```
[Out] Piecewise((x*(b*csc(e))**m*cot(e)**3, Eq(f, 0)), (Integral(cot(e + f*x)**3/
csc(e + f*x)**2, x)/b**2, Eq(m, -2)), (log(tan(e + f*x)**2 + 1)/(2*f) - log
(tan(e + f*x))/f - 1/(2*f*tan(e + f*x)**2), Eq(m, 0)), (-m*(b*csc(e + f*x))
**m*cot(e + f*x)**2/(f*m**2 + 2*f*m) + 2*(b*csc(e + f*x))**m/(f*m**2 + 2*f*
m), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^3, x)
```

Mupad [B]
time = 3.44, size = 92, normalized size = 2.14

$$\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m\,\left(m+4\sin\left(2\,e+2\,f\,x\right)^2+m\,\left(2\sin\left(2\,e+2\,f\,x\right)^2-1\right)-16\sin\left(e+f\,x\right)^2\right)}{f\,m\,\left(2\sin\left(2\,e+2\,f\,x\right)^2-8\sin\left(e+f\,x\right)^2\right)\,\left(m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(b/sin(e + f*x))^m,x)
```

```
[Out] ((b/sin(e + f*x))^m*(m + 4*sin(2*e + 2*f*x)^2 + m*(2*sin(2*e + 2*f*x)^2 - 1
) - 16*sin(e + f*x)^2))/(f*m*(2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2)*(m +
 2))
```

3.378 $\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx$

Optimal. Leaf size=69

$$-\frac{(b\csc(e+fx))^m}{fm}+\frac{2(b\csc(e+fx))^{2+m}}{b^2f(2+m)}-\frac{(b\csc(e+fx))^{4+m}}{b^4f(4+m)}$$

```
[Out] -(b*csc(f*x+e))^m/f/m+2*(b*csc(f*x+e))^(2+m)/b^2/f/(2+m)-(b*csc(f*x+e))^(4+
m)/b^4/f/(4+m)
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2686, 276}

$$-\frac{(b\csc(e+fx))^{m+4}}{b^4f(m+4)}+\frac{2(b\csc(e+fx))^{m+2}}{b^2f(m+2)}-\frac{(b\csc(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(b*Csc[e + f*x])^m,x]
```

```
[Out] -((b*Csc[e + f*x])^m/(f*m)) + (2*(b*Csc[e + f*x])^(2 + m))/(b^2*f*(2 + m))
- (b*Csc[e + f*x])^(4 + m)/(b^4*f*(4 + m))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\int \cot^5(e+fx)(b\csc(e+fx))^m\,dx &= -\frac{b\text{Subst}\left(\int (bx)^{-1+m}\left(-1+x^2\right)^2\,dx,x,\csc(e+fx)\right)}{f}\\
&= -\frac{b\text{Subst}\left(\int \left((bx)^{-1+m}-\frac{2(bx)^{1+m}}{b^2}+\frac{(bx)^{3+m}}{b^4}\right)\,dx,x,\csc(e+fx)\right)}{f}\\
&= -\frac{(b\csc(e+fx))^m}{fm}+\frac{2(b\csc(e+fx))^{2+m}}{b^2f(2+m)}-\frac{(b\csc(e+fx))^{4+m}}{b^4f(4+m)}
\end{aligned}$$

Mathematica [A]
time = 0.32, size = 63, normalized size = 0.91

$$-\frac{(b\csc(e+fx))^m\,(8+6m+m^2-2m(4+m)\csc^2(e+fx)+m(2+m)\csc^4(e+fx))}{fm(2+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(b*Csc[e + f*x])^m,x]
```

```
[Out] -(((b*Csc[e + f*x])^m*(8 + 6*m + m^2 - 2*m*(4 + m)*Csc[e + f*x]^2 + m*(2 +
m)*Csc[e + f*x]^4))/(f*m*(2 + m)*(4 + m)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.36, size = 9618, normalized size = 139.39

method	result	size
risch	Expression too large to display	9618

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(b*csc(f*x+e))^m,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [A]
time = 0.28, size = 83, normalized size = 1.20

$$-\frac{\frac{b^m\sin(fx+e)^{-m}}{m}-\frac{2\,b^m\sin(fx+e)^{-m}}{(m+2)\sin(fx+e)^2}+\frac{b^m\sin(fx+e)^{-m}}{(m+4)\sin(fx+e)^4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] -(b^m*sin(f*x + e)^(-m)/m - 2*b^m*sin(f*x + e)^(-m)/((m + 2)*sin(f*x + e)^2
) + b^m*sin(f*x + e)^(-m)/((m + 4)*sin(f*x + e)^4))/f
```

Fricas [A]
time = 0.39, size = 120, normalized size = 1.74

$$-\frac{\left((m^2+6\,m+8)\cos\left(fx+e\right)^4-4\,(m+4)\cos\left(fx+e\right)^2+8\right)\left(\frac{b}{\sin(fx+e)}\right)^m}{(fm^3+6\,fm^2+8\,fm)\cos\left(fx+e\right)^4+fm^3+6\,fm^2-2\,(fm^3+6\,fm^2+8\,fm)\cos\left(fx+e\right)^2+8\,fm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] -((m^2 + 6*m + 8)*cos(f*x + e)^4 - 4*(m + 4)*cos(f*x + e)^2 + 8)*(b/sin(f*x
 + e))^m/((f*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^4 + f*m^3 + 6*f*m^2 - 2*(f
*m^3 + 6*f*m^2 + 8*f*m)*cos(f*x + e)^2 + 8*f*m)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} x\left(b\csc\left(e\right)\right)^{m}\cot^{5}\left(e\right) & \text{for}\: f = 0 \\ \frac{\int \frac{\cot^{5}\left(e+fx\right)}{\csc^{4}\left(e+fx\right)}\, dx}{b^{4}} & \text{for}\: m = -4 \\ \frac{\int \frac{\cot^{5}\left(e+fx\right)}{\csc^{2}\left(e+fx\right)}\, dx}{b^{2}} & \text{for}\: m = -2 \\ -\frac{\log\left(\tan^{2}\left(e+fx\right)+1\right)}{2f} + \frac{\log\left(\tan\left(e+fx\right)\right)}{f} + \frac{1}{2f\tan^{2}\left(e+fx\right)} - \frac{1}{4f\tan^{4}\left(e+fx\right)} & \text{for}\: m = 0 \\ -\frac{m^{2}\left(b\csc\left(e+fx\right)\right)^{m}\cot^{4}\left(e+fx\right)}{fm^{3}+6fm^{2}+8fm} - \frac{2m\left(b\csc\left(e+fx\right)\right)^{m}\cot^{4}\left(e+fx\right)}{fm^{3}+6fm^{2}+8fm} + \frac{4m\left(b\csc\left(e+fx\right)\right)^{m}\cot^{2}\left(e+fx\right)}{fm^{3}+6fm^{2}+8fm} - \frac{8\left(b\csc\left(e+fx\right)\right)^{m}}{fm^{3}+6fm^{2}+8fm} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(b*csc(f*x+e))**m,x)
```

```
[Out] Piecewise((x*(b*csc(e))**m*cot(e)**5, Eq(f, 0)), (Integral(cot(e + f*x)**5/
csc(e + f*x)**4, x)/b**4, Eq(m, -4)), (Integral(cot(e + f*x)**5/csc(e + f*x
)**2, x)/b**2, Eq(m, -2)), (-log(tan(e + f*x)**2 + 1)/(2*f) + log(tan(e + f
*x))/f + 1/(2*f*tan(e + f*x)**2) - 1/(4*f*tan(e + f*x)**4), Eq(m, 0)), (-m*
*2*(b*csc(e + f*x))**m*cot(e + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) - 2*m*(b
*csc(e + f*x))**m*cot(e + f*x)**4/(f*m**3 + 6*f*m**2 + 8*f*m) + 4*m*(b*csc(
e + f*x))**m*cot(e + f*x)**2/(f*m**3 + 6*f*m**2 + 8*f*m) - 8*(b*csc(e + f*x
))**m/(f*m**3 + 6*f*m**2 + 8*f*m), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^5, x)
```

Mupad [B]
time = 7.63, size = 222, normalized size = 3.22

$$-\frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m\,\left(2\,\sin\left(2\,e+2\,f\,x\right)^2+\sin\left(4\,e+4\,f\,x\right)\,1{}\mathrm{i}-1\right)\,\left(\frac{2\,\left(2\,\sin(2\,e+2\,f\,x)^2-1\right)\,\left(-2\,\sin(2\,e+2\,f\,x)^2+\sin(4\,e+4\,f\,x)\,1{}\mathrm{i}+1\right)}{f\,m}-\frac{\left(-2\,\sin(2\,e+2\,f\,x)^2+\sin(4\,e+4\,f\,x)\,1{}\mathrm{i}+1\right)\,\left(6\,m^2+4\,m+48\right)}{f\,m\,\left(m^2+6\,m+8\right)}+\frac{2\,\left(2\,\sin(e+f\,x)^2-1\right)\,\left(-2\,\sin(2\,e+2\,f\,x)^2+\sin(4\,e+4\,f\,x)\,1{}\mathrm{i}+1\right)\,\left(4\,m^2+8\,m-32\right)}{f\,m\,\left(m^2+6\,m+8\right)}\right)}{16\,\sin\left(e+f\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(b/sin(e + f*x))^m,x)
```

```
[Out] -((b/sin(e + f*x))^m*(sin(4*e + 4*f*x)*1i + 2*sin(2*e + 2*f*x)^2 - 1)*((2*(
2*sin(2*e + 2*f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1))
/(f*m) - ((sin(4*e + 4*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*(4*m + 6*m^2 + 4
8))/(f*m*(6*m + m^2 + 8)) + (2*(2*sin(e + f*x)^2 - 1)*(sin(4*e + 4*f*x)*1i
- 2*sin(2*e + 2*f*x)^2 + 1)*(8*m + 4*m^2 - 32))/(f*m*(6*m + m^2 + 8))))/(16
*sin(e + f*x)^4)
```

3.379 $\int (b\csc(e+fx))^m \tan^4(e+fx)\,dx$

Optimal. Leaf size=63

$$\frac{(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);-\frac{1}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-3+m)}\tan^3(e+fx)}{3f}$$

```
[Out] 1/3*(b*csc(f*x+e))^m*hypergeom([-3/2, -3/2+1/2*m],[-1/2],cos(f*x+e)^2)*(sin
(f*x+e)^2)^(-3/2+1/2*m)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$\frac{\tan^3(e+fx)\sin^2(e+fx)^{\frac{m-3}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};-\frac{1}{2};\cos^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[-3/2, (-3 + m)/2, -1/2, Cos[e + f*x]^
2]*(Sin[e + f*x]^2)^((-3 + m)/2)*Tan[e + f*x]^3)/(3*f)
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\csc(e+fx))^m\tan^4(e+fx)\,dx = \frac{(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);-\frac{1}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}}}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 212 vs. 2(63) = 126.
time = 1.50, size = 212, normalized size = 3.37

$$-\frac{(b\csc(e+fx))^m\sec^2(e+fx)^{-m/2}\left({}_2F_1\left(\frac{1-m}{2},-1-\frac{m}{2};\frac{3-m}{2};-\tan^2(e+fx)\right)\sqrt{\sin^2(e+fx)}-2\,{}_2F_1\left(\frac{1-m}{2},-\frac{m}{2};\frac{3-m}{2};-\tan^2(e+fx)\right)\sqrt{\sin^2(e+fx)}+(-1+m)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3}{2};\cos^2(e+fx)\right)\sec^2(e+fx)^{m/2}\sin^2(e+fx)^{m/2}\right)\tan(e+fx)}{f(-1+m)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] -(((b*Csc[e + f*x])^m*(Hypergeometric2F1[(1 - m)/2, -1 - m/2, (3 - m)/2, -T
an[e + f*x]^2]*Sqrt[Sin[e + f*x]^2] - 2*Hypergeometric2F1[(1 - m)/2, -1/2*m
, (3 - m)/2, -Tan[e + f*x]^2]*Sqrt[Sin[e + f*x]^2] + (-1 + m)*Cos[e + f*x]^
2*Hypergeometric2F1[1/2, (1 + m)/2, 3/2, Cos[e + f*x]^2]*(Sec[e + f*x]^2)^(
m/2)*(Sin[e + f*x]^2)^(m/2))*Tan[e + f*x])/(f*(-1 + m)*(Sec[e + f*x]^2)^(m/
2)*Sqrt[Sin[e + f*x]^2]))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(b \csc \left(f x +e \right)\right)^{m} \left(\tan^{4}\left(f x +e \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e)^4,x)
```

```
[Out] int((b*csc(f*x+e))^m*tan(f*x+e)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \csc{\left(e + f x \right)}\right)^{m} \tan^{4}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e)**4,x)
```

```
[Out] Integral((b*csc(e + f*x))**m*tan(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^4\,{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(b/sin(e + f*x))^m,x)
```

```
[Out] int(tan(e + f*x)^4*(b/sin(e + f*x))^m, x)
```

3.380 $\int (b\csc(e+fx))^m \tan^2(e+fx)\,dx$

Optimal. Leaf size=58

$$\frac{(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(-1+m)}\tan(e+fx)}{f}$$

```
[Out] (b*csc(f*x+e))^m*hypergeom([-1/2, -1/2+1/2*m],[1/2],cos(f*x+e)^2)*(sin(f*x+
e)^2)^(-1/2+1/2*m)*tan(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$\frac{\tan(e+fx)\sin^2(e+fx)^{\frac{m-1}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{1}{2};\cos^2(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[-1/2, (-1 + m)/2, 1/2, Cos[e + f*x]^2
]*(Sin[e + f*x]^2)^((-1 + m)/2)*Tan[e + f*x])/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int (b\csc(e+fx))^m\tan^2(e+fx)\,dx = \frac{(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{1}{2}(}}{f}$$

Mathematica [A]
time = 0.68, size = 79, normalized size = 1.36

$$\frac{(b\csc(e+fx))^m\,{}_2F_1\left(1-\frac{m}{2},\frac{3}{2}-\frac{m}{2};\frac{5}{2}-\frac{m}{2};-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}\tan^3(e+fx)}{f(3-m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] ((b*Csc[e + f*x])^m*Hypergeometric2F1[1 - m/2, 3/2 - m/2, 5/2 - m/2, -Tan[e
 + f*x]^2]*Tan[e + f*x]^3)/(f*(3 - m)*(Sec[e + f*x]^2)^(m/2))
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(b \csc \left(f x +e \right)\right)^{m} \left(\tan^{2}\left(f x +e \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*tan(f*x+e)^2,x)
```

```
[Out] int((b*csc(f*x+e))^m*tan(f*x+e)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \csc \left(e + f x\right)\right)^{m} \tan^{2}\left(e + f x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*tan(f*x+e)**2,x)
```

```
[Out] Integral((b*csc(e + f*x))**m*tan(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m*tan(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \tan(e+f\,x)^2\,\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(b/sin(e + f*x))^m,x)
[Out] int(tan(e + f*x)^2*(b/sin(e + f*x))^m, x)
```

3.381 $\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx$

Optimal. Leaf size=63

$$-\frac{\cot^3(e+fx)(b\csc(e+fx))^m\,{}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{3+m}{2}}}{3f}$$

```
[Out] -1/3*cot(f*x+e)^3*(b*csc(f*x+e))^m*hypergeom([3/2, 3/2+1/2*m],[5/2],cos(f*x
+e)^2)*(sin(f*x+e)^2)^(3/2+1/2*m)/f
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$-\frac{\cot^3(e+fx)\sin^2(e+fx)^{\frac{m+3}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{3}{2},\frac{m+3}{2};\frac{5}{2};\cos^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(b*Csc[e + f*x])^m,x]
```

```
[Out] -1/3*(Cot[e + f*x]^3*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/2, (3 + m)/2, 5
/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((3 + m)/2))/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
 !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cot^2(e+fx)(b\csc(e+fx))^m\,dx=-\frac{\cot^3(e+fx)(b\csc(e+fx))^m\,{}_2F_1\left(\frac{3}{2},\frac{3+m}{2};\frac{5}{2};\cos^2(e+fx)\right)\sin^2(e+f}{3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(63) = 126.
time = 1.25, size = 186, normalized size = 2.95

$$-\frac{(b\csc(e+fx))^m\left(-4(1+m)\,{}_2F_1\left(1-m,\frac{1}{2}-\frac{m}{2};\frac{3}{2}-\frac{m}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(-1+m)\cot^2\left(\frac{1}{2}(e+fx)\right)\,{}_2F_1\left(-\frac{1}{2}-\frac{m}{2},-m;\frac{1}{2}-\frac{m}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(1+m)\,{}_2F_1\left(\frac{1}{2}-\frac{m}{2},-m;\frac{3}{2}-\frac{m}{2};-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec^2\left(\frac{1}{2}(e+fx)\right)^{-m}\tan\left(\frac{1}{2}(e+fx)\right)}{2f\left(-1+m^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(b*Csc[e + f*x])^m,x]
```

```
[Out] -1/2*((b*Csc[e + f*x])^m*(-4*(1 + m)*Hypergeometric2F1[1 - m, 1/2 - m/2, 3/
2 - m/2, -Tan[(e + f*x)/2]^2] + (-1 + m)*Cot[(e + f*x)/2]^2*Hypergeometric2
F1[-1/2 - m/2, -m, 1/2 - m/2, -Tan[(e + f*x)/2]^2] + (1 + m)*Hypergeometric
2F1[1/2 - m/2, -m, 3/2 - m/2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2])/(f*(-
1 + m^2)*(Sec[(e + f*x)/2]^2)^m)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(\cot^{2}\left(f x +e \right)\right) \left(b \csc \left(f x +e \right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(b*csc(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^2*(b*csc(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \csc{\left(e + f x \right)}\right)^{m} \cot^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(b*csc(f*x+e))**m,x)
```

```
[Out] Integral((b*csc(e + f*x))**m*cot(e + f*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(b*csc(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e+f\,x)^2\,\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(b/sin(e + f*x))^m,x)
```

```
[Out] int(cot(e + f*x)^2*(b/sin(e + f*x))^m, x)
```

3.382 $\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx$

Optimal. Leaf size=63

$$-\frac{\cot^5(e+fx)(b\csc(e+fx))^m\,{}_2F_1\left(\frac{5}{2},\frac{5+m}{2};\frac{7}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{\frac{5+m}{2}}}{5f}$$

```
[Out] -1/5*cot(f*x+e)^5*(b*csc(f*x+e))^m*hypergeom([5/2, 5/2+1/2*m],[7/2],cos(f*x
+e)^2)*(sin(f*x+e)^2)^(5/2+1/2*m)/f
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2697}

$$-\frac{\cot^5(e+fx)\sin^2(e+fx)^{\frac{m+5}{2}}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{5}{2},\frac{m+5}{2};\frac{7}{2};\cos^2(e+fx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(b*Csc[e + f*x])^m,x]
```

```
[Out] -1/5*(Cot[e + f*x]^5*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/2, (5 + m)/2, 7
/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^((5 + m)/2))/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \cot^4(e+fx)(b\csc(e+fx))^m\,dx = -\frac{\cot^5(e+fx)(b\csc(e+fx))^m\,{}_2F_1\left(\frac{5}{2},\frac{5+m}{2};\frac{7}{2};\cos^2(e+fx)\right)\sin^2(e+$$

Mathematica [A]
time = 0.28, size = 106, normalized size = 1.68

$$-\frac{\cot(e+fx)(b\csc(e+fx))^m\left({}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3}{2};\cos^2(e+fx)\right)-2\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{3}{2};\cos^2(e+fx)\right)+{}_2F_1\left(\frac{1}{2},\frac{5+m}{2};\frac{3}{2};\cos^2(e+fx)\right)\right)\sin^2(e+fx)^{\frac{1+m}{2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(b*Csc[e + f*x])^m,x]
```

```
[Out] -((Cot[e + f*x]*(b*Csc[e + f*x])^m*(Hypergeometric2F1[1/2, (1 + m)/2, 3/2,
Cos[e + f*x]^2] - 2*Hypergeometric2F1[1/2, (3 + m)/2, 3/2, Cos[e + f*x]^2]
+ Hypergeometric2F1[1/2, (5 + m)/2, 3/2, Cos[e + f*x]^2])*(Sin[e + f*x]^2)^
((1 + m)/2))/f)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\cot^{4}\left(fx+e\right)\right) \left(b\csc\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(b*csc(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^4*(b*csc(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\csc\left(e+fx\right)\right)^{m}\cot^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(b*csc(f*x+e))**m,x)
```

```
[Out] Integral((b*csc(e + f*x))**m*cot(e + f*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(b*csc(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m*cot(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(e+f\,x)^4\,\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(b/sin(e + f*x))^m,x)
[Out] int(cot(e + f*x)^4*(b/sin(e + f*x))^m, x)
```

3.383 $\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx$

Optimal. Leaf size=79

$$\frac{2\cos^2(e+fx)^{5/4}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(5-2m);\frac{1}{4}(9-2m);\sin^2(e+fx)\right)(d\tan(e+fx))^{5/2}}{df(5-2m)}$$

```
[Out] 2*(cos(f*x+e)^2)^(5/4)*(b*csc(f*x+e))^m*hypergeom([5/4, 5/4-1/2*m],[9/4-1/2
*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(5/2)/d/f/(5-2*m)
```

Rubi [A]
time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2698, 2682, 2657}

$$\frac{2\cos^2(e+fx)^{5/4}(d\tan(e+fx))^{5/2}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(5-2m);\frac{1}{4}(9-2m);\sin^2(e+fx)\right)}{df(5-2m)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(5/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[5/4, (5 - 2*
m)/4, (9 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(5/2))/(d*f*(5 - 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int (b\csc(e+fx))^m (d\tan(e+fx))^{3/2}\,dx = \left((b\csc(e+fx))^m \left(\frac{\sin(e+fx)}{b}\right)^m\right)\int \left(\frac{\sin(e+fx)}{b}\right)^{-m} (d\,\mathrm{ta}$$
$$= \frac{\left(\cos^{\frac{5}{2}}(e+fx)(b\csc(e+fx))^{3+m}\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}(d\tan(e+fx)\right.}{bd}$$
$$= \frac{2\cos^2(e+fx)^{5/4}(b\csc(e+fx))^{3+m}\,{}_2F_1\left(\frac{5}{4},\frac{1}{4}(5-2m);\frac{1}{4}(9-2m\right.}{b^3df(5-2m)}$$

Mathematica [A]
time = 5.92, size = 87, normalized size = 1.10

$$-\frac{2(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4}(5-2m),1-\frac{m}{2};\frac{1}{4}(9-2m);-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}(d\tan(e+fx))^{5/2}}{df(-5+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*(d*Tan[e + f*x])^(3/2),x]
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(5 - 2*m)/4, 1 - m/2, (9 - 2*m)/4,
 -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(5/2))/(d*f*(-5 + 2*m)*(Sec[e + f*x]^2)^
(m/2))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^m (d\tan(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x)
[Out] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*tan(f*x + e))^(3/2)*(b*csc(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m*d*tan(f*x + e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*(d*tan(f*x+e))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*tan(f*x + e))^(3/2)*(b*csc(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\tan(e+f\,x))^{3/2}\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(3/2)*(b/sin(e + f*x))^m,x)
[Out] int((d*tan(e + f*x))^(3/2)*(b/sin(e + f*x))^m, x)
```

3.384 $\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\,dx$

Optimal. Leaf size=79

$$\frac{2\cos^2(e+fx)^{3/4}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}(3-2m);\frac{1}{4}(7-2m);\sin^2(e+fx)\right)(d\tan(e+fx))^{3/2}}{df(3-2m)}$$

```
[Out] 2*(cos(f*x+e)^2)^(3/4)*(b*csc(f*x+e))^m*hypergeom([3/4, 3/4-1/2*m],[7/4-1/2
*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(3/2)/d/f/(3-2*m)
```

Rubi [A]
time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2698, 2682, 2657}

$$\frac{2\cos^2(e+fx)^{3/4}(d\tan(e+fx))^{3/2}(b\csc(e+fx))^m\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}(3-2m);\frac{1}{4}(7-2m);\sin^2(e+fx)\right)}{df(3-2m)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(3/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[3/4, (3 - 2*
m)/4, (7 - 2*m)/4, Sin[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(3 - 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int (b\csc(e+fx))^m\sqrt{d\tan(e+fx)}\,dx = \left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^m\right)\int\left(\frac{\sin(e+fx)}{b}\right)^{-m}\sqrt{d\tan}$$

$$= \frac{\left(\cos^{\frac{3}{2}}(e+fx)(b\csc(e+fx))^{2+m}\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}(d\tan(e+fx))^{3/}\right.}{bd}$$

$$= \frac{2\cos^2(e+fx)^{3/4}(b\csc(e+fx))^{2+m}\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}(3-2m);\frac{1}{4}(7-2m);\mathrm{s}\right.}{b^2df(3-2m)}$$

Mathematica [**A**]
time = 3.36, size = 87, normalized size = 1.10

$$-\frac{2(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4}(3-2m),1-\frac{m}{2};\frac{1}{4}(7-2m);-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}(d\tan(e+fx))^{3/2}}{df(-3+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m*Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(3 - 2*m)/4, 1 - m/2, (7 - 2*m)/4,
 -Tan[e + f*x]^2]*(d*Tan[e + f*x])^(3/2))/(d*f*(-3 + 2*m)*(Sec[e + f*x]^2)^
(m/2))
```

Maple [**F**]
time = 0.30, size = 0, normalized size = 0.00

$$\int (b\csc(fx+e))^m\sqrt{d\tan(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x)
```

```
[Out] int((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^m \sqrt{d\tan(e+fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m*(d*tan(f*x+e))**(1/2),x)
[Out] Integral((b*csc(e + f*x))**m*sqrt(d*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m*(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\tan(e+f\,x)}\,\left(\frac{b}{\sin(e+f\,x)}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^(1/2)*(b/sin(e + f*x))^m,x)
[Out] int((d*tan(e + f*x))^(1/2)*(b/sin(e + f*x))^m, x)
```

3.385 $\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\,(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4},\frac{1}{4}(1-2m);\frac{1}{4}(5-2m);\sin^2(e+fx)\right)\sqrt{d\tan(e+fx)}}{df(1-2m)}$$

```
[Out] 2*(cos(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^m*hypergeom([1/4, 1/4-1/2*m],[5/4-1/2
*m],sin(f*x+e)^2)*(d*tan(f*x+e))^(1/2)/d/f/(1-2*m)
```

Rubi [A]
time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2698, 2682, 2657}

$$\frac{2\sqrt[4]{\cos^2(e+fx)}\,\sqrt{d\tan(e+fx)}\,(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4},\frac{1}{4}(1-2m);\frac{1}{4}(5-2m);\sin^2(e+fx)\right)}{df(1-2m)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m/Sqrt[d*Tan[e + f*x]],x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(1/4)*(b*Csc[e + f*x])^m*Hypergeometric2F1[1/4, (1 - 2*
m)/4, (5 - 2*m)/4, Sin[e + f*x]^2]*Sqrt[d*Tan[e + f*x]])/(d*f*(1 - 2*m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{(b\csc(e+fx))^m}{\sqrt{d\tan(e+fx)}}\,dx = \left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^m\right)\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{\sqrt{d\tan(e+fx)}}\,dx$$
$$= \frac{\left(\sqrt{\cos(e+fx)}\,(b\csc(e+fx))^{1+m}\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}\sqrt{d\tan(e+fx)}\right)\int \sqrt{\cos(e}}{bd}$$
$$= \frac{2\sqrt[4]{\cos^2(e+fx)}\,(b\csc(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4}(1-2m);\frac{1}{4}(5-2m);\sin^2(e+fx)\right)}{bdf(1-2m)}$$

Mathematica [A]
time = 1.37, size = 87, normalized size = 1.10

$$-\frac{2(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4}(1-2m),1-\frac{m}{2};\frac{1}{4}(5-2m);-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}\sqrt{d\tan(e+fx)}}{df(-1+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m/Sqrt[d*Tan[e + f*x]],x]
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(1 - 2*m)/4, 1 - m/2, (5 - 2*m)/4,
 -Tan[e + f*x]^2]*Sqrt[d*Tan[e + f*x]])/(d*f*(-1 + 2*m)*(Sec[e + f*x]^2)^(m
/2))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{(b\csc(fx+e))^m}{\sqrt{d\tan(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x)
[Out] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^m/sqrt(d*tan(f*x + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m/(d*tan(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\csc{(e+fx)})^m}{\sqrt{d\tan{(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**m/(d*tan(f*x+e))**(1/2),x)
[Out] Integral((b*csc(e + f*x))**m/sqrt(d*tan(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^m/sqrt(d*tan(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{b}{\sin(e+f\,x)}\right)}^m}{\sqrt{d\,\tan\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(1/2),x)
[Out] int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(1/2), x)
```

3.386 $\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(-1-2m);\frac{1}{4}(3-2m);\sin^2(e+fx)\right)}{df(1+2m)\sqrt[4]{\cos^2(e+fx)}\,\sqrt{d\tan(e+fx)}}$$

```
[Out] -2*(b*csc(f*x+e))^m*hypergeom([-1/4, -1/4-1/2*m],[3/4-1/2*m],sin(f*x+e)^2)/
d/f/(1+2*m)/(cos(f*x+e)^2)^(1/4)/(d*tan(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2698, 2682, 2657}

$$-\frac{2(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(-2m-1);\frac{1}{4}(3-2m);\sin^2(e+fx)\right)}{df(2m+1)\sqrt[4]{\cos^2(e+fx)}\,\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^m/(d*Tan[e + f*x])^(3/2),x]
```

```
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[-1/4, (-1 - 2*m)/4, (3 - 2*m)/4, S
in[e + f*x]^2])/(d*f*(1 + 2*m)*(Cos[e + f*x]^2)^(1/4)*Sqrt[d*Tan[e + f*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{(b\csc(e+fx))^m}{(d\tan(e+fx))^{3/2}}\,dx = \left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^m\right)\int \frac{\left(\frac{\sin(e+fx)}{b}\right)^{-m}}{(d\tan(e+fx))^{3/2}}\,dx$$

$$= \frac{\left((b\csc(e+fx))^m\left(\frac{\sin(e+fx)}{b}\right)^{\frac{1}{2}+m}\right)\int \cos^{\frac{3}{2}}(e+fx)\left(\frac{\sin(e+fx)}{b}\right)^{-\frac{3}{2}-m}dx}{bd\sqrt{\cos(e+fx)}\sqrt{d\tan(e+fx)}}$$

$$= -\frac{2(b\csc(e+fx))^m\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4}(-1-2m);\frac{1}{4}(3-2m);\sin^2(e+fx)\right)}{df(1+2m)\sqrt[4]{\cos^2(e+fx)}\sqrt{d\tan(e+fx)}}$$

Mathematica [A]
time = 3.77, size = 87, normalized size = 1.10

$$-\frac{2(b\csc(e+fx))^m\,{}_2F_1\left(\frac{1}{4}(-1-2m),1-\frac{m}{2};\frac{1}{4}(3-2m);-\tan^2(e+fx)\right)\sec^2(e+fx)^{-m/2}}{df(1+2m)\sqrt{d\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^m/(d*Tan[e + f*x])^(3/2),x]
[Out] (-2*(b*Csc[e + f*x])^m*Hypergeometric2F1[(-1 - 2*m)/4, 1 - m/2, (3 - 2*m)/4
, -Tan[e + f*x]^2])/(d*f*(1 + 2*m)*(Sec[e + f*x]^2)^(m/2)*Sqrt[d*Tan[e + f*
x]])
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(b\csc(fx+e))^m}{(d\tan(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x)
[Out] int((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="maxima")
```

[Out] integrate((b*csc(f*x + e))^m/(d*tan(f*x + e))^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(d*tan(f*x + e))*(b*csc(f*x + e))^m/(d^2*tan(f*x + e)^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \csc(e+fx))^m}{(d \tan(e+fx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*csc(f*x+e))**m/(d*tan(f*x+e))**(3/2),x)

[Out] Integral((b*csc(e + f*x))**m/(d*tan(e + f*x))**(3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*csc(f*x+e))^m/(d*tan(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((b*csc(f*x + e))^m/(d*tan(f*x + e))^(3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\sin(e+f\,x)}\right)^m}{(d \tan(e+f\,x))^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(3/2),x)

[Out] int((b/sin(e + f*x))^m/(d*tan(e + f*x))^(3/2), x)

3.387 $\int (a \csc(e+fx))^m (b \tan(e+fx))^n \, dx$

Optimal. Leaf size=89

$$\frac{\cos^2(e+fx)^{\frac{1+n}{2}} (a \csc(e+fx))^m \, {}_2F_1\left(\frac{1+n}{2}, \frac{1}{2}(1-m+n); \frac{1}{2}(3-m+n); \sin^2(e+fx)\right) (b \tan(e+fx))^{1+n}}{bf(1-m+n)}$$

```
[Out] (cos(f*x+e)^2)^(1/2+1/2*n)*(a*csc(f*x+e))^m*hypergeom([1/2+1/2*n, 1/2-1/2*m
+1/2*n],[3/2-1/2*m+1/2*n],sin(f*x+e)^2)*(b*tan(f*x+e))^(1+n)/b/f/(1-m+n)
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2698, 2682, 2657}

$$\frac{\cos^2(e+fx)^{\frac{n+1}{2}} (a \csc(e+fx))^m (b \tan(e+fx))^{n+1} \, {}_2F_1\left(\frac{n+1}{2}, \frac{1}{2}(-m+n+1); \frac{1}{2}(-m+n+3); \sin^2(e+fx)\right)}{bf(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Csc[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] ((Cos[e + f*x]^2)^((1 + n)/2)*(a*Csc[e + f*x])^m*Hypergeometric2F1[(1 + n)/
2, (1 - m + n)/2, (3 - m + n)/2, Sin[e + f*x]^2]*(b*Tan[e + f*x])^(1 + n))/
(b*f*(1 - m + n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2682

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[a*Cos[e + f*x]^(n + 1)*((b*Tan[e + f*x])^(n + 1)/(b*
(a*Sin[e + f*x])^(n + 1))), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] && !IntegerQ[n]
```

Rule 2698

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Dist[(a*Csc[e + f*x])^FracPart[m]*(Sin[e + f*x]/a)^FracPar
t[m], Int[(b*Tan[e + f*x])^n/(Sin[e + f*x]/a)^m, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int (a\csc(e+fx))^m (b\tan(e+fx))^n\,dx = \left((a\csc(e+fx))^m \left(\frac{\sin(e+fx)}{a}\right)^m\right)\int \left(\frac{\sin(e+fx)}{a}\right)^{-m} (b\tan$$

$$= \frac{\left(\cos^{1+n}(e+fx)(a\csc(e+fx))^{1+m}\left(\frac{\sin(e+fx)}{a}\right)^{m-n}(b\tan(e+fx)\right.}{ab}$$

$$= \frac{\cos^2(e+fx)^{\frac{1+n}{2}}(a\csc(e+fx))^{1+m}\,{}_2F_1\left(\frac{1+n}{2},\frac{1}{2}(1-m+n);\frac{1}{2}(3-\right.}{abf(1-m+r}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 2.12, size = 287, normalized size = 3.22

$$-\frac{a(-3+m-n)F_1\left(\frac{1}{2}(1-m+n);n,1-m;\frac{1}{2}(3-m+n);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(a\csc(e+fx))^{-1+m}(b\tan(e+fx))^n}{f(-1+m-n)\left((-3+m-n)F_1\left(\frac{1}{2}(1-m+n);n,1-m;\frac{1}{2}(3-m+n);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left((-1+m)F_1\left(\frac{1}{2}(3-m+n);n,2-m;\frac{1}{2}(5-m+n);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+nF_1\left(\frac{1}{2}(3-m+n);1+n,1-m;\frac{1}{2}(5-m+n);\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Csc[e + f*x])^m*(b*Tan[e + f*x])^n,x]
```

```
[Out] -((a*(-3 + m - n)*AppellF1[(1 - m + n)/2, n, 1 - m, (3 - m + n)/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(a*Csc[e + f*x])^(-1 + m)*(b*Tan[e + f*x])
^n)/(f*(-1 + m - n)*((-3 + m - n)*AppellF1[(1 - m + n)/2, n, 1 - m, (3 - m
+ n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*((-1 + m)*AppellF1[(3
- m + n)/2, n, 2 - m, (5 - m + n)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + n*AppellF1[(3 - m + n)/2, 1 + n, 1 - m, (5 - m + n)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int (a\csc(fx+e))^m (b\tan(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

```
[Out] int((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\csc(e+fx))^m (b\tan(e+fx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csc(f*x+e))**m*(b*tan(f*x+e))**n,x)
[Out] Integral((a*csc(e + f*x))**m*(b*tan(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*csc(f*x+e))^m*(b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((a*csc(f*x + e))^m*(b*tan(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\tan(e+f\,x))^n \left(\frac{a}{\sin(e+f\,x)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tan(e + f*x))^n*(a/sin(e + f*x))^m,x)
[Out] int((b*tan(e + f*x))^n*(a/sin(e + f*x))^m, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False



def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```